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Plasma Physics

OSCILLATIONS OF AN UNBOUNDED MAGNETIZED PLASMA

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The perturbations in an unbounded three-component plasma (ions, electrons and neutrals) in the presence of longitudinal magnetic field produce longitudinal and transverse oscillations. The neutral component introduces a new mode of oscillations. The neutral component collision frequencies make the longitudinal and transverse modes of oscillations more damped and increase the damping range of collision frequencies of two component plasma. The magnetic field does not affect the longitudinal mode of oscillations but it shifts the lower collision frequency range to higher value for damping of the transverse modes.

Keywords: Oscillations; Resonance; Plasma Confinement; Universe

INTRODUCTION

EVEN though fully ionized plasma often occurs in the universe, there are several situations where the interaction between an ionized gas and neutral becomes important [Alfven (1954); Piddington (1954*a*, *b*) and Lehnert (1959*a*, *b*), (1970)]. The effect of plasma-neutral-gas interaction is also important in the laboratory plasma (confinement devices), if the plasma density is in the permeable range i.e. $n \leqslant \frac{10^{14}}{3L_c}$, L_c being the characteristic length of the Plasma [Lehnert (1973)]. The dynamics of a three-component plasma consisting of ions, electrons and neutrals has been dealt with by several authors [references in Lehnert (1973)].

The small amplitude disturbances excite oscillations in cold and collisionless plasma. The dispersion characteristics of such a plasma are governed by the bulk motion of the plasma, the finite collision frequencies and the pressure gradient. The astrophysical and the laboratory plasmas are inhomogeneous and hot for which the bulk motion is important. A plasma in thermal non-equilibrium has a finite bulk motion which acts opposite to the direction of the pressure gradient and tends to destroy the pressure gradient and spatial inhomogeneity. In certain cases, the phase term of the wave and hence the force acting on the system becomes independent of time. This happens when $\omega - \bar{k} \cdot \bar{v} = 0$ which is Cherenkov resonance condition.

The collisions between particles also affect the motion of particles. The longitudinal oscillations of a two-component plasma under no magnetic field have been discussed by Khosa and Singh (1974).

In the present paper, we have studied the plasma oscillations of an unbounded magnetized plasma, a three-component system of ions, electrons and neutrals. The effect of bulk motion of the plasma on the various modes of oscillations has also been discussed. It has been found that the magnetice field has no effect on the longitudinal mode of oscillations. The neutral component introduces a new mode of oscillations. The bands of longitudinal and transverse modes of oscillations become wider due to the neutral component collisions. The magnetic field shifts the lower colision frequency-range to higher value for damping of the transverse oscillations.

Basic Equations and the Dispersion Relation

The equations of motions together with Maxwell equations for electrons, ions and neutrals components of a plasma are given by

$$\frac{d\bar{V}_e}{dt} = -\frac{e}{m_e}\left[\bar{E} + \frac{\bar{V}_e \times \bar{B}}{c}\right] - \alpha_{ei}\,[\bar{V}_e - \bar{V}_i] - \alpha_{en}\,[\bar{V}_e - \bar{V}_n] - \frac{\nabla P_e}{N_e m_e}, \quad \ldots(1)$$

$$\frac{d\bar{V}_i}{dt} = \frac{e}{m_i}\left[\bar{E} + \frac{\bar{V}_i \times \bar{B}}{c}\right] - \alpha_{ie}\,[\bar{V}_i - \bar{V}_e] - \alpha_{in}\,[\bar{V}_i - \bar{V}_n] - \frac{\nabla P_i}{N_i m_i}, \quad \ldots(2)$$

$$\frac{d\bar{V}_n}{dt} = \alpha_{ne}\,[\bar{V}_n - \bar{V}_e] - \alpha_{ni}\,[\bar{V}_n - \bar{V}_i] - \frac{\nabla P_n}{N_n m_n}, \quad \ldots(3)$$

$$\frac{\partial N_j}{\partial t} + \nabla \cdot (N_j \bar{V}_j) = 0, \quad \ldots(4)$$

$$\frac{d(P_j \bar{N}_j^{\alpha_j})}{dt} = 0, \quad \ldots(5)$$

for $j = e, i$ and n,

where $\alpha_{ie} = \frac{m_e \alpha_{ei}}{m_i}$; $\alpha_{in} = \frac{m_n \alpha_{ni}}{m_i}$ and $\alpha_{ne} = \frac{m_e \alpha_{en}}{m_n}$,

