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Relativity

TACHYONS IN STATIONARY FIELDS

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The authors have investigated here whether Tachyons can move in stable circular orbits in Kerr, Kerr-Newmann and Lewis fields owing to their dragging effect. It is found that stable circular orbits are possible only in a Lewis field.

Keywords : Special Relativity Tachyons; General Relativity; Stationary Fields

INTRODUCTION

HETTEL and Helliwell (1973) have shown that no stable circular orbit of a tachyon is possible in a Schwarzschild field. It has further been shown by Banerjee and Dutta Choudhury (1977) that a tachyon cannot move in stable circular orbits in Reissner-Nordstrom ($m^2 > e^2$) and Marder fields also. The reason for instability in all these cases is perhaps that there is repulsion between ordinary matter and tachyons (Raychaudhuri, 1974). Now one may ask whether the dragging effect of the corresponding stationary fields, viz., Kerr, Kerr-Newmann and Lewis fields, may cause stability to circular tachyonic orbits. We have investigated this problem in this paper and have found that stable circular orbits are possible only in a Lewis field.

CALCULATIONS

Kerr and Kerr-Newmann Fields

We proceed to our calculation with the more general field viz., Kerr-Newmann field.

The Kerr-Newmann metric at $\theta = \frac{\pi}{2}$ is given by

$$ds^2 = \left(1 - \frac{2m - e^2}{r^2}\right) dt^2 - \left[\frac{r^2}{r^2 + a^2 - (2mr - e^2)}\right] dr^2$$
$$- \left[r^2 + a^2 + \frac{(2mr - e^2)\, a^2}{r^2} \right] d\phi^2 - \frac{2a\,(2mr - e^2)}{r^2}\, d\phi\, dt \quad \ldots(1)$$

where m, e and a are respectively the mass, charge and angular momentum per unit mass of the Kerr-Newmann source. The expressions for the total energy E and the angular momentum L of a test particle are given by the first integrals of the geodesic equations as

$$\left(1 - \frac{2mr - e^2}{r^2}\right)\dot{t} - a\frac{(2mr - e^2)}{r^2}\dot{\phi} = E \qquad \text{...(2)}$$

$$\left[r^2 + a^2\frac{(2mr - e^2)a^2}{r^2}\right]\dot{\phi} + a\frac{(2mr - e^2)}{r^2}\dot{t} = L \qquad \text{...(3)}$$

where the dot refers to differentiation with respect to s for a tardyon or a tachyon and to some affine parameter for a photon. Now using (2) and (3) in (1), we get

$$\dot{r}^2 = \frac{1}{x^4}\left[x^4 + d^2(x^2 + 2x - p^2)\right]\left[E^2 + \frac{2dk(2x - p^2)E}{x^4 + d^2(x^2 + 2x - p^2)} + \left\{-\frac{\Omega(x^4 - 2x^3 + d^2x^2 + p^2x^2)}{x^4 + d^2(x^2 + 2x - p^2)} - \frac{k^2(x^2 - 2x + p^2)}{x^4 + d^2(x^2 + 2x - p^2)}\right\}\right] \qquad \text{...(4)}$$

where $\Omega = +1, 0$ and -1 for tardyons, photons and tachyons respectively and

$$x = \frac{r}{m},\ d = \frac{a}{m},\ k = \frac{L}{m} \text{ and } p = \pm\frac{e}{m}.$$

Following Adler *et al.* (1975) from (4), we can find the potential energy function V from the equation (with $\Omega = -1$ for tachyons)

$$AV^2 + BV + C = 0 \qquad \text{...(5)}$$

where

$$\left.\begin{array}{l} A = x^4 + d^2(x^2 + 2x - p^2),\ B = 2dk(2x - p^2) \\ C = x^4 - 2x^3 + d^2x^2 + p^2x^2 - k^2(x^2 - 2x + p^2) \end{array}\right\} \qquad \text{...(6)}$$

The potential energy function

$$V = \frac{-dk(2x - p^2) \pm \sqrt{x^4 - 2x^3 + d^2x^2 + p^2x^2}\sqrt{k^2x^2 - [x(x^3 + d^2x + 2d^2) - p^2d^2]}}{x^4 + d^2(x^2 + 2x - p^2)} \qquad \text{...(7)}$$

As Adler *et al.* have pointed out, the positive sign must be taken. This agrees with the condition (Bardeen *et al.*, 1972)

$$2AV + B > 0 \qquad \text{...(8)}$$

Now, we proceed to obtain the condition for stable circular orbits. On double differentiation of (5) with respect to x, we get

$$V''(AV_m + B) + A''V_m^2 + C'' = 0 \qquad \text{...(9)}$$

where V_m is a maximum or minimum of V. For V_m to represent a minimum, V'' must be positive at the minimum. Then from (9) taking into account the inequality (8), we obtain the condition

$$V_m^2 < -\frac{C''}{A''} \qquad \text{...(10)}$$

If this condition is to be satisfied, C'' must be negative since A'' is positive. Now from (6), we find that C'' is negative between the following value of x

$$x_{\pm} = \frac{1 \pm \sqrt{1 - \frac{2}{3}(d^2 + p^2 - k^2)}}{2} \qquad \text{...(11)}$$

A tachyon may move in a stable circular orbit, if at all, between these values of x. So we shall have to calculate V (given by (7)) in this range of x only. We have done this with a large number of sets of values of the parameters d, k and p (*vide* Table I) under the physical assumption : $d^2 < 1$, $k^2 < 1$ and $p^2 < 1$. But we have failed to find any minimum of V. We have also repeated the calculations for all these cases with $p = 0$ (Kerr field) but have not come across any minimum of V. So, we may conclude that stable circular orbits are not possible in the Kerr and Kerr-Newmann fields under the above mentioned physical assumptions.

