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Plasma Physics

COLLISIONAL ABSORPTION OF INTENSE ELECTROMAGNETIC RADIATION PROPAGATING IN EXTRAORDINARY MODE

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Inverse bremsstrahlung (IB) is an important mechanism to heat the plasma in laser fusion process. Since there is a self generated d.c. magnetic field with its direction perpendicular to the polarization and propagation vactors of the incident laser radiations, the effect of magnetic field on IB process needs investigation. We have derived the expressions for the effective electron-ion collision frequency in the weak ($\nu_{Th} >> \nu_E$) and strong ($\nu_{Th} << \nu_E$) field regimes (where ν_{Th} is the thermal velocity of the electrons and ν_E is the velocity of oscillation of the electrons in high frequency field). The explicit expressions for the time averaged power absorbed per unit volume in the two field regimes in presence of magnetic field have been obtained. Our results show that the effect of magnetic field is to increase the absorption in the weak field regime and to decrease the same in the strong field regime.

Keywords : Intense Electromagnetic Radiation; Inverse Bremsstrahlung; Laser Fusion; Magnetic Field

INTRODUCTION

THE study of absorption of laser beams has good prospects owing to its immense practical utility in the generation of controlled thermonuclear power (Engelhardt *et al.*, 1972, Dawson, 1964, and Basov *et al.*, 1964). The laser energy is deposited in the plasma via process like resonance absorption (Woo *et al.*, 1978) and plasma instabilities (Kaw & Dawson 1969). It has also been emphasised that the absorption of energy in a real plasma also occurs via inverse bremsstrahlung (IB) (Johnson & Dawson 1973). In IB process a plasma electron gains energy from electromagnetic field of the leser beam by absorbing a laser photon during collision with a nucleus. The most complete classical calculation of IB is due to Silin (1965). He has developed a nonlinear theory of the conductivity of fully ionized plasma in a strong high frequency field and has investigated the dependence of the nonlinear current on the polarization and

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on the constant magnetic field. Catto and Thomas (1977) have derived Silin's result in a simple manner using Lorentz collision operator appropriate for Merkoffian description. Seely (1974) has studied the laser heating of plasma electrons in a uniform static magnetic field via IB process using quantum mechanical approach and has found that the multiphoton absorption coefficient decreases at laser frequencies close to electron cyclotron frequencies. A similiar problem has also been studied by Ford and Connell (1980) using Drude model resulting int he dependence of the absorption rate on the relative orientation of the polarization vector of the laser beam in a magnetoplasma. Both Seely (1974) and Ford and Connell (1980) have con sidered the propagation of the electromagnetic beam perallel to the magnetic field where asin the present paper we have considered the propagation of the electromagnetic beam perpendicular to the magnetic field in the extraordinary mode. Our case is more realistic because self generated d.c. megnetic field with its direction perpendicular to the polarization and propagation vectors of the incident laser beam has been experimentally measured (Stamper & Ripin 1975, Stamper *et al.*, 1978, and Sakagami *et al.*, 1979) and theoretically predicted (Woo and De Groot 1978, Max *et al.* 1978) in laser pellet experiments. Our aim in this paper is to investigate the effect of magnetic field on the IB process when the laser beam is propagating prependicular to the magnetic field in extraordinary mode.

In section 2, an expression for current density in terms of basic frequencies has been obtained by making use of Boltzmann equation. We have also derived the expressions for the effective electron-ion collision frequency in the weak ($v_{Th} \gg v_E$) and strong ($v_{Th} \ll v_E$) field regimes (where v_{Th} is the thermal velocity of the electrons and v_E is the velocity of oscillation of the electrons in the high frequency field). In section 3, explicit expressions for the time averaged power absorbed per unit volume in the two field regimes have been obtained. Discussion of results has been given in Section 4. Our results show that the effect of magnetic field is to increase the absorption in the weak field regime and to decrease in the strong field regime. This behaviour can be explained by noting that in the strong field regime directed motion dominates over the thermal motion and consequently electron ion collision frequency gets modified. This causes reduction in power absorbed in strong field regime. However in the weak field regime the magnetic field influences the photon propagation by affecting the plasma frequency and therefore the power absorption. It should be noted that magnetic field does not affect electron ion collision frequency in the weak regime.

