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METAL INSULATOR TRANSITIONS

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A metal may turn insulator due to increasing electron correlation (Mott) or disorder (Anderson) or electron lattice coupling, or a combination of these. The first two mechanisms are reviewed here briefly. Attempts to describe what happens when both are present, are also summarized. Finally some comments are made on criteria for the transition, effects of electronic (in) commensuration and disorder as well as on heavy fermions.

Key Words : Metal Insulator Transitions; Mott Electron Correlation; Anderson Disorder; Electron Lattice Coupling; Heavy Fermions; Couloment Interaetions; Orbital Effects; Doped Semiconductors; Electronic Incommensuration

INTRODUCTION

THE transformation of a substance from metal to insulator on varying a physical parameter such as temperature or pressure or disorder is often due to a basic change in electronic bonding. In the solid state, the change is best studied through physical properties and thus the subject is of interest both as chemistry and physics. In spite of extensive work in this field by chemists and physicists, and a general agreement on model patterns, we are far from a full understanding of the phenomena. In this brief review, I shall sketch some simple models, and will conclude with some comments. The subject has been extensively discussed many times; the book by Mott[1] is a comprehensive and unique survey of the experimental and theoretical situation as of a decade ago. A recently published Festschrift for Mott describes the present state of the subject.[2]

Some of the questions involved are most easily brought out by considering a model system, namely a periodic lattice of hydrogen atoms, first discussed by Mott.[3] The lattice structure is assumed to remain unchanged, so that the single relevant variable is the lattice constant b. If b is small, electronic overlap is large, and the system is a metal according to one electron band theory, the highest occupied band being half full. The electronic states are best described as molecular orbitals. On the other hand, suppose b is very large (i.e. $b \gg a_0$ the Bohr radius), the system is then best viewed (to zeroeth order) as a lattice of isolated hydrogen atoms, i.e. a Heitler London or atomic description is a good starting point. In this limit, electron motion is activated, the activation process being the removal of an electron from 'its' proton and attachment to a nearby neutral atom. The activation energy is $U = I - A$, I being the ionization energy and A the electron affinity. The system is thus an insulator. In the band picture, the system

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always ought to be a halffilled band metal, albeit with a greatly reduced bandwidth as b increases. In fact, however, one expects to pass from a metal to an insulator, because the nature of electronic states changes from extended and uncorrelated (band like) to localized and correlated (atomic like). What is the nature of this transition? Does it occur at a critical value b_c of the lattice constant b ? Is it continuous or discontinuous? How do measured properties such as electrical conductivity, electron excitation spectrum and magnetic susceptibility vary across the transition? Is it accompanied by structural and magnetic changes?

One can sense, from the fact that the two phases are most simply described in very different chemical terms, that the theoretical problem is difficult. Experimentally, the above questions have been extensively investigated for a large number of substances; an important question is the relationship between these observations and predictions of the above simplified model. Another question, first discussed by Mott,[4] is of experimentally studying hydrogenic systems so that the basic phenomenon of correlation driven metal insulator transition (Mott transition) can be most clearly exhibited. Mott pointed out that since shallow donor and acceptor states are large and hydrogenic, the insulator metal transition that occurs with increasing dopant concentration ought to be an attractive possibility.

However, even in the relatively simple model case of a hydrogen atom lattice, a number of complications are possible, and do occur. For example, the actual low density ground state is a lattice of hydrogen molecules. In the above description, this possibility, namely that of unit cell doubling by lattice deformation has been ignored. Since now the unit cell has two electrons, the system can be insulating even in one electron band theory. The metal insulator transition can thus be driven by electron ion or electron lattice interaction rather than electron correlation. The actual nature of the proposed model system, e.g., phosphorus-doped silicon, points to another factor, namely disorder. The physical parameter in Si : P is the phosphorus density n, the critical value for the insulator metal transition being $n_c = 3.8 \times 10^{18}$/ c.c. The system is hydrogenic and the electron lattice interaction effect is weak since the basic electronic scale is $a_0^* \simeq 30$Å, much larger than the lattice constant. However, since phosphorus atoms *randomly* substitute Si atoms, one does not have a periodic system. The quantum mechanical matrix elements for an electron to hop from one P atom to the next vary over a *very* wide range. It was first realized by Anderson[5] that this kind of randomness by itself (i.e. even in the absence of correlation) can localize electronic states. The question of whether the observed metal insulator transition in P doped Si arises from correlation or disorder or from a combination, is still not settled, as we shall see.

