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Relativity

CHARGED NUT FIELD : I. MOTION OF TEST PARTICLES AND II. COSMIC CENSORSHIP

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In this paper, we study some properties of the charged NUT field. In the first part of the paper, some general aspects of the charged NUT field have been investigated using uncharged and charged particles. The behaviour of the particles near the singularity has also been considered. In the second part of the paper, we study the charged NUT sources in the context of cosmic censorship hypothesis. We consider motion of charged particles in the equatorial plane and along the axis. We discover from our investigation the interesting result that by such a bombardment of charged test particles, the existing event horizons cannot be destroyed but, in contrast to the Reissner-Nordstrom field, naked singularities do not get enveloped by event horizons.

Keywords : General Relativity; NUT Field; Test Particles; Cosmic Censorship

INTRODUCTION

NEWMAN *et al.* (1965) derived a generalised Schwarzschild metric which is the so-called NUT metric. Although the metric is not asymtotically flat, it has been useful in developing and understanding certain areas of general relativity (Misner, 1967). In this paper, we study some properties of the charged NUT field. We divide our work into two parts.

The characteristics of a field can be visualised from the behaviour of test particles in that field. This is the primary motive behind the study of the motions of test particles in a field. Krori and Chaudhary (1978) have investigated the restrictions caused to the motion of particles (material particles and photons) by the rotation of cylindrical bodies about their axis. Karade and Rao (1975) have investigated the motion of charged particles in R-N metric. Karade and Rao (1976) have also studied the motion of test particles in the NUT field. In the first part of this paper, we have discussed the motion of test particles (both uncharged and charged) in the charged NUT field (MacGuire Ruffini, 1975; and Krori *et al.*, 1980). In section I (A). we have discussed how the angular momentum of uncharged test particles and photons decides their turning, trapping and escape to infinity. In section I (B), we have discussed the radial motion of charged particles in the charged NUT field and studied their behaviour near the metric singularity.

Now we proceed to the second part of the paper. An interesting question confronting relativistic astrophysics at present is whether the naked singularities and/or event horizons can be created or destroyed. This is closely related to Penrose's cosmic censorship hypothesis according to which naked singularities have a tendency to get enveloped by event horizons as the universe evolves (Cohen & Gautreau, 1979). While no direct theoretical proof of the hypothesis has yet been offered, it is widely accepted as true owing to the failure of numerous attempts to construct counter-examples. Perhaps, one of the main reasons for believing in this hypothesis is the disquieting nature of the alternative : the existence of the naked singularities, and hence loss of predictability, the possibility of closed time like lines, etc. (Hiscock, 1979).

Recently, Cohen and Gautreau (1979) have investigated the motion of charged test particles in a Reissner-Nordstrom field. They have shown that naked singularities can be destroyed by shooting in suitably charged test particles to produce an event horizon around the source where none previously existed. They have also shown that existing event horizons cannot be destroyed by bombardment with charged test particles. Thus, naked singularities can be destroyed but not created while event horizons can be created but not destroyed.

In the second part of this paper, we study charged NUT sources in the context of cosmic censorship hypothesis. For this purpose, we analyse the motion of charged test particles in the equatorial plane and along the axis. We consider situations both with and without event horizons and we find that radial test particle oscillations are possible, a result which has no classical (Newtonian) analogue. Also conditions for the creation or destruction of event horizons and naked singularities by bombardment with charged test particles are investigated. We discover from our investigation the interesting result that, by such bombardment, the existing event horizons cannot be destroyed but, in contrast to the Reissner-Nordstrom field, naked singularities do not get envoloped by an event horizon.

I. Motion of Test Particles

A. Uncharged Particles and Photons — Turning, Trapping and Escape to Infinity

We have the line-element for a charged NUT field in the form (MacGuire & Ruffini, 1975; and Krori *et al.*, 1980)

$$ds^2 = -A^{-1}dr^2 - (r^2 + a^2)(d\theta^2 + \sin^2\theta d\phi^2) + A\left(dt + 4a\sin^2\frac{\theta}{2}\,d\phi\right)^2 \quad \ldots(1)$$

where $A = \dfrac{r^2 - 2mr + e^2 - a^2}{r^2 + a^2}$, m is the mass,

e is the charge and a is the NUT parameter of the source.

The first integrals of the geodesic equations in the $\theta = \frac{\pi}{2}$, plane are

$$C = 2Aa\dot{t} + \{4a^2A - (r^2 + a^2)\}\dot{\phi} \qquad ...(2)$$

$$E = A\dot{t} + 2aA\dot{\phi} \qquad ...(3)$$

$$\Omega = -A^{-1}\dot{r}^2 + \{4a^2A - (r^2 + a^2)\}\dot{\phi}^2 + A\dot{t}^2 + 4aA\dot{t}\dot{\phi} \qquad ...(4)$$

where C and E are the total angular momentum and total energy respectively of a test particle. $\Omega = 1$ and 0 for material particles and photons respectively. Here a dot means differentiation with respect to t.