General Formalism : Calculation of Current Density and Electron Ion Collision Frequency

Consider an extraordinary electromagnetic pump wave $E = E_x\hat{x} + E_y\hat{y})\, e^{i(\omega t - k0z)}$ propagating in a plasma with static magnetic field B along Z-axis. In the presence of the beam the electron distribution is governed by Boltzmann transfer equation which in the presence of the static magnetic field can be written as

$$\frac{\partial f}{\partial t} + v.\Delta_r f + \frac{e}{m}\left(\mathbf{E} + \frac{1}{C}\mathbf{v} \times B\right)\Delta_v f + S = 0 \qquad \ldots(1)$$

where $\mathbf{r}$ is the spatial coordinate vector, $\mathbf{v}$ is the velocity of electrons, m is the mass of the electron, $\mathbf{E}$ is the electric field of the pump wave, S is the collision integral that determines the change of function f resulting from the collisions of the electrons with ions.

The distribution function f satisfies the condition,

$$\int_{-\infty}^{\infty} f(t, \mathbf{r}, \mathbf{v})\, d\mathbf{v} = n$$

where n is the concentration of the electrons at point $\mathbf{r}$, at time t.

In equilibrium and in the absence of the fields, the distribution is well known to be Maxwellian,

$$f = f_{00}(v) = n\left(\frac{m}{2\pi kT}\right)^{3/2} e^{-mv^2/2kT}$$

Also $\int f_{00}(v)\, d\mathbf{v} = 4\pi \int f_{00}\, v^2 dv = n$

The electron distribution can be written as

$$f(v) = f_0(v) + \mathbf{f}_1(v).\frac{\mathbf{v}}{v} \qquad \ldots(2)$$

$$|\, \mathbf{f}_1(v)\, | \ll f_0(v)$$

Using eqn. (2) in eqn. (1) the perturbed part $\mathbf{f}_1$ satisfies the following equation,

$$\frac{\partial \mathbf{f}_1}{\partial t} + \frac{e\mathbf{E}}{m}.\frac{\partial f_{00}}{\partial v} + \frac{e}{mc}\mathbf{B} \times \mathbf{f}_1 + \nu \mathbf{f}_1 = 0 \qquad \ldots(3)$$

where electron-electron collisions have been neglected because the dominant role is played only by the electron-ion collisions. Considering the ions to be fixed the electron-ion collision frequency ν is given by

$$\nu = N \left\langle \int d\Omega v(1 - \cos \chi) \frac{dQ}{d\Omega} \right\rangle \qquad \ldots(4)$$

where N represents the number density of ions and $\frac{dQ}{d\Omega}$ is the differential cross section for scattering of electrons moving with velocity v through angle χ. The cross-section for non-relativistic electrons is given by

$$\frac{dQ}{d\Omega} = \left[\frac{Z.e^2}{2mv^2 \sin^2 \frac{\chi}{2}}\right]^2 \qquad \ldots(5)$$

Using eqn. (5) is eqn. (4) we obtain

$$\nu(v) = 2\pi N \left(\frac{Ze^2}{m}\right)^2 \left\langle \frac{L}{v^3} \right\rangle$$

where L is coulomb logrithm

$$L = -2 \text{ in } \sin\frac{\chi_{min}}{2};\ \sin\frac{\chi_{min}}{2} \simeq \frac{ze^2}{mv^2}\left(\frac{4\pi \text{ Noe. } e^2}{KT}\right)^{1/2}$$

The angle χ_{min} is the minimum scattering angle corresponding to maximum impact parameter. Maximum impact parameter to is taken be the Debye length. As L is a weakly dependent function of velocity, ν veries as v^{-3}. The velocity consists of two contributions, one from the random thermal motion and other from the directed motion due to electric field.

