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Thermoelasticity

THERMOELASTIC INTERACTIONS IN AN INFINITE ELASTIC SOLID WITH THERMAL RELAXATION

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In this paper, the generalised dynamical theory of thermoelasticity is employed to solve the problem of determination of the distribution of temperature, deformation, stress and strain, in an infinite isotropic elastic solid containing distributed time-dependent heat sources. The solutions are obtained by the use of the Laplace transform on time and the Fourier transform on space. Wave fronts and small time approximations are obtained and compared with the previous results deduced from classical coupled thermoelastic theory.

Keywords : Thermoelastic Interactions; Infinite Elastic Solid; Thermal Relaxation.

INTRODUCTION

PARIA (1968) considered a coupled thermoelastic problem concerning an infinite isotropic elastic solid containing distributed instantaneous heat sources. The solutions for temperature distribution and deformation consisted of two parts—one has the wave-like behaviour travelling with the velocity of the dilatational wave, while the other part is of diffused nature. The solutions for both temperature distribution and deformation were found to be continuous at the displacement wave front. In the present paper, the author has discussed a similar problem using the generalised dynamical theory of thermoelasticity of Lord and Shulman (1967) by introducing into the field equations the terms representing the thermal relaxation in time. It is seen that in presence of thermal relaxation in time, the solutions are non-diffusive in nature and the solutions for temperature, deformation, strain, stress consist of two waves propogating with velocities v_+ and v_- where $v_+ < v_-$.

As the expressions for temperature, strain etc. are not, in general, readily invertible with respect to the Laplace transform variable p, our attention is directed to the consideration of possible discontinuities at both the wave fronts and small time approximations. It is seen that though the displacement is continuous at both the wave fronts, temperature, strain and stress are discontinuous and the magnitudes of these discontinuities at the wave fronts are studied, which are in fact, valid for relatively short times. In the classical coupled theory, both temperature distribution and deformation are continuous at the displacement wave front. It may be mentioned that through transient boundary value problems relating to elastic half-space (Popov, 1967) and surface waves in generalised thermoelasticity (Puri, 1973; and Agarwal 1978) have been considered, the problem of an infinite elastic solid, as the author is aware, with distributed time-dependent heat sources considering thermal relaxation in time has not been discussed earlier.

STATEMENT OF THE PROBLEM: BASIC EQUATIONS

We consider a homogenous, isotropic and thermally conducting infinite elastic solid, unstrained and unstressed initially, with uniform reference temperature T^*. The infinite solid is then subjected to the heat sources distributed over a plane area. The problem is to discuss the subsequent distribution of temperature, deformation, strain and stress considering thermal relaxation in time and possible discontinuities at the wave fronts, which are valid for short times.

The basic equations of linear thermoelasticity with thermal relaxation are the following :

(*a*) The principle of balance of linear momentum gives the equations of motion (without the body forces)

$$\tau_{ij,j} = \rho \ddot{u}_i, \quad \begin{pmatrix} i = 1,2,3, \\ j = 1,2,3, \end{pmatrix} \qquad \text{...(1)}$$

(*b*) The principle of local energy balance gives the linearized energy equation

$$-q_{i,i} + Q = \rho c_v \dot{T} + \beta T^* \dot{\triangle} \qquad \text{...(2)}$$

(*c*) The modified form of Fourier's law of heat conduction considering the thermal relaxation in time (Lord & Shulman, 1967) is

$$q_i + \tau_0 \dot{q} = -K T_{,i} \qquad \text{...(3)}$$

(*d*) The stress-strain-temperature relation with the strain-displacement relations are

$$\tau_{ij} = (\lambda \triangle - \beta T)\, \delta_{ij} + 2\mu e_{ij} \qquad \text{...(4)}$$

and

$$2e_{ij} = u_{i,j} + u_{j,i} \qquad \text{...(5)}$$

where : τ_{ij} = components of stress tensor;

u_i = components of displacement vector;

$\triangle$ = dilatation = $u_{i,i}$;

q_i = components of heat-flux vector;

ρ = constant mass density;

C_v = specific heat of the solid at constant strain;

δ_{ij} = Kronecker delta;

T = increase of temperature above the reference, temperature, T^*;

λ, μ = Lame' constants;

α_t = coefficient of linear thermal-expansion;

τ_0 = thermal relaxation in time;

K = coefficient of thermal conductivity of the solid;

and Q is the source term.