$$\nabla \times \nabla \times \bar{E} = -\frac{4\pi}{c^2}\frac{\partial \bar{J}}{\partial t} - \frac{1}{c^2}\frac{\partial^2 \bar{E}}{\partial t^2}, \quad \ldots(6)$$

$$\bar{J} = e\,[N_i \bar{V}_i - N_e \bar{V}_e], \quad \ldots(7)$$

$$\nabla \cdot \bar{B} = 0 \quad \ldots(8)$$

$$\nabla \times \bar{E} = -\frac{1}{c}\frac{\partial \bar{B}}{\partial t} \qquad ...(9)$$

The perturbed physical quantities are denoted by

$$\bar{B} = B_e + \bar{b} = [B_e, 0, 0] + \bar{b}$$

$$\bar{V}_j = \bar{U}_e + \bar{v}_j = [u_e, 0, 0] + \bar{v}_j,$$

$$\bar{E} = \bar{E} = [0, 0, 0] + \bar{E},$$

$$N_j = N_0 + n_j,$$

$$P_j = P_e + p_j,$$

$$\bar{J} = [0, 0, 0] + \bar{J}, \qquad ...(10)$$

where equilibrium quantities are denoted by a subscript zero.

The linearized perturbation equations become

$$\frac{d\bar{v}_e}{dt} = -\frac{e}{m_e}\left[\bar{E} + \frac{\bar{v}_e \times \bar{B}_e}{c} + \frac{\bar{u}_e \times \bar{b}}{c}\right] - \alpha_{ei}[\bar{v}_e - \bar{v}_i] - \alpha_{en}[\bar{v}_e - \bar{v}_n] - \frac{\nabla p_e}{N_0 m_e}, \qquad ...(11)$$

$$\frac{d\bar{v}_i}{dt} = \frac{e}{m_i}\left[\bar{E} + \frac{\bar{v}_i \times \bar{B}_e}{c} + \frac{\bar{u}_e \times \bar{b}}{c}\right] - \alpha_{ie}[v_i - v_e] - \alpha_{in}[\bar{v}_i - \bar{v}_n] - \frac{\nabla p_i}{N_0 m_i}, \qquad ...(12)$$

$$\frac{d\bar{v}_n}{dt} = -\alpha_{ne}[\bar{v}_n - \bar{v}_e] - \alpha_{ni}[v_n - v_i] - \frac{\nabla p_n}{N_0 m_n}, \qquad ...(13)$$

$$\nabla \times \nabla \times \bar{E} = -\frac{4\pi}{c^2}\frac{\partial \bar{J}}{\partial t} - \frac{1}{c^2}\frac{\partial^2 \bar{E}}{\partial t^2}, \qquad ...(14)$$

$$\nabla \times \bar{E} = -\frac{1}{c}\frac{\partial \bar{b}}{\partial t}, \qquad ...(15)$$

$$\bar{J} = e[N_0\bar{v}_i + \bar{u}_0 n_i - n_e\bar{v}_e - \bar{u}_e n_e], \qquad ...(16)$$

Assuming that the perturbations in physical quantities vary as exp $[i(nt - kx)]$, equations (4) and (5) give

$$n_j = \frac{N_e k\bar{v}_j}{(\omega - ku_0)}, \qquad ...(17)$$

$$\frac{\nabla p_j}{m_j N_0} = \frac{-ik^2 u_j^2 \bar{v}_j}{(\omega - ku_0)}, \qquad \ldots(18)$$

where $$u_j^2 = \frac{\alpha_j k T_j}{m_j}. \qquad \ldots(19)$$

Equations of motion (11), (12), (13) combined with equations (15) and (18) reduce to

$$\left[i(\omega - ku_0) - \frac{ik^2 u_0^2}{(\omega - ku_0)} + \alpha_{ei} + \alpha_{en}\right]\bar{v}_e$$

$$= \frac{-e\bar{E}}{m_e} - \frac{ie\bar{u}_0 \times (\nabla \times \bar{E})}{m_e \omega} - \frac{\bar{v}_e \times e\bar{B}_0}{m_e c} + \alpha_{ei}\bar{v}_i + \alpha_{en}\bar{v}_n, \qquad \ldots(20)$$

$$\left[i(\omega - ku_0) - \frac{ik^2 u_i^2}{(\omega - ku_0)} + \alpha_{ie} + \alpha_{in}\right]\bar{v}_i$$