At a turning point $r = r_0$, $\dot{r} = 0$ and hence from (4) using the values of $\dot{\phi}$ and $\dot{t}$ from (2) and (3), we have

$$\left(\frac{c}{m}\right)_{\pm} = 2\left(\frac{a}{m}\right)E \pm \sqrt{\left(\frac{r_0}{m}\right)^2 + \left(\frac{a}{m}\right)^2} \sqrt{\frac{E^2\left[\left(\frac{r^2}{m}\right)^2 + \left(\frac{a}{m}\right)^2\right]}{\left(\frac{r_0}{m}\right)^2 - 2\left(\frac{r_0}{m}\right) + \left(\frac{e}{m}\right)^2\left(\frac{a}{m}\right)^2} - \Omega} \qquad ...(5)$$

We now consider the variations of $\left(\frac{c}{m}\right)_+$ and $\left(\frac{c}{m}\right)_-$ for (*i*) material particles ($\Omega = +1$) and (*ii*) for photons ($\Omega = 0$), with $E \geqslant 1$ and $E < 1$ (*see* Figs. 1 – 6). Such variations are considered for various values of $\frac{a}{m}$ and $\frac{e}{m}$. The continuous curves are for material particles and the dotted ones are for photons. The curves have been drawn *only broadly* (not exactly). The shaded areas are the regions in which the particles are trapped. The following are some of the interesting features of the curves :

(1) In each of the curves considered (Figs. 1 – 6), the possibility exists for photons being trapped. Photons may escape to infinity only if their $\left(\frac{c}{m}\right)_{\pm}$ lies between two limits α_1 and α_2.

(2) In the first two cases (Figs. 1 and 2), material particles may escape to infinity only if their $\left(\frac{c}{m}\right)_{\pm}$ tends to 0. In the next two cases (Figs. 3 and 4) the escape is possible for their $\left(\frac{c}{m}\right)_{\pm}$ tending to a certain value β. In the last two cases (Figs. 5 and 6) the possibility exists for material particles being trapped. In these two cases, there is a wider probability for escape also. They may escape if their $\left(\frac{c}{m}\right)_{\pm}$ lies between two limits (β_1 and β_2).

B. Radial Motion of Charged Particles

The equation of motion of a charged particle of specific charge $\frac{q}{m_0}$ in the charged NUT field is given by

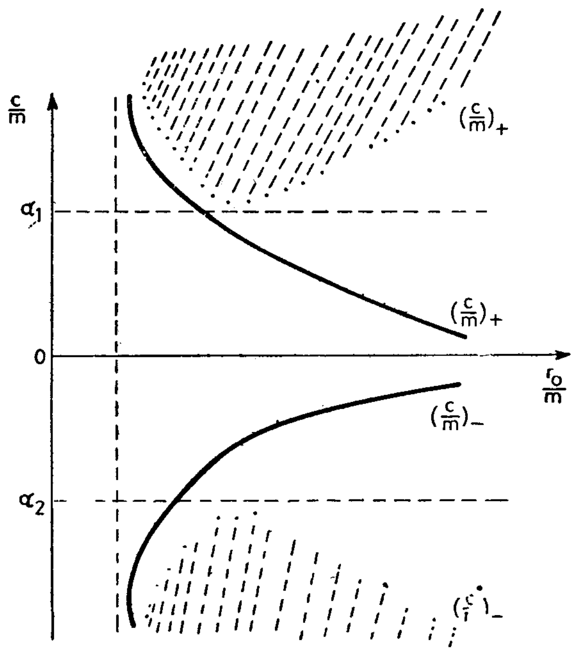


FIG. 1.

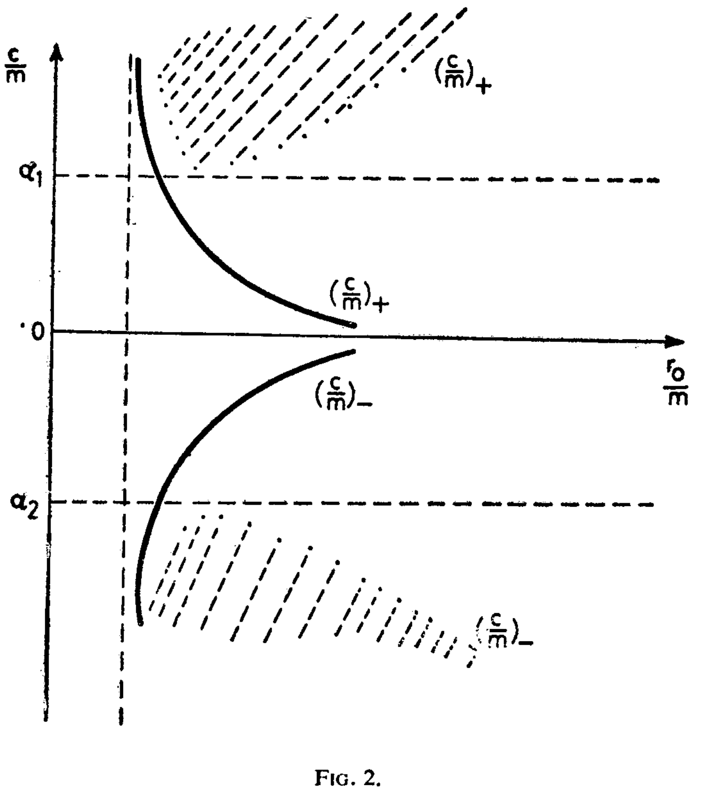


FIG. 2.