Thus $\mathbf{v} = \mathbf{v}_{Th} + \mathbf{v}_D$

In case of nondegenerate magnetoplasma, the thermal velocity has a root mean value given by

$$v_{Th} = \left(\frac{3kT}{m}\right)^{1/2}$$

The directed velocity is given by

$$v_D^2 = v_x v_x^* + v_y v_y^* \qquad ...(6)$$

where v_x and v_y are the components of pump wave induced velocity and are given by

$$v_x = \frac{e}{m(\nu + i\omega)}\left[E_x + \frac{\omega_c}{(\nu + i\omega)}E_y\right]\left[1 + \frac{\omega_o^2}{(\nu + i\omega)^2}\right]^{-1} \qquad ...(7)$$

$$v_y = \frac{e}{m(\nu + i\omega)}\left[E_y - \frac{\omega_c}{(\nu + i\omega)}E_x\right]\left[1 + \frac{\omega_o^2}{(\nu + i\omega)^2}\right]^{-1} \qquad ...(8)$$

where $\omega_c = \dfrac{eB}{mc}$ = Electron cyclotron frequency. For upper hybrid mode to propagate,

$$E_x = -\left[\frac{\omega_c\omega_p^2\ \{2\nu\omega - i(\omega^2 - \omega_c^2)\}}{\omega(\omega^2 - \omega_c^2)(\omega^2 - \omega_{uh}^2) - i\nu(\omega^2 + \omega_c^2)\omega_p^2}\right]E_y \qquad ...(9)$$

where $\omega_{uh} = \left(\omega_o^2 + \omega_p^2\right)^{1/2}$ = Upper hybrid frequency.

$\omega_p = \left(\dfrac{4\pi ne^2}{m}\right)^{1/2}$ = Plasma frequency.

Using eqns. (7) and (8) in eqn (6) and then using eqn. (9) we obtain (for $\nu \ll \omega$),

$$v_D^2 = v_E^2\left\{\frac{\omega^2\left(\omega^2 + \omega_o^2\right)}{\left(\omega^2 - \omega_o^2\right)^2}\right\} \qquad ...(10)$$

where

$$v_E = \frac{eE}{m\omega} \qquad \ldots(11)$$

The ensemble averaged value of velocity is given by

$$\langle v^2 \rangle = v_{\mathrm{Th}}^2 + v_E^2 \left\{ \frac{\omega^2 \left(\omega^2 + \omega_c^2 \right)}{\left(\omega^2 - \omega_c^2 \right)^2} \right\} \qquad \ldots(12)$$

In the weak field regime $v_E \ll v_{\mathrm{Th}}$ so that the effective electron-ion collision frequency is essentially proportional to v_{Th}^{-3} and is given by

$$\nu_w = \frac{2\pi NLZ^2e^4}{m^{1/2}\{3kT\}^{3/2}} \qquad \ldots(13)$$

In the strong field regime $v_E \gg v_{\mathrm{Th}}$ so that the effective electron-ion collision frequency is proportional to v_D^{-3} and is given by

$$\nu_s = \nu_E \left\{ \frac{\omega^2 - \omega_c^2}{\omega \left(\omega^2 + \omega_c^2 \right)^{1/2}} \right\}^3 \qquad \ldots(14)$$

where

$$\nu_E = \frac{2\pi NLemZ^2\omega^3}{E_y^3} \qquad \ldots(15)$$

Now from eqn. (3)

$$f_{1x} \pm i f_{1y} = \frac{-e\,[E_x \pm iE_y]\,\dfrac{\partial f_{00}}{\partial v}}{m\,[\nu + i(\omega \mp \omega_c)]} \qquad \ldots(16)$$