$$= \frac{e\bar{E}}{m_i} + \frac{ie\bar{u}_0 \times (\nabla \times \bar{E})}{m_i \omega} + \frac{\bar{v}_i \times e\bar{B}_0}{m_i c} + \alpha_{ie}\bar{v}_e + \alpha_{in}\bar{v}_n, \qquad \ldots(21)$$

$$\left[i(\omega - ku_0) - \frac{ik^2 u_n^2}{(\omega - ku_0)} + \alpha_{ne} + \alpha_{ni}\right]\bar{v}_n = \alpha_{ne}\bar{v}_e + \alpha_{ni}\bar{v}, \qquad \ldots(22)$$

and equation (14) with (16) and (17) gives

$$\nabla \times (\nabla \times \bar{E}) - \left(\frac{\omega^2}{c^2}\right)\bar{E} = -\frac{4\pi N_0 e}{c^2}\frac{i\omega^2}{(\omega - ku_0)}[\bar{v}_i - \bar{v}_e]. \qquad \ldots(23)$$

Solving equations (20) and (21) with equations (22) and (23), we get

$$\left[A - \frac{\omega_{pe}^2}{(\omega - ku_0)} + \frac{\alpha_{en}\alpha_{ne}}{D}\right] v_{ex} + \left[\frac{\omega_{pe}^2}{(\omega - ku_0)} + \frac{\alpha_{en}\alpha_{ni}}{D} + i\alpha_{ei}\right] \times v_{ix} = 0, \qquad \ldots(24)$$

$$\left[A - \frac{\omega_{pe}^2\,\omega}{(\omega^2 - k^2c^2)} + \frac{\alpha_{en}\alpha_{en}}{D}\right] v_{ey} + \left[\frac{\omega_{pe}^2}{(\omega^2 - k^2c^2)} + \frac{\alpha_{en}\alpha_{ni}}{D} + i\,\alpha_{ie}\right] v_{iy} + i\Omega_e v_{ez} = 0, \qquad \ldots(25)$$

$$\left[A - \frac{\omega_{pe}^2\,\omega}{(\omega^2 - k^2c^2)} + \frac{\alpha_{en}\alpha_{ne}}{D}\right] v_{ez} + \left[\frac{\omega_{pe}^2\,\omega}{(\omega^2 - k_0^2c^2)} + \frac{\alpha_{en}\alpha_{ni}}{D} + i\alpha_{ei}\right] \times v_{iz} - i\Omega_e v_{ey} = 0 \qquad \ldots(26)$$

$$\left[B-\frac{\omega_{pi}^2}{(\omega-ku_0)}+\frac{\alpha_{in}\alpha_{ni}}{D}\right]v_{in}+\left[\frac{\omega_{pi}^2}{(\omega-ku_0)}+\frac{\alpha_{in}\alpha_{ne}}{D}+i\alpha_{ie}\right]v_{ex}=0, \quad ...(27)$$

$$\left[B-\frac{\omega_{pi}^2\,\omega}{(\omega^2-k^2c^2)}+\frac{\alpha_{in}\alpha_{ni}}{D}\right]v_{iy}+\left[\frac{\omega_{pi}^2\,\omega}{(\omega^2-k^2c^2)}+\frac{\alpha_{in}\alpha_{ne}}{D}+i\alpha_{ie}\right]\times v_{ey}+i\Omega_i\,v_{iz}=0 \quad ...(28)$$

and

$$\left[B-\frac{\omega_{pi}^2\,\omega}{(\omega^2-k^2c^2)}+\frac{\alpha_{in}\alpha_{ni}}{D}\right]v_{iz}+\left[\frac{\omega_{pi}^2\,\omega}{(\omega^2-k^2c^2)}+\frac{\alpha_{in}\alpha_{ne}}{D}+i\alpha_{ie}\right]\times v_{ez}-i\Omega_i\,v_{iy}=0 \quad ...(29)$$

where

$$\left.\begin{aligned}
A&=\left[(\omega-ku_0)-\frac{k^2u_e^2}{(\omega-ku_0)}-i\alpha_{ei}-i\alpha_{en}\right]\\
B&=\left[(\omega-ku_0)-\frac{k^2u_i^2}{(\omega-ku_0)}-i\alpha_{ie}-i\alpha_{in}\right]\\
D&=\left[(\omega-ku_0)-\frac{k^2u_n^2}{(\omega-ku_0)}-i\alpha_{ne}-i\alpha_{ni}\right]\\
\Omega_e&=\frac{-eB_0}{m_ec}\text{ and }\Omega_i=\frac{eB_0}{m_ic}.
\end{aligned}\right\} \quad ...(30)$$

We note that the above equations decouple into two sets: one in which only the x-components occur and the other in which y and z components occur. The set of equations of x-component gives the longitudinal mode of oscillations and the other gives the transverse mode.