A comma (,) followed by a suffix denotes differentiation with respect to that suffix and superposed dots designate differentiation with respect to time.

From equations (1), (4) and (5), on elimination of τ_{ij}, we obtain

$$\mu \nabla^2 u_i + (\lambda + \mu) \Delta_{,i} - (3\lambda + 2\mu) \alpha_t T_{,i} = \rho \ddot{u}_i, \; (i = 1, 2, 3) \qquad ...(6)$$

Elimination of q_i between (2) and (3) yields (Dhaliwal & Singh, 1980)

$$KT_{,ii} + Q + \tau_0 \dot{Q} = \rho C_v (\dot{T} + \tau_0 \ddot{T}) + (3 . \lambda + 2\mu) \alpha_t T^* (\dot{\Delta} + \tau_0 \ddot{\Delta}) \qquad ...(7)$$

Equations (6) and (7) are basic for the displacement and temperature fields in the generalized dynamical theory of thermoelasticity with thermal relaxation in time in the form of two coupled partial differential equations.

We seek the disturbances in the x direction so all quantities depend on spatial co-ordinates x and time t.

Let the heat sources be continuously distributed over the plane $x = 0$ and the solid occupy the whole space $-\infty < x < \infty$. The displacement vector $\vec{u}$ has the components $(u(x, t), 0, 0)$ parallel to the co-ordinate axes and the temperature increase is $T = T(x, t)$. Then the basic equations reduce to

$$(\lambda + 2\mu) \frac{\partial^2 u}{\partial x^2} = \rho \frac{\partial^2 u}{\partial t^2} + \beta \frac{\partial T}{\partial x} \qquad ...(7)'$$

$$K \frac{\partial^2 T}{\partial x^2} + Q + \tau_0 \dot{Q} = \rho C_v (\dot{T} + \tau_0 \ddot{T}) + \beta T^* (\dot{\Delta} + \tau_0 \ddot{\Delta}) \qquad ...(8)$$

We represent the strength of the heat source by $Q(x, t) = Q_0 \, \delta(x) \, H(t)$, where Q_0 is the constant strength of the source, $\delta(x)$ is Dirac's delta function and $H(t)$ is the Heaviside unit function.

We now introduce the following dimensionless qnantities,

$$\xi = \frac{ax}{\kappa}, \eta = \frac{a^2 t}{\kappa}, \theta = \frac{T}{T^*}, U = \frac{a(\lambda + 2\mu) u}{\kappa \beta T^*},$$

κ being the thermal diffusivity and $a^2 = \dfrac{\lambda + 2\mu}{\rho}$.

Then equations (7)′ and (8) reduce to the dimentionless forms

$$\frac{\partial^2 U}{\partial \xi^2} = \frac{\partial^2 U}{\partial \eta^2} + \frac{\partial \theta}{\partial \xi} \qquad \text{...(9)}$$

and

$$\frac{\partial^2 \theta}{\partial \xi^2} + K_0 \delta(\xi) H(\eta) + K_0 \tau_0 \delta(\xi) \delta(\eta)$$

$$= \frac{\partial \theta}{\partial \eta} + \tau_0' \frac{\partial^2 \theta}{\partial \eta^2} + \epsilon \frac{\partial^2 U}{\partial \xi \partial \eta} + \epsilon \tau_0' \frac{\partial^3 U}{\partial \xi \partial \eta^2} \qquad \text{...(10)}$$

where k_0 is a constant depending on relaxation constant τ_0', density and the thermal properties of the medium. Here $\epsilon = \beta^2 T^*/\rho C_v (\lambda + 2\mu)$ is the thermoelastic coupling constant and $\tau_0' = a^2 \tau_0/\kappa$.

Application of Laplace and Fourier Transforms for the Solution of the Problem in the Transformed Domain

We use Laplace transform

$$\bar{U}(\xi, p) = \int_0^\infty U(\xi, \eta) e^{-p\eta} d\eta, \; \bar{\theta}(\xi, p) = \int_0^\infty \theta(\xi, \eta) e^{-p\eta} d\eta$$

where p is Laplace parameter and use Fourier transform

$$\bar{U}_1 = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \bar{U} \exp(i\zeta\xi) \, d\xi, \; \bar{\theta}_1 = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \bar{\theta} \exp(i\zeta\xi) \, d\xi$$

where ζ is the Fourier transform parameter.