The current density is defined as

$$j_i = e \int \frac{v_i(\mathbf{f}_1.\mathbf{v})\,dv}{\nu} \qquad \ldots(17)$$

Using eqn. (16) in eqn. (17) we obtain

$$j_x \pm i j_y = \frac{\omega_p^2 \left(E_x \pm iE_y \right) \left(\omega \pm \omega_c \right) \left[\nu \left(\omega \pm \omega_c \right) - i \left(\omega^2 - \omega_c^2 \right) \right]}{4\pi \left(\omega^2 - \omega_c^2 \right)^2} \qquad \ldots(18)$$

Using eqn. (9) in eqn. (18) the expression for the current density is obtained as

$$j = \frac{\omega_p^2 E_y e^{i\omega t}}{4\pi\left(\omega^2 - \omega_c^2\right)^2}\left[\left\{\nu\left(\omega^2 + \omega_c^2\right) - i\omega\left(\omega^2 - \omega_c^2\right)\right\}\right.$$

$$\times\left\{-\frac{\omega_c\omega_p^2\left\{2\nu\omega - i\left(\omega^2 - \omega_c^2\right)\right\}}{\omega\left(\omega^2 - \omega_{uh}^2\right)\left(\omega^2 - \omega_c^2\right) - i\nu\omega_p^2\left(\omega^2 + \omega_c^2\right)}\hat{x} + \hat{y}\right\}$$

$$+\left\{\omega_c\left(\omega^2 - \omega_c^2\right) + 2i\nu\omega\,\omega_c\right\}$$

$$\left.\times\left\{\hat{x} + \frac{\omega_c\omega_p^2\left\{2\nu\omega - i\left(\omega^2 - \omega_c^2\right)\right\}}{\omega\left(\omega^2 - \omega_{uh}^2\right)\left(\omega^2 - \omega_c^2\right) - i\nu\omega_p^2\left(\omega^2 + \omega_c^2\right)}\hat{y}\right\}\right] \qquad ...(19)$$

Absorption of Radiation Propagating in the Extraordinary Mode and the Laser Fusion Process

The time averaged power absorbed per unit volume is given by

$$p = \langle \mathbf{j}(t).\mathbf{E}(t)\rangle = \tfrac{1}{2}\,\text{Real}\,(\mathbf{j}.\mathbf{E}^*) \qquad ...(20)$$

Using eqn (9) the expression for electric field is written as

$$E = E_y.e^{i\omega t}\left[-\frac{\omega_c\omega_p^2\left\{2\nu\omega - i\left(\omega^2 - \omega_c^2\right)\right\}}{\omega\left(\omega^2 - \omega_c^2\right)\left(\omega^2 - \omega_{uh}^2\right) - i\nu\left(\omega^2 + \omega_c^2\right)\omega_p^2}\hat{x} + \hat{y}\right] \qquad ...(21)$$

Using eqns. (19) and (21) in eqn. (20) we obtain (for $\nu^2 \ll \omega^2$)

$$p = \frac{\omega_p^2 \cdot \nu}{8\pi} \cdot \frac{E_y^2\left(\omega^2 + \omega_c^2\right)}{\left(\omega^2 - \omega_c^2\right)^2}\left[1 + \frac{4\omega_c^2\,\omega_p^2}{\left(\omega^2 - \omega_{uh}^2\right)\left(\omega^2 + \omega_c^2\right)}\right.$$

$$\left. + \frac{\omega_c^2\,\omega_p^4}{\omega^2\left(\omega^2 - \omega_{uh}^2\right)^2}\right] \qquad ...(22)$$

In absence of the magnetic field, this expression reduces to the usual expression

$$p = \frac{\omega_p^2 \cdot \nu}{8\pi} \cdot \frac{E_y^2}{\omega^2} \qquad ...(23)$$

The absorption of laser light in the atmosphere of pellet produces a temperature of the order of $10^8\,°K$ (or 10 Kev.) for which $v_{Th} \sim 7 \times 10^9$ cm/sec, If $E \sim 1.5 \times 10^9$