LONGITUDINAL OSCILLATIONS

The dispersion relation for the longitudinal mode is given by the determinant

$$\begin{vmatrix}
A-\dfrac{\omega_{pe}^2}{\omega-ku_0}+\dfrac{\alpha_{en}\alpha_{ne}}{D} & \dfrac{\omega_{pe}^2}{(\omega-ku_0)}+\dfrac{\alpha_{en}\alpha_{ni}}{D}+i\alpha_{ei}\\
\dfrac{\omega_{pi}^2}{(\omega-ku_0)}+\dfrac{\alpha_{in}\alpha_{ne}}{D}+i\alpha_{ie} & B-\dfrac{\omega_{pi}^2}{(\omega-ku_0)}+\dfrac{\alpha_{in}\alpha_{ni}}{D}
\end{vmatrix}=0 \quad ...(31)$$

The above determinant reduces to the result found by Khosa and Singh (1974) if the effect of neutral component is made zero i.e.,

$$\alpha_{en} = \alpha_{ne} = \alpha_{ni} = \alpha_{in} = 0 \text{ and } u_n = 0.$$

In the case of no collisions the dispersion relation (31) reduces to

$$\frac{k^2 u_n^2}{\omega^2} = \left(1 - \frac{k u_0}{\omega}\right)^2 \qquad \ldots(32)$$

and

$$\begin{vmatrix} (\omega - k u_0) - \dfrac{k^2 u_e^2}{(\omega - k u_0)} - \dfrac{\omega_{pe}^2}{(\omega - k u_0)} & \dfrac{\omega_{pe}^2}{(\omega - k u_0)} \\ \dfrac{\omega_{pi}^2}{(\omega - k u_0)} & (\omega - k u_0) - \dfrac{k^2 u_i^2}{(\omega - k u_0)} - \dfrac{\omega_{pi}^2}{(\omega - k u_0)} \end{vmatrix} = 0 \qquad \ldots(33)$$

The inclusion of neutral component introduces a new mode of oscillations given by relation (32). The magnetic field does not have any effect on the longitudinal oscillations.

Considering the electron oscillations fast as compared to the ion oscillations due to external perturbations, we can take $\nu_i \simeq 0$ and obtain from equation (24) after some simplification and neglecting terms of higher order than second, the dispersion relation as

$$\frac{k^2 u_e^2}{(\omega - k u_0)^2} = 1 - \frac{k^2 u_n^2}{(\omega - k u_0)^2} - \frac{\omega_{pe}^2}{(\omega - k u_0)^2} - \frac{\alpha_{ne}\alpha_{ei} + \alpha_{ne}\alpha_{ei} + \alpha_{in}\alpha_{ni}}{(\omega - k u_0)^2} - \frac{i(\alpha_{ei} + \alpha_{en} + \alpha_{ne} + \alpha_{ni})}{(\omega - k u_0)} = 0 \qquad \ldots(34)$$

which for no bulk motion ($u_0 = 0$) reduces to

$$\frac{k^2 u_e^2}{\omega^2} = 1 - \frac{k^2 u_n^2}{\omega^2} - \frac{\omega_{pe}^2}{\omega^2} - \frac{\alpha_{ne}\alpha_{ei} + \alpha_{ni}\alpha_{ei} + \alpha_{en}\alpha_{ni}}{\omega^2} - \frac{i(\alpha_{ei} + \alpha_{en} + \alpha_{ne} + \alpha_{ni})}{\omega} = 0. \qquad \ldots(35)$$

From equation (35) we find that the range of frequencies of electron longitudinal waves is reduced far below the frequencies of electron plasma, neutral component and also the collision frequencies of inter-particles. The damping frequencies band width increases further due to the presence of neutral component.