Then first applying Laplace transform on time and then by Fourier transform on space, we have

$$(p^2 + \zeta^2) \bar{U}_1 = i\zeta \bar{\theta}_1 \qquad \text{...(11)}$$

and

$$(p + \tau_0' p^2 + \zeta^2) \bar{\theta}_1 - (\epsilon p i\zeta + \epsilon \tau_0' p^2 i\zeta) \bar{U}_1 = \frac{K_0}{\sqrt{2\pi}} \left((\tau_0 + \frac{1}{p} \right) \qquad \text{...(12)}$$

Solving for $\bar{U}_1$, $\bar{\theta}_1$, we get

$$\bar{U}_1 = \frac{K_0 i\zeta \left(\tau_0 + \frac{1}{p} \right)}{\sqrt{2\pi} M} \quad \text{and} \quad \bar{\theta}_1 = \frac{K_0 \left(\zeta_0 + \frac{1}{p} \right) (p^2 + \zeta^2)}{\sqrt{2\pi} \; M}$$

where

$$M = (\tau_0' p^2 + p + \zeta^2)(p^2 + \zeta^2) + \zeta^2 (\epsilon p + \epsilon \zeta_0' p^2)$$

$$= (\zeta^2 + \zeta_3^2)(\zeta^2 + \zeta_4^2)$$

where $-\zeta_{3,4}^2$ are the roots of the equation

$$\zeta^4 + (k_1 + pk_2)\, p\zeta^2 + (p^3 + \tau_0' \, p^4) = 0$$

Thus

$$\tau_3^2 + \zeta_4^2 = p(k_1 + pk_2),$$

$$\zeta_3^2 \, \zeta_4^2 = p^3 + \tau_0' \, p^4, \; k_1 = 1 + \epsilon, \; k_2 = 1 + \epsilon\tau_0' + \tau_0'$$

$$\therefore \quad \zeta_{3,4}^2 = \frac{p}{2}\left[(k_1 + p\,k_2) \pm \sqrt{R}\,\right],$$

where $R = \{ (k_2^2 - 4\tau_0')\, p^2 + (2k_1k_2 - 4) \cdot p + k_1^2\}.$

We write $$\bar{\theta}_1 = \frac{k_0\left(\zeta_0 + \dfrac{1}{p}\right)}{\sqrt{2\pi}}\left[\frac{A}{\zeta^2 + \zeta_3^2} + \frac{B}{\zeta^2 + \zeta_4^2}\right]$$

where $$A = \frac{p^2 - \zeta_3^2}{\zeta_4^2 - \zeta_3^2}, \quad B = \frac{p^2 - \zeta_4^2}{\zeta_3^2 - \zeta_4^2}.$$

Also $$\bar{U}_1 = \frac{ik_0}{\sqrt{2\pi}} \; \frac{\left(\zeta_0 + \dfrac{1}{p}\right)}{(\zeta_4^2 - \zeta_3^2)}\left[\frac{\zeta}{\zeta^2 + \zeta_3^2} - \frac{\zeta}{\xi^2 + \zeta_4^2}\right]$$

By taking the Inverse Fourier transforms, we find

$$\bar{U}(\xi, p) = \frac{k_0\left(\zeta_0 + \dfrac{1}{p}\right)}{2\,(\zeta_4^2 - \zeta_3^2)}\,[e^{-\xi\zeta_3} - e^{-\xi\zeta_4}]$$

and

$$\bar{\theta}\,(\xi, p) = \frac{k_0\left(\zeta_0 + \dfrac{1}{p}\right)}{2\zeta_3\zeta_4}\,[\,A\,\zeta_4\, e^{-\xi\zeta_3} + B\,\zeta_3\, e^{-\xi\zeta_4}\,], \text{ for } \xi > 0.$$

As we are interested in small time approximations, we make use of Abel's theorem

$$\lim_{t \to 0} f(t) = \lim_{p \to \infty} \{ p\bar{f}(p)\}.$$

Small values of the time correspond to large values of the parameter p. We now expand $\bar{U}$, $\bar{\theta}$ in ascending power of $1/p$. For this, we put $s = 1/p$ and expand by Maclaurin's theorem about $s = 0$ and then replace $1/s$ by p to obtain

$$\zeta_{3,4} = \frac{p}{v_\pm} + \beta_\pm + \frac{\varphi_\pm}{p} + \text{terms of order } (1/p^2)$$