V/cm, $v_E \sim 10^9$ cm/sec i.e. v_E is slightly smaller than v_{Th}. In such a case we are in the weak field regime. It is however interesting to note that keeping all parameters to be the same we shift to the strong field regime ($v_E > v_{Th}$) by considering the temperatures less than 2×10^6 °K as at this temperature v_E and v_{Th} are exactly equal. The average oscillatory energy in the electric field is given by

$$\epsilon_{osc} = \tfrac{1}{2} m v_E^2 = \tfrac{1}{2} \cdot \frac{e^2 \cdot E_y^2}{m\omega^2}$$

The thermal contribution to the total energy ϵ_T increases in accordance with the relation

$$\frac{d\langle \epsilon_T \rangle}{dt} = \int d^3v \cdot \frac{mv^2}{2} \cdot \frac{\partial f(v)}{\partial t} \nu \, \epsilon_{osc}.$$

In the weak field regime ($v_E \ll v_{Th}$)

$$\frac{d\langle \epsilon_T \rangle}{dt} = \nu_w \, \epsilon_{osc} = \frac{\pi NLZ^2 e^6}{(3mkT)^{3/2}} \cdot \frac{E_y^2}{\omega^2}$$

In the strong field regime ($v_E \gg v_{Th}$)

$$\frac{d\langle \epsilon_T \rangle}{dt} = \nu_s \cdot \epsilon_{osc} = \frac{\pi NLZ^2 e^3 \omega}{E_y}$$

Thus in the weak field regime $\frac{d\langle \epsilon_T \rangle}{dt}$ increases with increasing values of E_y and decreasing values of ω where as reverse is true for strong field regime.

Using eqns. (13) and (14) in eqn. (22), the expressions for time averaged power absorbed per unit volume in the weak and strong field regimes are obtained as

$$p_w = \frac{\pi NnLZ^2 e^6 E_y^2}{(3mkT)^{3/2}} \left\{ \frac{\omega^2 + \omega_c^2}{\left(\omega^2 - \omega_c^2\right)^2} \right\} \left[1 + \frac{4\omega_c^2 \omega_p^2}{\left(\omega^2 - \omega_{uh}^2\right)\left(\omega^2 + \omega_c^2\right)} + \frac{\omega_c^2 \omega_p^4}{\omega^2 \left(\omega^2 - \omega_{uh}^2\right)^2} \right] \quad \ldots(24)$$

$$p_s = \frac{\pi NnLZ^2 e^3}{E_y} \left\{ \frac{\left(\omega^2 - \omega_c^2\right)}{\left(\omega^2 + \omega_c^2\right)^{1/2}} \right\} \left[1 + \frac{4\omega_c^2 \omega_p^2}{\left(\omega^2 - \omega_{uh}^2\right)\left(\omega^2 + \omega_c^2\right)} + \frac{\omega_c^2 \omega_p^4}{\omega^2 \left(\omega^2 - \omega_{uh}^2\right)^2} \right] \quad \ldots(25)$$

Eqns. (24) and (25) have been used for computing the power absorbed in weak and strong field regimes respectively.

Discussion

Following set of parameters has been used for calculations.

$n = 10^{15}$ cm^{-3}

$N = 10^{15}$ cm^{-3}

$Z = 1$ (Singly ionized plasma)

$E_v = 1.5 \times 10^9$ V/cm.