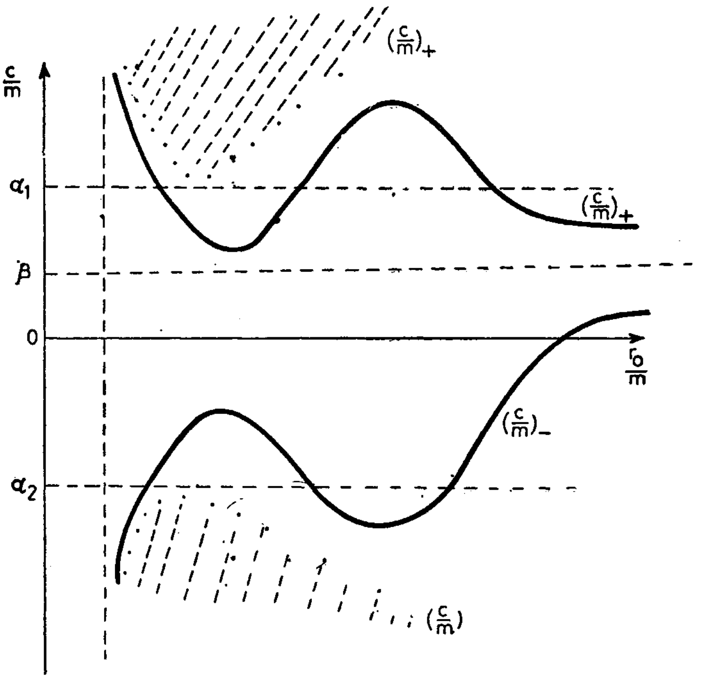

$\frac{c}{m}$
$(\frac{c}{m})_+$
α_1
$(\frac{c}{m})_+$
β
0
$\frac{r_0}{m}$
$(\frac{c}{m})_-$
α_2
$(\frac{c}{m})$

FIG. 3.

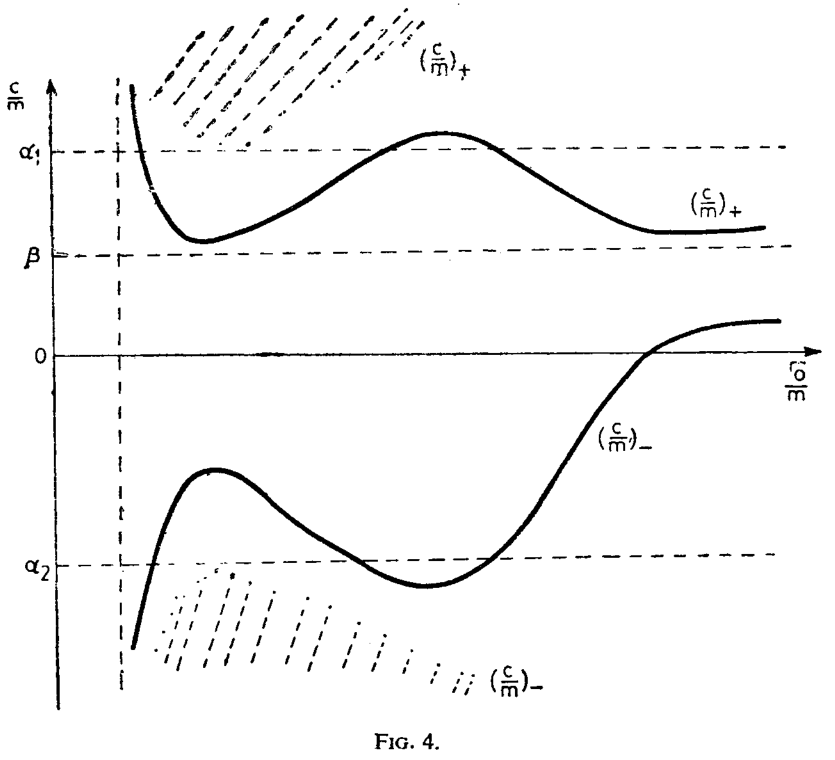

$\frac{c}{m}$
$(\frac{c}{m})_+$
α_1
$(\frac{c}{m})_+$
β
0
$\frac{r_0}{m}$
$(\frac{c}{m})_-$
α_2
$(\frac{c}{m})_-$

FIG. 4.

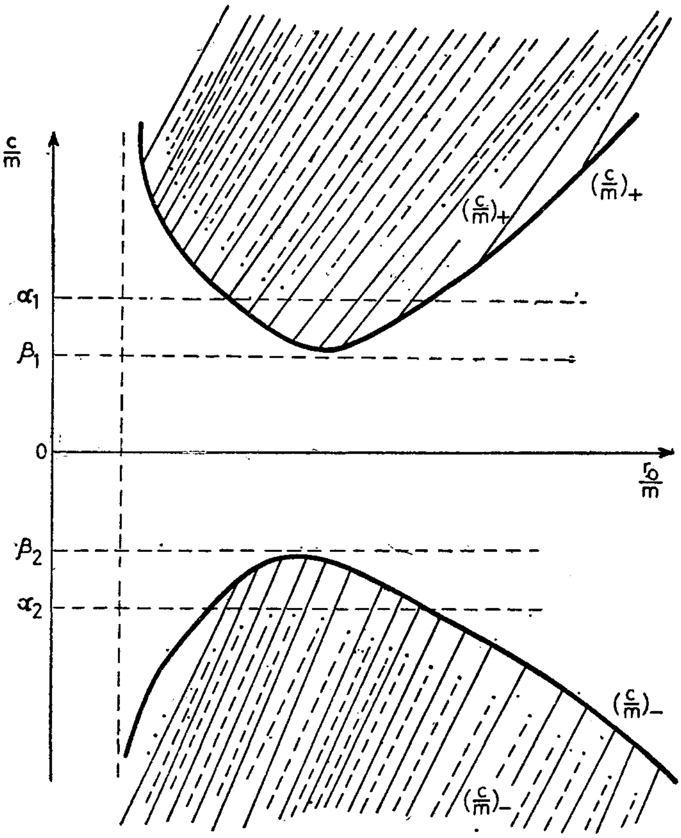


FIG. 5.