where $$\frac{1}{v_\pm} = \frac{1}{\sqrt{2}}\,\{k_2 \pm \sqrt{k_2^2 - 4\tau_0'}\,\}^{1/2},$$

$$\beta \pm = \frac{\frac{1}{2\sqrt{2}}\left\{ k_1 \pm \frac{1}{2}\,\frac{2k_1k_2 - 4}{\sqrt{k_2^2 - 4\tau_0'}} \right\}}{\{ k_2 \pm \sqrt{k_2^2 - 4\,\tau_0'}\,\}^{1/2}}$$

$$\varphi_\pm = \frac{1}{4\sqrt{2}\,\{k_2 \pm \sqrt{(k_2^2 - 4\tau_0')}\}^{3/2}} \Big[\pm \{k_2 \pm \sqrt{k_2^2 - 4\tau_0'}\,\}\times$$

$$\left\{ \frac{2k_1^2}{(k_2^2 - 4\tau_0')^{1/2}} - \frac{\frac{1}{2}\,(2k_1k_2 - 4)^2}{(k_2^2 - 4\tau_0')^{3/2}} \right\} \Big]$$

$$\therefore \qquad \zeta_3 = \frac{p}{v_+} + \beta_+ + \frac{\varphi_+}{p} + 0\,(1/p^2)$$

and $$\zeta_4 = \frac{p}{v_-} + \beta_- + \frac{\varphi_-}{p} + 0\,(1/p^2)$$

Also $$\frac{2}{v_\pm^2} = 1 + \epsilon\tau_0' + \tau_0' \pm \sqrt{\Gamma}\ ,$$

where $$\Gamma = k_2^2 - 4\,\tau_0' = (1 + \epsilon\tau_0' + \tau_0')^2 - 4\,\tau_0'$$

$$= (1 + \epsilon\tau_0' - \tau_0')^2 + 4\epsilon\tau_0'^2,$$

so that Γ is a positive quantity.

Since $\tau_0' > 0,\ (1 + \epsilon\tau_0' + \tau_0')^2 > \Gamma$

i.e. $1 + \epsilon\tau_0' + \tau_0' > \Gamma$

$\therefore \quad \frac{1}{v_+^2} > \frac{1}{v_-^2}$ or, $v_+ < v_-$ where v_+ is the velocity of the slowest wave and v_- corresponds to fastest wave.

Now $\frac{1}{\zeta_4^2 - \zeta_3^2} = -\frac{1}{p\sqrt{R}} \simeq \left[-\frac{1}{p^2}\cdot\frac{1}{\sqrt{\Gamma}} + \frac{k_1k_2 - 2}{\Gamma^{3/2}}\cdot\frac{1}{p^3} + 0\left(\frac{1}{p^4}\right) \right]$, for large p.

We thus find

$$\bar{U}\,(\xi, p) = \frac{k_0\left(\tau_0 + \frac{1}{p}\right)}{2\,(\zeta_4^2 - \zeta_3^2)}\,[e^{-\xi\zeta_3} - e^{-\xi\zeta_4}]$$

$$\simeq \frac{k_0}{2}\left[-\frac{\tau_0}{\sqrt{\Gamma}}\,\frac{1}{p^2} + \left\{ \frac{(k_1k_2 - 2)\,\tau_0}{\Gamma^{3/2}} - \frac{1}{\sqrt{\Gamma}} \right\} \frac{1}{p^3} - 0 \right.$$

$$\left. + 0\left(\frac{1}{p^4}\right) \right] \times \left[\exp\left(-\xi\left(\frac{p}{v_+} + \beta_+\right)\right) - \exp\left(-\xi\left(\frac{p}{v_-}\right) + \beta_-\right) \right].$$

Again

$$\frac{A}{\zeta_3} \simeq -\nu_+\Bigg[\left(1-\frac{k_2+\sqrt{\Gamma}}{2}\right)\cdot\frac{1}{\sqrt{\Gamma}}\cdot\frac{1}{p}-$$

$$\left\{\frac{1}{2\sqrt{\Gamma}}\left(k_1-\frac{k_1k_2-2}{\sqrt{\Gamma}}\right)+\frac{\beta_+\nu_+}{\sqrt{\Gamma}}\left(1-\frac{k_2+\sqrt{\Gamma}}{2}\right)\right.$$

$$\left.+\frac{(k_1k_2-2)}{\Gamma^{3/2}}\left(1-\frac{k_2+\sqrt{\Gamma}}{2}\right)\right\}\frac{1}{p^2}+0\left(\frac{1}{p^3}\right)\Bigg]$$

Similar expressions can be obtained for B/ζ_4 and $\bar{\theta}$ (ξ, p).