$\omega = 1.8 \times 10^{15}$ Sec^{-1}

$T = 10^8$ °K (Weak field regime)

$T = 10^6$ °K (Strong field regime)

Keeping all parameters the same we have studied the absorption in weak and strong field regimes by assigning two different values of temperature. When temperature is of the order of 10^8 °K, the condition $v_{Th} \gg v_E$ is satisfied and so we are in the weak field regime. For temperature of the order of 10^6 °K, $v_{Th} \ll v_E$ and so we are in the strong field regime. Fig. 1 shows the variation of power absorbed with the

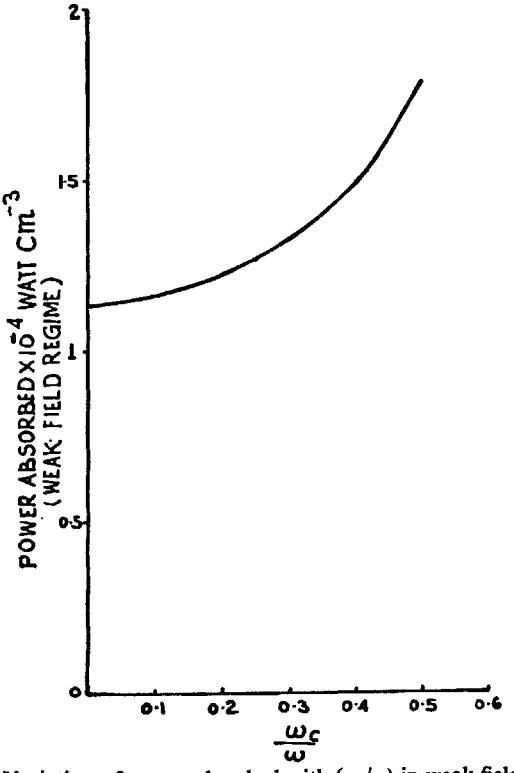

Fig. 1. Variation of power absorbed with (ω_c/ω) in weak field regime.

magnetic field in the weak field regime. The results show that the magnetic field enhances the power absorbed in the weak field regime. In Fig. 2 we present the variation of power absorbed with the magnetic field in strong field regime. The results show that the effect of magnetic field is to reduce the power absorbed. The effect of magnetic field in the weak field regime is to affect the plasma frequency indirectly in the sense that the photon propagation will depend on the direction of propagation relative to the magnetic field. It can be deduced from eqn. (24) that in presence of the magnetic field the plasma frequency gets modified to

$$\Omega_p = \frac{\omega_p\left(1 + \dfrac{\omega_c^2}{\omega^2}\right)^{1/2}}{\left(1 - \dfrac{\omega_c^2}{\omega^2}\right)} \cdot R.$$

Where $R \simeq 1$ for the parameters under consideration. The variation of effective plasma frequency Ω_p with magnetic field in weak field regime is shown in Fig. 3. The magnetic field increases the effective plasma frequency which in turn increases the power absorbed in the weak field regime. In the strong field regime the magnetic field influences the absorption directly by increasing the directed velocity of electrons.

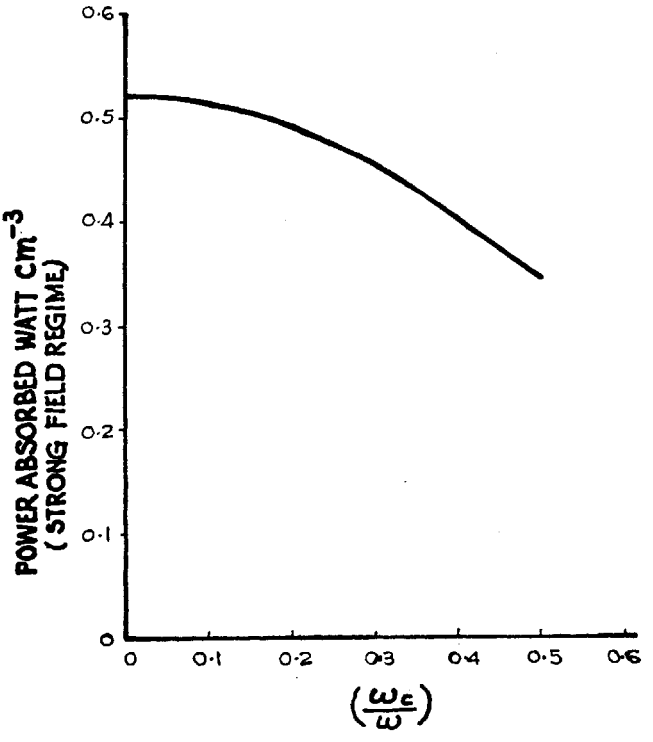

FIG. 2. Variation of power absorbed $\alpha 10^{-5}$ with (ω_c/ω) in strong field regime.