$$\frac{d^2x^i}{ds^2} + \{jk, i\} \frac{dx^j}{ds} \frac{dx^k}{ds} + \frac{q}{m_0} F^i{}_j \frac{dx^j}{ds} = 0 \qquad \ldots(6)$$

where i, j, k take the value from 1 to 4. The components of the electromagnetic field tensor (MacGuire & Ruffini 1975; and Krori *et al.*, 1980) are

$$F_{41} = -\frac{e(r^2 - a^2)}{(r^2 + a^2)^2} \qquad \ldots(7)$$

$$F_{31} = -\frac{4ae(r^2 - a^2)\sin^2\theta/2}{(r^2 + a^2)^2} \qquad \ldots(8)$$

$$F_{32} = \frac{2aer\sin\theta}{r^2 + a^2} \qquad \ldots(9)$$

Taking initial conditions as $\theta = \frac{\pi}{2}$ and $\phi = 0$ we have from equation (6)

$$\ddot{\theta} = \ddot{\phi} = 0 \qquad \ldots(10)$$

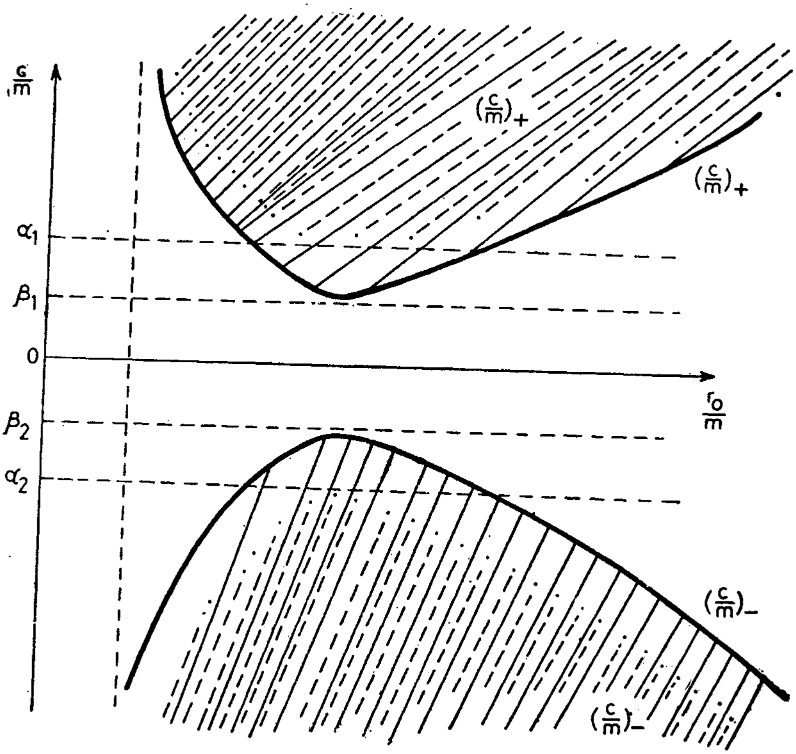


FIG. 6.

Thus the equations to be satisfied are

$$\dot{S}^2 = -A^{-1}\dot{r}^2 + A \quad \ldots(11)$$

$$\ddot{r} - A^{1/2}(A^2 - \dot{r}^2)\frac{q}{m_0}\frac{\dot{S}\,e}{A}\frac{(r^2 - a^2)}{(r^2 + a^2)^2} - \frac{3A'\dot{r}^2}{2A} + \frac{AA'}{2} = 0 \quad \ldots(12)$$

where a dot and a dash denote differentiation with respect to t and r respectively.

Using (11) in (12), we have

$$\ddot{r} - A^{3/2}(A^2 - \dot{r}^2)^{3/2}\frac{q}{m_0}\frac{e(r^2a^2)}{(r^2 + a^2)^2} - \frac{3A'}{2A}\dot{r}^2 + \frac{AA'}{2} = 0 \quad \ldots(13)$$

The solutions of (13) are

$$\dot{r} = A \quad \ldots(14a)$$

and

$$\dot{r}^2 = A^2\left[1 - A\left\{1 - \frac{eqr}{m_0(r^2 + a^2)}\right\}^{-2}\right] \quad \ldots(14b)$$

The solution (14a) is trivial; it corresponds to the trajectory of photons. We shall not discuss it here.

Now if in addition to the initial condition $\theta = \frac{\pi}{2}$ and $\phi = 0$ we suppose $\dot{r} = 0$ initially, we shall then obtain from (13)

$$m\ddot{r} = -\frac{mm_0(r^2 - a^2)}{(r^2 + a^2)^2} + \frac{qe(r^2 - a^2)}{(r^2 + a^2)^2} - \frac{mm_0(e^2 - 2a^2 - 2mr) + m_0 r(e^2 - 2a^2)}{(r^2 + a^2)^2} + \frac{r(e^2 - 2a^2 - 2mr)^2}{(r^2 + a^2)^3} \quad ...(15)$$

The first term in equation (15) is the gravitational force of attraction, the second term is the electrostatic force of repulsion and the remaining ones are relativistic correction terms.