Thus

$$\frac{B}{\zeta_4} \simeq \nu_-\Bigg[\left(1-\frac{k_2+\sqrt{\Gamma}}{2}\right)\frac{1}{\sqrt{\Gamma}}\cdot\frac{1}{p}-\left\{\tfrac{1}{2}\left(k_1+\frac{k_1k_2-2}{\sqrt{\Gamma}}\right)\frac{1}{\sqrt{\Gamma}}\right.$$

$$+\frac{\beta_-\nu_-}{\sqrt{\Gamma}}\left(1-\frac{k_2+\sqrt{\Gamma}}{2}\right)+\frac{(k_1k_2-2)}{\Gamma^{3/2}}\times$$

$$\left.\left(1-\frac{k_2+\sqrt{\Gamma}}{2}\right)\right\}\frac{1}{p^2}+0\left(\frac{1}{p^3}\right)\Bigg].$$

$$\bar{\theta}=\frac{k_0}{2}\left(\tau_0+\frac{1}{p}\right)\left[\frac{A}{\zeta_3}e^{-\xi\zeta_3}+\frac{B}{\zeta_2}e^{-\xi\zeta_4}\right],$$

$$\simeq -\frac{k_0\nu_+}{2}\Bigg[\frac{\tau_0}{\sqrt{\Gamma}}\left(1-\frac{k_2+\sqrt{\Gamma}}{2}\right)\frac{1}{p}-\zeta_0\left\{\frac{1}{2\sqrt{\Gamma}}\left(k_1-\frac{k_1k_2-2}{\sqrt{\Gamma}}\right)\right.$$

$$\left.+\frac{\beta_+\nu_+}{\sqrt{\Gamma}}\left(1-\frac{k_2+\sqrt{\Gamma}}{2}\right)+\frac{k_1k_2-2}{\Gamma^{3/2}}\left(1-\frac{k_2+\sqrt{\Gamma}}{2}\right)\right\}\frac{1}{p^2}$$

$$+0\left(\frac{1}{p^3}\right)\zeta_0+\left(1-\frac{k_2+\sqrt{\Gamma}}{2}\right)\frac{1}{\sqrt{\Gamma}}\cdot\frac{1}{p^2}-0\left(\frac{1}{p^3}\right)\Bigg]$$

$$\exp\left\{-\xi\left(\frac{p}{\nu_+}+\beta_+\right)\right\}+\frac{k_0}{2}\cdot\nu_-\Bigg[\frac{\tau_0}{\sqrt{\Gamma}}\left(1-\frac{k_2+\sqrt{\Gamma}}{2}\right)\frac{1}{p}$$

$$-\tau_0\left\{\tfrac{1}{2}\left(k_1+\frac{k_1k_2-2}{\sqrt{\Gamma}}\right)\cdot\frac{1}{\sqrt{\Gamma}}+\frac{\beta_-\nu_-}{\sqrt{\Gamma}}\left(1-\frac{k_2+\sqrt{\Gamma}}{2}\right)\right.$$

$$\left.+\frac{k_1k_2-2}{\Gamma^{3/2}}\left(1-\frac{k_2+\sqrt{\Gamma}}{2}\right)\right\}\frac{1}{p^2}+0\left(\frac{1}{p^3}\right)\cdot\tau_0$$

$$+\left(1-\frac{k_2+\sqrt{\Gamma}}{2}\right)\cdot\frac{1}{\sqrt{\Gamma}}\cdot\frac{1}{p^2}+0.\left(\frac{1}{p^3}\right)\Bigg]$$

$$\times\exp\left\{-\xi\left(\frac{p}{\nu_-}+\beta_-\right)\right\}$$

On making use of these expansions, we obtain for the non-dimensionless stress

$$\bar{\sigma}_{\xi\xi} = \frac{d\bar{U}}{d\xi} - \bar{\theta}$$

$$\simeq \frac{k_0}{2} \cdot \exp(-\xi\beta_+)\left[\frac{v_+}{\sqrt{\Gamma}} \cdot \tau_0 \cdot \frac{\exp\left(\frac{-\xi p}{v_+}\right)}{p}\right.$$