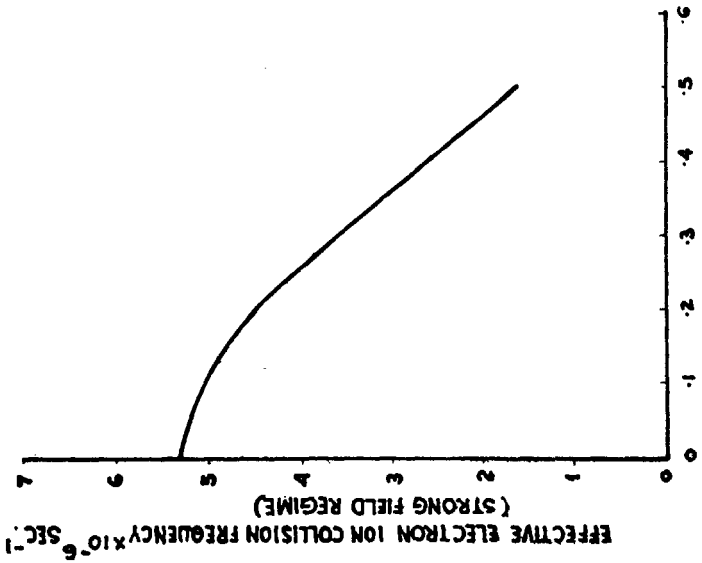

FIG. 4. Variation of effective electron-ion collision frequency with (ω_e/ω) in strong field regime.

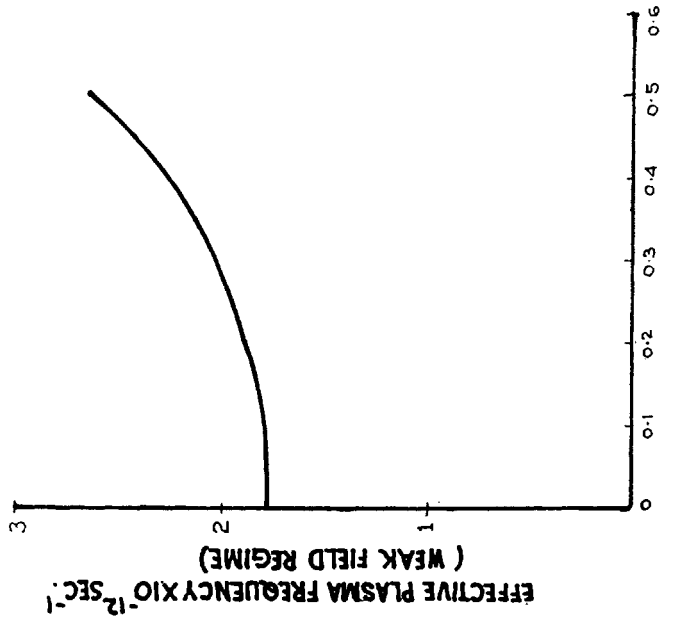

FIG. 3. Variation of effective plasma frequency with (ω_e/ω) in the weak field regime.

Due to increased velocity the electrons escape without interacting much with the ions and thereby reducing the effective electron-ion collision frequency. This behaviour of effective electronion collision frequency with magnetic field in the strong field regime is shown in Fig. 4. Due to the decrease in effective electron-ion collision frequency the power absorbed reduces with the magnetic field in the strong field regime. Our results are similar to that of Seely (1974).

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