Now at a very large distance away from the symmetry axis, we have

$$m\ddot{r} \approx -\frac{mm_0}{r^2} + \frac{qe}{r^2} \quad ...(16)$$

From (14*b*) it is seen that velocity and acceleration of the charged particle will vanish at the metric singularity $r = m + b$ where $b = \sqrt{m^2 + a^2 - e^2}$. But the situation is quite different for a proper observer.

Writing

$$r = r(R) \text{ and } \frac{dr}{dR} = A, \quad ...(17)$$

we have

$$R = r + m \log \frac{(r - m)^2 - b^2}{(R - m)^2 - b^2} + b \log \frac{(r - m - b)(R - m + b)}{(r - m + b)(R - m - b)} \quad ...(18)$$

It is obvious that $r(R)$ is a monotonically increasing function. Now the charged NUT metric becomes

$$ds^2 = A(R)(dt^2 - dR^2) - \{r^2(R) + a^2\}(d\theta^2 + \sin^2\theta\, d\phi^2) + 8A(R)\, a \sin^2\frac{\theta}{2}\, d\phi \left(2a \sin^2\frac{\theta}{2}\, d\phi + dt\right) \quad ...(19)$$

From equation (19) we have

$$f = \left(\frac{ds}{dt}\right)^2 = A(1 - \dot{R}^2) - (r^2 + a^2)(\dot{\theta}^2 + \sin^2\theta\dot{\phi}^2) + 8aA \sin^2\frac{\theta}{2}\, \dot{\phi}\{2a \sin^2(\theta/2)\, \dot{\phi} + 1\} \quad ...(20)$$

The motion of a test particle satisfies

$$\delta \int ds = 0 \qquad \text{...(21)}$$

and so from equations (19) and (21) its path is described by

$$\delta \int f^{1/2}\, dt = 0$$

where f is given by equation (20).

The Lagrangian for the moving particle is then given by

$$L = -\alpha f^{1/2} \qquad \text{...(22)}$$

where the constant α characterises the moving particle and is always positive. Now taking the initial conditions as noted earlier we have the Lagrangian L as

$$L = -\alpha\sqrt{A(1-\dot{R}^2)} \qquad \text{...(23)}$$

The canonical momentum conjugate to R is

$$P_R = \alpha \dot{R}\sqrt{\frac{A}{1-\dot{R}^2}} \qquad \text{...(24)}$$

The Hamiltonian H is given by

$$H = \alpha\sqrt{\frac{A}{1-\dot{R}^2}} \qquad \text{...(25)}$$

For sufficiently large values of R, we have

$$H = \frac{\alpha}{\sqrt{1-\dot{R}^2}}\left[1 - \frac{m}{r} - \frac{a^2}{r^2} - \frac{m^2}{2r^2} + \frac{e^2}{2r^2}\right] \qquad \text{...(26)}$$

Since H in equation (26) does not involve time, it is a constant of motion and it is denoted by αE.

Thus

$$E = \sqrt{\frac{A}{1-\dot{R}^2}} \qquad \text{...(27)}$$

Now since E and A are positive quantities, $\dot{R}$ must be less than unity. Again at singularity since $A = 0$, from equation (27) $\dot{R} = 1$. Thus the velocity of the particle approaches that of light at the metric singularity. The acceleration of the particle from equation (27) is given by

$$\ddot{R} \approx (1-\dot{R}^2)\left(-\frac{m}{r^2} - \frac{2a^2}{r^3} + \frac{3ma^2}{r^4} + \frac{e^2}{r^3}\right) \qquad \text{...(28)}$$

and hence the gravitational force on the particle is

$$F = \frac{\alpha}{\sqrt{1-\dot{R}^2}}\left(-\frac{m}{r^2}\right) + \frac{\alpha}{\sqrt{1-\dot{R}^2}}\left(-\frac{2a^2}{r^3} + \frac{3ma^2}{r^4} + \frac{e^2}{r^3}\right)$$

...(29)

where $\frac{\alpha}{\sqrt{1-\dot{R}^2}}$ is the relativistic mass of the particle.

From equation (29), it is seen that the force on the particle due to the charge e of the source is repulsive in nature. For $e^2 = 2a^2$, the contribution to the force (29) and the energy (26) due to the charge and the NUT parameter of the source will vanish provided the term containing r^{-4} is negligible.

Now to study the behaviour of the charged particle in the neighbourhood of the singularity ($r = m + b$) let us introduce two new variables u and v such that

$$r-(m+b) = u = \frac{v^2}{4(m+b)}$$

The space-times given by (11) in the neigbourhood of $r = m + b$ is given in terms of u aud v respectively by

$$ds^2 = -\frac{(m+b)^2+a^2}{2ub}\,du^2 + \frac{2ub}{(m+b)^2+a^2}\,dt^2 \qquad ...(30)$$

and

$$ds^2 = -\frac{2(m+b)}{bv^2}\,[(m+b)^2+a^2]\,dv^2 + \frac{bv^2}{2(m+b)}\,\frac{dt^2}{[(m+b)^2+a^2]} \qquad ...(31)$$

From equation (30), we have

$$\left(\frac{du}{dt}\right)^2 = \frac{4b^2u^2}{(m+b)}\,(1-\alpha u) \qquad ...(32)$$

where

$$\alpha = 2bm_0^2\left[1+\frac{a^2}{(m+b)^2}\right]\left[mm_0+m_0b-eq+\frac{m_0a^2}{m+b}\right] \qquad ...(33)$$

The solution of equation (32) gives

$$\alpha u = 1-\left(\frac{1+ke^{-lt}}{1-ke^{-lt}}\right)^2 \qquad ...(34)$$

where $l = \frac{4b^2}{[(m+b)^2+a^2]^2}$ and k is a constant of integration.