$$- \tau_0\left(\frac{k_1k_2 - 2}{\Gamma^{3/2}} \cdot v_+ + \frac{v_+^2\,\beta_+}{\sqrt{\Gamma}}\right) \times \frac{\exp\left(\frac{-\xi p}{v_+}\right)}{p^2} + v_+ \cdot \frac{1}{\sqrt{\Gamma}} \times$$

$$\left.\frac{\exp\left(\frac{-\xi p}{v_+}\right)}{p^2} + 0\left(\frac{1}{p^3}\right)\right] - \frac{k_0}{2}\exp(-\xi\beta_-)$$

$$\left[\frac{v_-}{\sqrt{\Gamma}} \cdot \tau_0 \frac{\exp\left(\frac{-\xi p}{v_-}\right)}{p} - \tau_0\left\{\frac{k_1k_2 - 2}{\Gamma^{3/2}} \cdot v_- + \frac{v_-^2\beta_-}{\sqrt{\Gamma}}\right\} \times\right.$$

$$\left.\frac{\exp\left(\frac{-\xi p}{v_-}\right)}{p^2} \cdot + v_- \frac{1}{\sqrt{\Gamma}} \cdot \frac{\exp\left(\frac{-\xi p}{v_-}\right)}{p^2} + 0\left(\frac{1}{p^2}\right)\right].$$

We find for the strain $\epsilon(\xi, \eta)$,

$$\bar{\epsilon} = \frac{\beta T^*}{\lambda + 2\mu} \frac{d\bar{U}}{d\xi}$$

$$\simeq \frac{\beta T^*}{\lambda + 2\mu} \cdot \frac{k_0}{2}\left[\tau_0\left\{\frac{1}{v_+\sqrt{\Gamma}} \cdot \frac{1}{p} + \frac{1}{p^2}\left(\frac{\beta_+}{\sqrt{\Gamma}} - \frac{k_1k_2 - 2}{v_+\,\Gamma^{3/2}}\right.\right.\right. + \text{terms}$$

$$\text{of order}\left.\left.\left(\frac{1}{p^3}\right)\right\} + \frac{1}{v_+\sqrt{\Gamma}} \cdot \frac{1}{p^2} + \text{term of order } (1/p^3\right]\exp\left\{-\xi\left(\frac{p}{v_+} + \beta_+\right)\right\}$$

$$\frac{-\beta T^*}{\lambda + 2\mu} \cdot \frac{k_0}{2}\left[\tau_0 \cdot \left\{\frac{1}{v_-\sqrt{\Gamma}} \cdot \frac{1}{p} + \frac{1}{p^2}\left(\frac{\beta_-}{\sqrt{\Gamma}} - \frac{k_1k_3 - 2}{v_- \cdot \Gamma^{3/2}}\right) + \text{terms of}\right.\right.$$

$$\text{order}\quad \left.\left(\frac{1}{p^3}\right) + \frac{1}{v_-\sqrt{\Gamma}} \cdot \frac{1}{p^2} + 0\left(\frac{1}{p^3}\right)\right]\exp -\left\{\xi\left(\frac{p}{v_-} + \beta_-\right)\right\}$$

Inverting with respect to Laplace transform, we arrive at the following expressions for U, θ, ϵ, $\sigma_{\xi\xi}$ valid respectively for relatively short times.

$$U(\xi,\eta) \simeq \frac{k_0}{2}\left[-\frac{\tau_0}{\sqrt{\Gamma}} \cdot (\eta - \xi \cdot v_+^{-1})\, H(\eta - \xi\, v_+^{-1}) \cdot e^{-\xi\beta_+}\right.$$

$$\left.+ \frac{\tau_0}{\sqrt{\Gamma}}(\eta - \xi \cdot v_-^{-1}) \cdot H(\eta - \xi \cdot v_-^{-1}) \cdot e^{-\xi\beta_-}\right]$$

$$\theta(\xi,\eta) \simeq \frac{k_0}{2} \cdot \nu_+ \cdot e^{-\xi\beta_+} \cdot \Big[-\frac{\tau_0}{\sqrt{\Gamma}}\Big(1 - \frac{k_2+\sqrt{\Gamma}}{2}\Big) \cdot H(\eta - \xi \cdot \nu_+^{-1})$$

$$+ \tau_0 \Big\{ \frac{1}{2\sqrt{\Gamma}}\Big[k_1 - \frac{k_1 k_2 - 2}{\sqrt{\Gamma}}\Big) + \frac{\beta_+ \nu_+}{\sqrt{\Gamma}}\Big(1 - \frac{k_2+\sqrt{\Gamma}}{2}\Big)$$