Similar calculations with equation (27) with $\nu_e \simeq 0$ gives the dispersion relation for ion-plasma oscillations as

$$\frac{k^2u_i^2}{(\omega - ku_0)^2} = 1 - \frac{k^2u_n^2}{(\omega - ku_0)^2} - \frac{\omega_{pi}^2}{(\omega - ku_0)^2} - \frac{\alpha_{ie}\alpha_{ne} + \alpha_{ie}\alpha_{ni} + \alpha_{in}\alpha_{ne}}{(\omega - ku_0)} - \frac{i(\alpha_{ie} + \alpha_{in} + \alpha_{ne} + \alpha_{ni})}{(\omega - ku_0)} = 0 \qquad ...(36)$$

which for $u_0 = 0$ reduces to

$$\frac{k^2u_i^2}{\omega^2} = 1 - \frac{k^2u_n^2}{\omega^2} - \frac{\omega_{pi}^2}{\omega^2} - \frac{\alpha_{ie}\alpha_{ne} + \alpha_{ie}\alpha_{ni} + \alpha_{in}\alpha_{ne}}{\omega^2} - \frac{i(\alpha_{ie} + \alpha_{in} + \alpha_{ne} + \alpha_{ni})}{\omega} = 0, \qquad ...(37)$$

under the conditions

$$ku_n,\ \omega_{pi},\ \alpha_{ie},\ \alpha_{ne},\ \alpha_{ni},\ \alpha_{en},\ \alpha_{ei} \ll \omega \text{ or of the order of } < \tfrac{1}{2}\omega.$$

The relation (37) indicates that the range of frequencies of ion longitudinal waves is reduced by the presence of neutral component and also the damping frequencies pand width increases.

TRANSVERSE MODE

The set of equations (25), (26), (28) and (29) gives the dispersion relations for transverse modes of oscillations. We consider the following limiting cases:

Case (A)

Considering the case of ions to be immobile in comparison to electrons ($v_{iy} = v_{iz} = 0$) due to the external perturbations we solve equations (25) and (26) and obtain the dispersion relation as

$$\left(A - \frac{\omega_{pe}^2\ \omega}{(\omega^2 - k^2c^2)} + \frac{\alpha_{en}\alpha_{ne}}{D} \right) \pm \Omega_e = 0 \qquad ...(38)$$

In the absence of collisions the dispersion relation (38) reduces to

$$\frac{k^2u_n^2}{\omega^2} = \left(1 - \frac{ku_0}{\omega}\right)^2 \qquad ...(39)$$

and

$$\frac{k^2u_e^2}{(\omega - ku_0)^2} = 1 - \frac{\omega_{pe}^2\ \omega}{(\omega^2 - k^2c^2)(\omega - ku_0)} \pm \frac{\Omega_e}{(\omega - ku_0)}. \qquad ...(40)$$

The dispersion relation (39) is same as found in the case of longitudinal modes. It indicates that in collisionless plasma the oscillations of the neutral component remain unchanged and are not influenced by the longitudinal magnetic field.

Considering $\omega^2 \gg k^2c^2$ and ω_{pe}^2, the relation (40) reduces to

$$\frac{k^2u_e^2}{\omega^2} = \left[\left(1 - \frac{ku_0}{\omega}\right) \pm \frac{\Omega_e}{1}\right]\left(1 - \frac{ku_0}{\omega}\right). \qquad ...(41)$$

The phase velocity of waves is obtained as

$$\frac{\omega}{k} = \frac{u_0}{2}\left(1 + \frac{1}{1 \pm \Omega_e}\right) \pm \left[\frac{u_0}{4}\,\frac{\Omega_e^2}{(1 \pm \Omega_e)^2} + \frac{u_e^2}{1 \pm \Omega_e}\right]. \qquad ...(42)$$

For zero magnetic field ($\Omega_e = 0$), we get the phase velocity as

$$\frac{\omega}{k} = u_0 + u_e, \qquad ...(43)$$

which under no bulk motion reduces to

$$\frac{\omega}{k} = u_e, \qquad ...(44)$$

indicating that the phase velocity equals to the sonic speed.