From (34), it is seen that at $t \to \infty$, $u \to 0$. Therefore, the co-ordinate time elapsed to reach the singularity $r = m + b$ is infinite for a charged test particle.

If we use $u = \frac{1}{\alpha}$ at $t = 0$ as the initial condition, then $k = -1$.

From equation (34), we have

$$\alpha u = 1-\left(\frac{1-e^{-lt}}{1+e^{-lt}}\right)^2 \qquad ...(35)$$

If we now suppose $b = 0$ i.e., $m^2 = l^2 - a^2$, then we have from (32)

$$\frac{du}{dt} = 0 \quad \text{and} \quad \frac{d^2u}{dt^2} = 0.$$

Therefore, in the neighbourhood of the singularity $r = m$ (where $b = 0$), the velocity and acceleration of the charged particle vanish. But for this value of $m^2(= e^2 - a^2)$, the velocity and acceleration of the charged particle from equation (14*b*) will not be zero showing that no Newtonian-analogue is found in this case.

We can study the behaviour of uncharged test particles in the charged NUT field by putting $q = 0$ in the relevant equations.

II. Cosmic Censorship

A. Motion of Charged Test Particles

We have line-element for a charged NUT source in the form (MacGuire & Ruffini, 1975; and Krori *et al.*, 1980)

$$ds^2 = -A^{-1}dR^2 - (R^2 + a^2)(d\theta^2 + \sin^2\theta \, d\phi^2) + A\left(dt + 4a \sin^2 \frac{\theta}{2} \, d\phi\right)^2 \quad \ldots(36)$$

where

$$A = \frac{R^2 - 2MR + Q^2 - a^2}{R^2 + a^2}.$$

Q is the charge, M the mass and a the NUT parameter of the source. The motion of a charged test particle is given by the Lorentz force equation

$$\frac{d}{d\tau}\frac{dx^\mu}{d\tau} + \Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau} = -\epsilon_0 F^{\mu\nu} g_{\nu\beta}\frac{dx^\beta}{d\tau} \quad \ldots(37)$$

where $\epsilon_0 = \frac{q}{m_0}$ is the ratio of the test particle charge to rest mass. The components of electromagnetic fields tensor are

$$F_{41} = -\frac{Q(R^2 - a^2)}{(R^2 + a^2)^2} \quad \ldots(38)$$

$$F_{31} = -\frac{4aQ(R^2 - a^2)\sin^2\theta/2}{(R^2 + a^2)^2} \quad \ldots(39)$$

$$F_{32} = \frac{2aQR\sin\theta}{R^2 + a^2} \quad \ldots(40)$$

Now taking the initial condition as $\theta = \pi/2$ and $\phi = 0$, we have from (36) and (37)

$$\left(\frac{dR}{d\tau}\right)^2 = -A + (k - \epsilon_0\psi)^2 \quad \ldots(39)$$

$$\frac{d^2R}{d\tau^2} + \tfrac{1}{2} A_1 = \frac{\epsilon_0\theta(R^2 - a^2)(k - \epsilon_0\psi)}{(R^2 + a^2)^2} \qquad \ldots(40)$$

where $\psi = \dfrac{QR}{R^2 + a^2}$ and k is a constant of integration related to energy per unit mass of the test particle. The suffix 1 indicates differentiation w.r.t. r. Putting the values of A and ψ the equation (39) can be written as

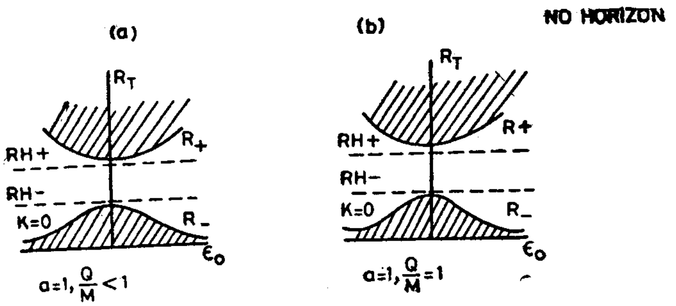


FIG. 7.

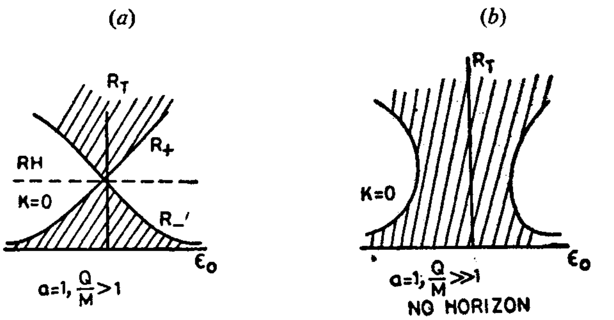


FIG. 8.