$$+ \frac{k_1 k_2 - 2}{\Gamma^{3/2}}\Big(1 - \frac{k_2\sqrt{\Gamma}}{2}\Big)\Big\} (\eta - \xi \cdot \nu_+^{-1}) \cdot H(\eta - \xi \cdot \nu_+^{-1})$$

$$-\Big(1 - \frac{k_2+\sqrt{\Gamma}}{2}\Big) \cdot \frac{1}{\sqrt{\Gamma}} (\eta - \xi \cdot \nu_+^{-1}) H(\eta - \xi \cdot \nu_+^{-1}) \Big]$$

$$+ \frac{k_0}{2} \nu_- \cdot e^{-\xi\beta_-} \cdot \Big[\frac{\tau_0}{\sqrt{\Gamma}}\Big(1 - \frac{k_2+\sqrt{\Gamma}}{2}\Big) H(\eta - \xi \nu_-^{-1}) - \tau_0$$

$$\times \Big\{ \frac{1}{2\sqrt{\Gamma}}\Big(k_1 + \frac{k_1 k_2 - 2}{\sqrt{\Gamma}}\Big) + \frac{\beta_- \nu_-}{\sqrt{\Gamma}}\Big(1 - \frac{k_2+\sqrt{\Gamma}}{2}\Big)$$

$$+ \frac{k_1 k_2 - 2}{\Gamma^{3/2}}\Big(1 - \frac{k_2+\sqrt{\Gamma}}{2}\Big)\Big\} (\eta - \xi . \nu_-^{-1}) H(\eta - \xi . \nu_-^{-1})$$

$$+\Big(1 - \frac{k_2+\sqrt{\Gamma}}{2}\Big) \cdot \frac{1}{\sqrt{\Gamma}} \cdot (\eta - \xi \cdot \nu_-^{-1}) \cdot H(\eta - \xi \cdot \nu_-^{-1}) \Big].$$

$$\epsilon \simeq \frac{\beta T^*}{\lambda + 2\mu} \cdot \frac{k_0}{2} \cdot \Big[e^{-\xi\beta_+} \cdot \Big\{ \frac{\tau_0}{\nu_+ \sqrt{\Gamma}} \cdot H(\eta - \xi \cdot \nu_+^{-1})$$

$$+ \Big(\frac{\beta_+}{\sqrt{\Gamma}} - \frac{k_1 k_2 - 2}{\nu_+ . \Gamma^{3/2}}\Big) (\eta - \xi \cdot \nu_+^{-1}) \cdot H(\eta - \xi \cdot \nu_+^{-1})$$

$$+ \frac{1}{\nu_+ . \sqrt{\Gamma}} \cdot \Big(\eta - \xi . \nu_+^{-1}\Big) \cdot H\Big(\eta - \xi \cdot \nu_+^{-1}\Big)\Big\}$$

$$- \Big\{ \frac{\tau_0}{\nu_- \sqrt{\Gamma}} \cdot H\Big(\eta - \xi \cdot \nu_-^{-1}\Big) + \Big(\frac{\beta_-}{\sqrt{\Gamma}} - \frac{k_1 k_2 - 2}{\nu_- \Gamma^{3/2}}\Big) \times$$

$$\Big(\eta - \xi \cdot \nu_-^{-1}\Big). H\Big(\eta - \xi \cdot \nu_-^{-1}\Big) + \frac{1}{\nu_- \sqrt{\Gamma}} \cdot \Big(\eta - \xi . \nu_-^{-1}\Big).$$

$$H\Big(\eta - \xi \cdot \nu_-^{-1}\Big)\Big\} e^{-\xi \beta_-}\Big]$$

$$\sigma_{\xi\xi} \simeq \frac{k_0}{2}\Big[e^{-\xi \cdot \beta_+} \Big\{ \frac{\nu_+}{\sqrt{\Gamma}} \cdot \tau_0 \cdot H\Big(\eta - \xi . \nu_+^{-1}\Big) - \tau_0 \Big(\frac{k_1 k_2 - 2}{\Gamma^{3/2}} \nu_+$$

$$+ \frac{\nu_+^2 \beta_+}{\sqrt{\Gamma}}\Big(\eta - \xi \cdot \nu_+^{-1}\Big) \cdot H\Big(\eta - \xi \cdot \nu_+^{-1}\Big) + \nu_+ \cdot \frac{1}{\sqrt{\Gamma}} \times$$

$$\Big(\eta - \xi \cdot \nu_+^{-1}\Big) \cdot H\Big(\eta - \xi \cdot \nu_+^{-1}\Big)\Big\} - e^{-\xi \cdot \beta_-} \cdot \Big\{ \frac{\nu_-}{\sqrt{\Gamma}} \cdot \tau_0$$