Now making use of the fact that the collision frequencies are very much less than ω, the relation (38) reduces to

$$\frac{k^2u_e^2}{(\omega - ku_0)} = 1 - \frac{k^2u_n^2}{(\omega - ku_0)^2} - \frac{\omega_{pe}^2}{(\omega^2 - k^2c^2)(\omega - ku_0)} \pm \frac{\Omega_e}{(\omega - ku_0)}$$
$$- \frac{\alpha_{ei}\alpha_{ne} + \alpha_{ei}\alpha_{ni} + \alpha_{en}\alpha_{ni}}{(\omega - ku_0)^2} - i\left[\frac{\alpha_{ne} + \alpha_{ni} + \alpha_{ei} + \alpha_{en}}{(\omega - ku_0)}\right.$$
$$\left.\pm \frac{\Omega_e(\alpha_{ne} + \alpha_{ni})}{(\omega - ku_0)^2}\right]. \qquad ...(45)$$

Here we have neglected the terms higher than second order.

If we assume no streaming ($u_0 = 0$) and $k^2c^2 \ll \omega^2$ the relation (45) reduces to

$$\frac{k^2u_e^2}{\omega^2} = 1 - \frac{k^2u_n^2}{\omega^2} - \frac{\omega_{pe}^2}{\omega^2} \pm \frac{\Omega_e}{\omega} - \frac{\alpha_{ei}\alpha_{ne} + \alpha_{ei}\alpha_{ni} + \alpha_{en}\alpha_{ni}}{\omega^2}$$
$$- i\left[\frac{\alpha_{ne} + \alpha_{ni} + \alpha_{ei} + \alpha_{en}}{\omega} \pm \frac{\Omega_e(\alpha_{ne} + \alpha_{ni})}{\omega^2}\right]. \qquad ...(46)$$

The above dispersion relation gives four characteristic roots

$$\omega = \frac{i(a_n + a_e \pm \gamma^{1/2}\sin\theta/2) + (\gamma^{1/2}\cos\theta/2 - \alpha_{en}\alpha_{ne})}{2} \qquad ...(47)$$

where

$$\gamma^2 = [\Omega_e + 4b_e - (a_n - a_e)^2 - 4\alpha_{en}\alpha_{ne}]^2 + [2\Omega_e(a_n - a_e)]^2,$$

$$\tan\,\theta = \frac{2\Omega_e(a_n - a_e)}{[\Omega_e + 4b_e - (a_n - a_e)^2 - 4\alpha_{en}\alpha_{ne}]},$$

$$a_n = \alpha_{ne} + \alpha_{ni},$$

$$a_e = \alpha_{ei} + \alpha_{en},$$

$$b_e = k^2u_n^2 + k^2u_e^2 + \omega_{pe}^2.$$

From (47) we get the condition for damping of waves as

$$(a_n + a_e)^2 > \gamma. \qquad ...(48)$$

The inclusion of neutral component collisions is to increase the damping of waves in the transverse direction also make waves damped at lower collision frequencies. The effect of magnetic field is to reduce the range of collision frequencies for damping the transverse waves.

In the case of two-component plasma (electrons and ions) with zero magnetic field, the condition (48) reduces to

$$\alpha_{ei}^2 > 2[k^2u_e^2 + k^2u_n^2 + \omega_{pe}^2], \qquad ...(49)$$

as in the case of longitudinal modes.

Case (B)

To study the ion plasma oscillations we can neglect $v_{ey} = v_{ez}$ in comparison to ion motion due to external perturbations. Solving equations (28) and (29) under these assumptions, we get the dispersion ralation as

$$B - \frac{\omega_{pi}^2\ \omega}{(\omega^2 - k^2c^2)} + \frac{\alpha_{in}\alpha_{ni}}{D} \pm \Omega_i = 0. \qquad ...(50)$$

Now considering the collision frequencies, ω_{pi} and Ω_i very much less than n, the relation (50) reduces to

$$\frac{k^2u_i^2}{(\omega - ku_0)^2} = 1 - \frac{k^2u_n^2}{(\omega - ku_0)^2} - \frac{\omega_{pi}^2\ \omega}{(\omega^2 - k^2c^2)(\omega - ku_0)} \pm \frac{\Omega_i}{(\omega - ku_0)}$$
$$- \frac{\alpha_{ie}\alpha_{ne} + \alpha_{in}\alpha_{ne} + \alpha_{ie}\alpha_{ni}}{(\omega - ku_0)^2} - i\left[\frac{\alpha_{ie} + \alpha_{in} + \alpha_{ne} + \alpha_{ni}}{(\omega - ku_0)}\right.$$
$$\left.\pm \frac{\Omega_i(\alpha_{ne} + \alpha_{ni})}{(\omega - ku_0)^2}\right]. \qquad ...(51)$$