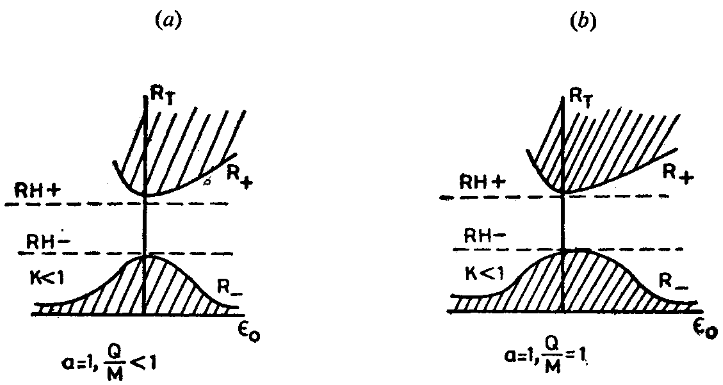
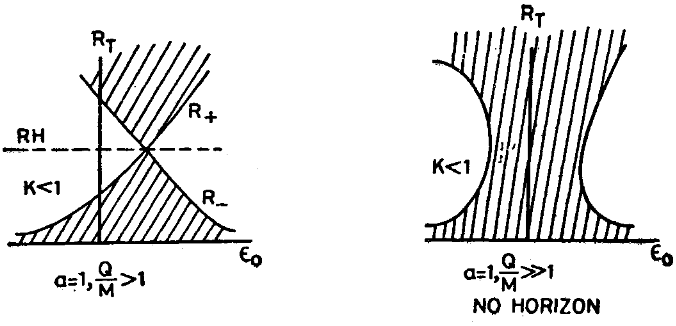


FIG. 9.

$$\left(\frac{dR}{d\tau}\right)^2 = \frac{(k^2-1)\,R^2}{(R^2+a^2)^2}\left[\left(\frac{R^2+a^2}{R}\right)^2 + \frac{2}{k^2-1}\,(M - k\epsilon_0 Q)\,\frac{R^2+a^2}{R} + \frac{\epsilon_0^2\,Q^2+2a^2-Q^2}{k^2-1} - \frac{Q^2-2a^2}{k^2-1}\,\frac{a^2}{R^2}\right] \qquad \ldots(41)$$



(*a*) (*b*)

FIG. 10.

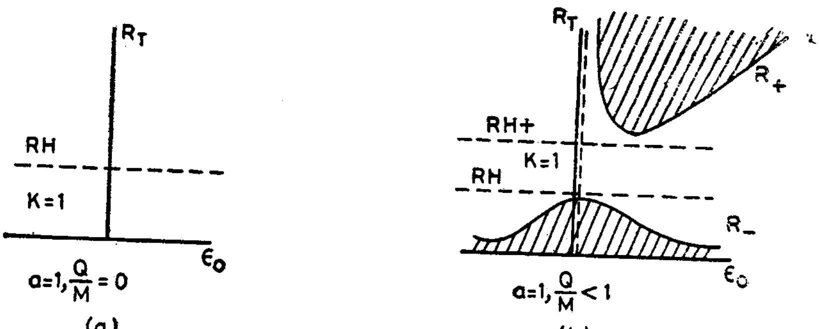


FIG. 11.

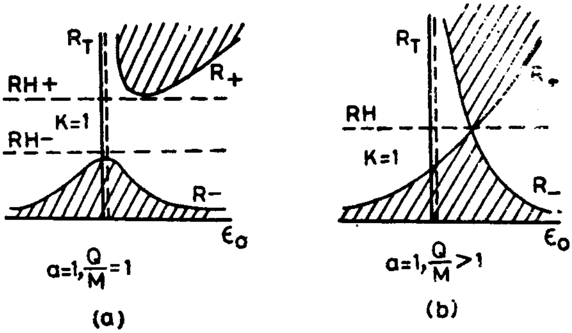


FIG. 12.

To study the properties of radially moving charged particles, we have plotted in the Figures 7-16 the turning radii R_T where $\frac{dR}{d\tau} = 0$ as a function of ϵ_0 for various

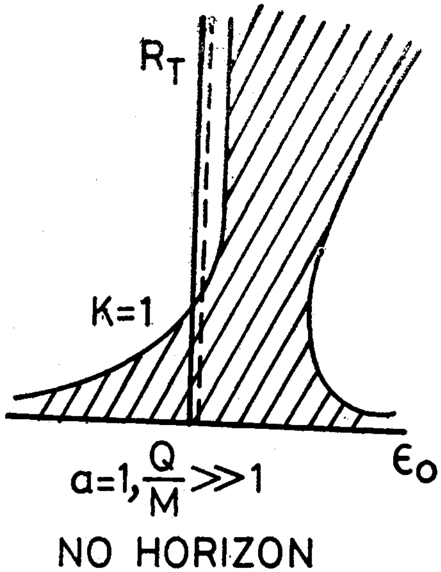


FIG. 13.