(*equation continued on p.* 508)

$$H\left(\eta - \xi \, . \, v_-^{-1}\right) - \tau_0\left(\frac{k_1 k_2 - 2}{\Gamma^{3/2}} \, . \, v_- + \frac{v_-^2 \, . \, \beta_-}{\sqrt{\Gamma}}\right)\left(\eta - \xi \, . \, v_-^{-1}\right).$$

$$H\left(\eta - \xi \, . \, v_-^{-1}\right) + \frac{v_-}{\sqrt{\Gamma}} \cdot \left(\eta - \xi \, . \, v_-^{-1}\right).$$

$$H\left(\eta - \xi \, . \, v_-^{-1}\right)\Big\}\Big].$$

Discussion

In the above expressions valid for short times, we observe that the terms containing $H\left(\eta - \xi \, . \, v_-^{-1}\right)$ gives the contribution of the fastest wave v_- near its wave fronts i.e., when $v_- \eta \simeq \xi$ and those with $H\left(\eta - \xi \, . \, v_+^{-1}\right)$ represent contribution of the slowest wave v_+ in the vicinity of the wave front $v_+ \eta \simeq \xi$. The above expressions, valid for small times, indicate jumps in temperature, strain and stress but not in deformation which is continuous at both the wave fronts, $\xi = \eta v_{+,-}$. The jumps are defined by $[(\theta)^+ - (\theta)^-]_{\xi=\eta v_+}$ at the wave front $\xi = \eta \, v_+$ and by $[(\theta)^+ - (\theta)^-]_{\xi=\eta v_-}$ at the wave frout $\xi = \eta \, v_-$. We observe that

$$[(\theta)^+ - (\theta)^-]_{v_-\eta=\xi} = \frac{k_0}{2} \, . \, v_- \, . \, e^{-v_-\eta\beta_-}\left(1 - \frac{k_2 + \sqrt{\Gamma}}{2}\right) \cdot \frac{\tau_0}{\sqrt{\Gamma}}.$$

and

$$[(\theta)^+ - (\theta)^-]_{v_+\eta=\xi} = -\frac{k_0}{2} \, . \, v_+ \, . \, e^{-v_+\eta\beta_+}\left(1 - \frac{k_2 + \sqrt{\Gamma}}{2}\right) \cdot \frac{\tau_0}{\sqrt{\Gamma}}.$$

$$[(\epsilon)^+ - (\epsilon)^-]_{v_-\eta=\xi} = -\frac{\beta T^*}{\lambda + 2\mu} \cdot \frac{k_0}{2} \, . \, e^{-v_-\eta\beta_-} \, . \, \frac{\tau_0}{v_- . \sqrt{\Gamma}}.$$

$$[(\epsilon)^+ - (\epsilon)^-]_{v_+\eta=\xi} = \frac{\beta T^*}{\lambda + 2\mu} \cdot \frac{k_0}{2} \, . \, e^{-v_+\eta\beta_+} \, . \, \frac{\tau_0}{v_+\sqrt{\Gamma}}.$$

$$[(\sigma_{\xi\xi})^+ - (\sigma_{\xi\xi})^-]_{v_-\eta=\xi} = -\frac{k_0}{2} \, . \, e^{-v_-\eta\beta_-} \, . \, \frac{v_-}{\sqrt{\Gamma}} \, . \, \tau_0.$$

and

$$[(\sigma_{\xi\xi})^+ - (\sigma_{\xi\xi})^-]_{v_+\eta=\xi} = \frac{k_0}{2} \, . \, e^{-v_+\eta\beta_+} \, . \, \frac{v_+}{\sqrt{\Gamma}} \, . \, \tau_0.$$

In the absence of relaxation time, $\tau_0' = 0$, then $\Gamma = 1$ and $k_2 = 1$. Then there is no jump in temperature at the wave fronts which shows that θ is continuous at both the wave fronts. The results then agree with classical coupled thermoelastic theory.

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