TABLE I

Calculations for Kerr-Newmann Field

d^2	P^2	K^2	x_+	x_-	Value of x between x_+ and x_-	Value of V
(1) 0.1	0.8	0.1	0.841565	0.158435	.68	−0.1761975
					.70	−0.0620254
					.71	−0.0318227
					.75	+0.0458174
					.79	+0.0938076
					.8	+0.1031706
(2) 0.16	.09	0.1	0.9743 416	0.0256584	.4	+.0397493
					.5	+.0931555
					.6	+.3395339
					.7	+.4315887
					.8	+.4893564
					.9	+.518704
(3) .09	.04	0.1	.9949747	.0050253	.1	−.7947792
					.2	−.4407101
					.3	−.11206
					.4	+.1359068
					.5	+.3475204
					.6	+.4978445
					.7	+.5919195
					.8	+·6614173

Lewis Field

The Lewis metric for a Z-constant plane is given by (Lewis, 1932)

$$ds^2 = f\,dt^2 - \rho\, dr^2 - l d\phi^2 + 2m d\phi dt \qquad ...(12)$$

where

$$f = \gamma^2 (r^{2k} - \omega^2 r^{2-2k})$$

$$\rho = A^2 r^{2k(k-1)}$$

$$l = \gamma^2 (r^{2-2k} - \omega^2 r^{2k})$$

$$m = \gamma^2 \omega (r^{2-2k} - r^{2k}) \qquad ...(13)$$

Here k is the mass per unit length of rotating cylinder and $\gamma^2 = (1 - \omega^2)^{-1}$, ω being related to the angular velocity of the cylinder.

The expressions for the total energy E and the angular momentum L of a test particle are given, as before, by the first integrals of the geodesic equations as

$$f\dot{t} + m\dot{\phi} = E \qquad ...(14)$$

$$l\dot{\phi} - m\dot{t} = L \qquad ...(15)$$

Using (14) and (15) in (12), we get

$$\dot{r}^2 = -\frac{\Omega}{\rho} + \frac{E^2}{\rho f} - \frac{(Lf + mE)^2}{\rho f r^2} \qquad ...(16)$$

The equation for the potential energy function V (with $\Omega = -1$) for (16) is

$$r^2 V^2 - (Lf + mV)^2 + fr^2 = 0 \qquad ...(17)$$

From this equation, we obtain

$$V = \frac{Lm \pm r\sqrt{L^2 - l}}{l} \qquad ...(18)$$

In deriving (16) use has been made of the relation

$$fl + m^2 = r^2 \qquad ...(19)$$

As in the earlier case, we adopt positive sign in (18) also. Taking two sets of values for the parameters k, ω and L, we have presented here two curves for V (Fig. 1) which show minimum (at A, B, C, D and E in curve 1 and at F in curve II). The values of k, w and L are respectively 0.6, 0.4 and 2.2390002 for curve I and 0.7, 0.7 and 2.2298116 for curve II. We may, therefore, conclude that it is possible for tachyons to move in stable circular orbits in a Lewis field.

Conclusion

We have seen above that tachyons may move in stable circular orbits in a Lewis field. But this is a quasi-physical field. Physically interesting fields are asymptotically flat.

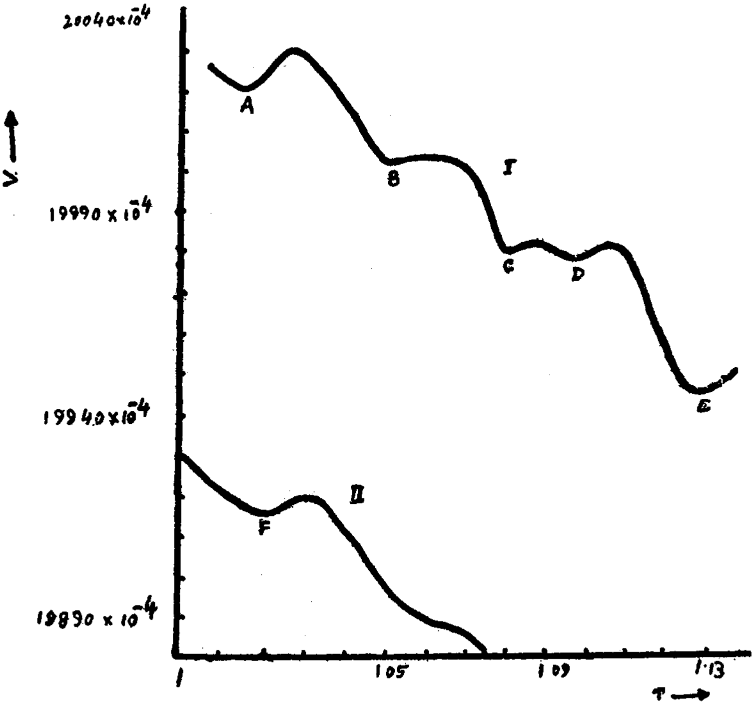

FIG. 1.

In this sense, Schwarzschild, Kerr and Kerr-Newmann fields are all physical. So one may feel tempted to conclude from all results obtained till now that stable circular orbits are not possible for tachyons in any physical field.

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