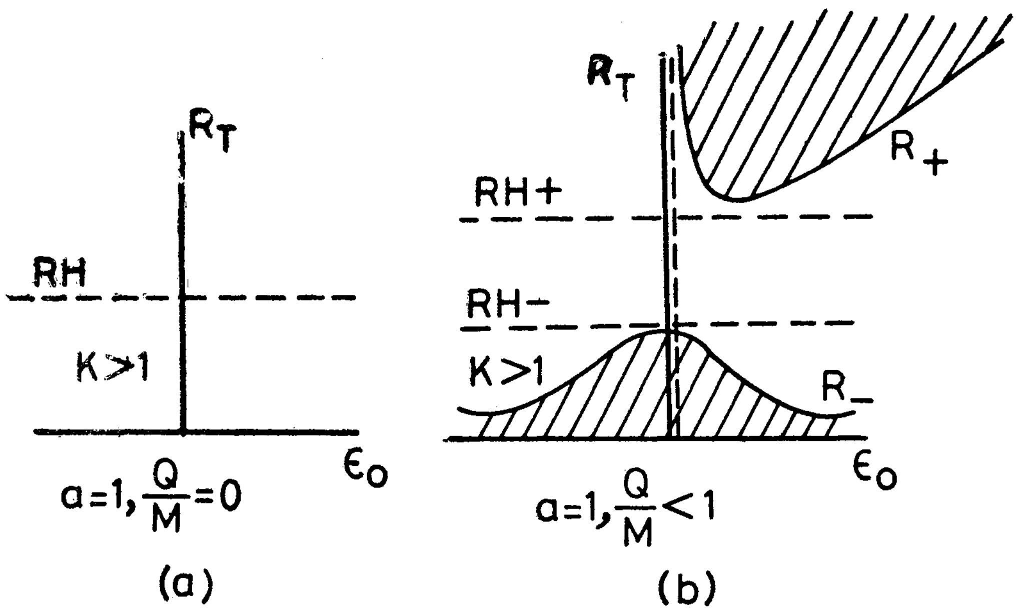


FIG. 14.

values of k and $\frac{Q}{M}$ with $a = 1$. The shaded regions in the figure are forbidden regions where $\left(\frac{dR}{d\tau}\right)^2 < 0$.

For $Q^2 \leqslant M^2 + a^2$, the horizons ($g_{44} = 0$) given by

$$R_{H\pm} = M\left[1 \pm \sqrt{1 + \frac{a^2}{M^2} - \frac{Q^2}{M^2}}\right]$$

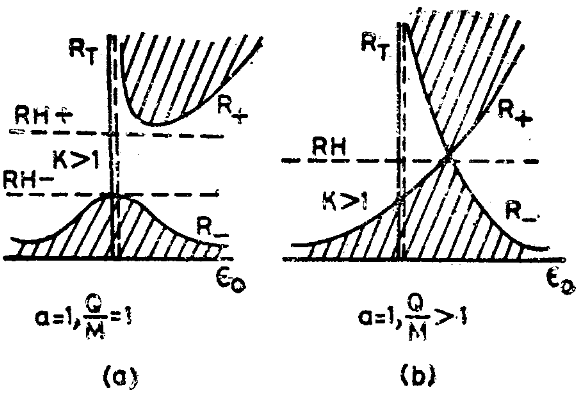


FIG. 15.

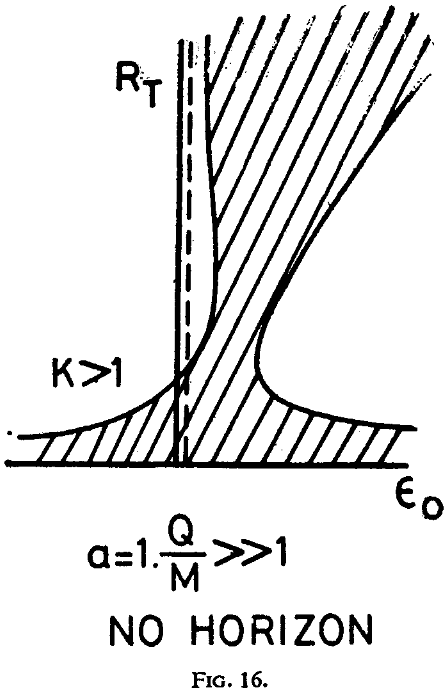


FIG. 16.

are shown by horizontal dashed lines in the figures. It is seen from the curves of the figures that for all values of k (for $k < 1$ only in the Reissner-Nordstrom field) radial oscillations of charged test particles occur in the allowed regions for all values of $\frac{Q}{M}$. Such radial oscillations have no classical (Newtonian) analogue. Classically, the force on a charged test particle points either outword or inword only and does not change direction. If $k < 1$, a test particle does not have sufficient energy to reach $R = \infty$ while if $k \geqslant 1$ a test particle can travel to $R = \infty$.

B. Stability of Naked Singularity against Bombardment of Charged Particles

Figs. 13 and 16 represent naked singularities. They may be bombarded with negatively charged test particles but they will all be turned back before reaching $R = 0$. This means that the nakedness of such a singularity is stable against this bombardment. This is contrary to the result obtained for the Reissner-Nordstrom field (Cohen & Gautreau, 1979). If however the source is not a singularity but has some size, then by such bombardment and event horizon as shown in Figs. (12*b*) and (16*b*) may be developed, because in this case the possibility of the test particles being turned back will be, for some values of ϵ_0, eliminated. All that has been said here hold also for axially moving charged test particles.

C. Non-Destructibility of Event Horizons

On the other hand, if an event horizon as shown in Figs. 11(*a*) and 16 already exists, it will not be destroyed as can be seen from Figs. 11(*b*) and 14(*b*) by bombardment of charged particles. This also holds for axially moving charged test particles.

Conclusion

We may now come to an interesting conclusion for charged test particles bombarding a charged NUT source in the equatorial plane and along the axis. An existing naked singularity cannot be destroyed by such bombardment to produce an event horizon nor can an existing event horizon be destroyed by this process to create a naked singularity.

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