Here we have neglected the terms higher than second order. If there is no bulk motion and $k^2c^2 \ll \omega^2$, the relation (51) reduces to

$$\frac{k^2u_i^2}{\omega^2} = -\frac{k^2u_n^2}{\omega^2} - \frac{\omega_{pi}^2}{\omega^2} \pm \frac{\Omega_i}{\omega} - \frac{\alpha_{ie}\alpha_{ne} + \alpha_{in}\alpha_{ne} + \alpha_{ie}\alpha_{ni}}{\omega^2}$$

$$- i\left[\frac{\alpha_{ie} + \alpha_{in} + \alpha_{ne} + \alpha_{ni}}{\omega} \pm \frac{\Omega_i(\alpha_{ne} + \alpha_{ni})}{\omega^2}\right]. \quad \ldots(52)$$

The above dispersion relation for ion plasma oscillations gives four characteristics roots as

$$\omega = \frac{i(a_n + a_i \pm \gamma^{1/2}\sin\theta/2) \pm (\gamma^{1/2}\cos\theta/2 - \Omega_i)}{2}, \quad \ldots(53)$$

where

$$\gamma^2 = [\Omega_i^2 + 4b_i - (a_n - a_i)^2 - 4\alpha_{in}\alpha_{ni}]^2 + [2\Omega_i(a_n - a_i)]^2,$$

$$a_n = \alpha_{ne} + \alpha_{ni},$$

$$a_i = \alpha_{ie} + \alpha_{in}$$

and

$$b_i = k^2u_n^2 + k^2u_i^2 + \omega_{pi}^2.$$

From (53) we get the condition for damping of waves as

$$(a_n + a_i)^2 > \gamma. \quad \ldots(54)$$

The inclusion of neutral component collision frequencies is to make the waves more damped at lower collision frequencies. But the effect of longitudinal magnetic field is to reduce the band width of collision frequencies for damping i.e., the magnetic field shifts the lower collision frequency range to higher value.

For two component plasma under no magnetic field the condition (54) reduces to

$$\alpha_{ie}^2 > 2[k^2u_i^2 + k^2u_n^2 + \omega_{pi}^2]. \quad \ldots(55)$$

Case (C)

The equations (25), (26), (28) and (29) on solving give the general dispersion relations of transverse modes for three-component plasma with collisions as

$$\begin{vmatrix} A_1 & a_1 & i\Omega_e & 0 \\ -i\Omega_e & 0 & A_1 & a_1 \\ b_1 & B_1 & 0 & i\Omega_i \\ 0 & -i\Omega_i & b_1 & B_1 \end{vmatrix} = 0, \quad \ldots(56)$$

where

$$A_1 = \left(A - \frac{\omega_{pe}^2 \, \omega}{(\omega^2 - k^2c^2)} + \frac{\alpha_{en}\alpha_{ne}}{D} \right)$$

$$B_1 = \left(B - \frac{\omega_{pi}^2 \, \omega}{(\omega^2 - k^2c^2)} + \frac{\alpha_{in}\alpha_{ni}}{D} \right)$$

$$a_1 = \left(\frac{\omega_{pe}^2 \, \omega}{(\omega^2 - k^2c^2)} + \frac{\alpha_{en}\alpha_{ni}}{D} + i\alpha_{ei} \right)$$

and

$$b_1 = \left(\frac{\omega_{pi}^2 \, \omega}{(\omega^2 - k^2c^2)} + \frac{\alpha_{in}\alpha_{ne}}{D} + i\alpha_{ie} \right).$$

i.e.,

$$(A_1^2 + i^2\Omega_e^2)(B_1^2 + i^2\Omega_i^2) - 2a_1b_1A_1B_1 + a_1^2b_1^2 - 2a_1b_1\Omega_e\Omega_i = 0. \quad \ldots(57)$$

In the case of no bulk motion and $u_n = u_i = u_e = 0$ and also assuming no collisions take place, the dispersion relation (57) reduces to the well familiar dispersion relation for transverse mode in a longitudinal magnetic field for two-components (electrons and ions) as

$$(\omega^2 - k^2c^2) - \frac{\omega_{pe}^2 \, \omega}{(\omega \pm \Omega_e)} - \frac{\omega_{pi}^2 \, \omega}{(\omega \pm \Omega_i)} = 0, \quad \ldots(58)$$

which is a standard result given in Montgomery and Tidman (1964).

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