In the succeeding section of this paper, we review the Mott transition (Section II) and the Anderson transition (Section III). The former is discussed mainly in a simple model due to Hubbard[6,7] and a number of realistic features ignored in the model are mentioned. In Section IV, we outline the present theoretical understanding of metal insulator transitions with both disorder and interaction effects present. The properties of the experimentally well-studied Si : P system are then described (Section V). We conclude with comments on the possible

range of phenomena near the metal insulator transition, and point out how detailed experimental studies can point to the mechanism or mechanisms involved (Section VI).

II Mott Transition

Perhaps the simplest model for correlation effects in solids goes by the name of Hubbard. [6,7] One imagines an orbitally non-degenerate atomic state at each lattice site i ; thus there can be zero, one or two electrons (one with spin up and the other with spin down) at the site i. The energies are 0, $\epsilon_{i\sigma}$ and $(\epsilon_{i\uparrow} + \epsilon_{i\downarrow} + U)$ respectively. The extra energy cost U of a two electron configuration is the sole measure of correlation. It is effective only on site. An electron hops from site i to site j with an amplitude t_{ij}. Usually t_{ij} is taken to be nonvanishing only between nearest neighbours, z in number, and to have the same value of t for all of them. The model is then characterized by single dimensionless ratio (zt/U) and by a (kinetic) energy scale zt. As (zt/U) decreases, one goes over continuously from a molecular orbital to a Heitler London limit. The Hamiltonian is

$$H = \sum_{i,\sigma} \epsilon_{i\sigma} n_{i\sigma} + \sum_{i,j,\sigma} t_{ij}\, a^{+}_{i\sigma}\, a_{j\sigma} + U \sum n_{i\uparrow} n_{i\downarrow} \qquad \ldots(1)$$

The electron density or the average number n_e of electrons per site is another imporant parameter.

The system described by the above model Hamiltonian clearly has a metal insulator transition for exactly $n_e = 1$ (one electron per site) when U is sufficiently large. In general, a system can be insulating for any commensurate filling, i.e. integer number of electrons per site, if correlation effects are strong. In the insulating phase, since the spin of the electron is localized as well, the solid has magnetic moments. They interact antiferromagnetically, with the nearest neighbour coupling energy (t_{ij}^2/U) so that the ground state is antiferromagnetically ordered. It is an interesting but scarcely explored prediction that for $n_e \neq 1$, the system is a metal no matter how strongly correlated it is. The reason is that the extra electrons or holes (for $n_e > 1$ or $n_e < 1$ respectively) are always mobile, there being more equivalent sites than carriers.

The possible phases of the Hubbard model have been investigated mainly in the one body or Hartree-Fock approximation.[8] The correlation term $Un_{i\uparrow}n_{i\downarrow}$ which depends on the actual number of up and down spin electrons at site i is approximated by the spin potential due to the *average* number of electrons of the other spin, i.e. one writes

$$Un_{i\uparrow}n_{i\downarrow} = Un_{i\uparrow} < n_{i\downarrow} > + Un_{i\downarrow} < n_{i\uparrow} > - U < n_{i\uparrow} > < n_{i\downarrow} < \qquad \ldots(2)$$

Now since the number of electrons per site $< n_{i\uparrow} > + < n_{i\downarrow} > = n_3$ is fixed, there is one unknown parameter, i.e., the magnetization $m_i = < n_{i\uparrow} > - < n_{i\downarrow} >$ per site. This parameter is determined in the usual way, i.e. the ground state of the approximate Hamiltonian of eqs. (1) and (2) is found, the quantities $\{m_i\}$ calculated from this, and the process iterated to self-consistency. It is not possible to do this

exactly, the problem being one of solving the electronic structure of a system with an arbitrary random one body potential (Anderson localization). Several simplifications are usually made. It is assumed that the magnitude m of the moment is the same at each site and that only the direction (up or down) is a random variable. An approximate method of calculating electronic structure of random systems, e.g. the CPA (which misses localization altogether) is then used. Detailed calculations of the ground state energy, the optimum value of m, the density of states, conductivity, etc. for spatially disordered as well as ordered configurations of local magnetic fields have been made.[8] We now mention some of the conclusions.

The one electron density of states developes a gap when $(U/zt) > U_c \sim 1$. This (Hubbard) gap increases with U, and for $U \gg U_c(zt)$, goes over to the limiting value namely U. If the lower band of states is exactly full, as happens for $n_e = 1$, then one can have activated carrier excitation and transport for $U > U_c\,(zt)$, and thus an insulator. Calculations indicate the possibility of a magnetic metal, i.e. a non-zero moment m, but no gap in the excitation spectrum. A schematic phase diagram for the half-filled case is shown in Fig. 1. In the weak correlation limit one has a Pauli paramagnetic metal (marked M). With increasing U, moments develop (M') and the ground state may be a magnetically ordered metal. As U increases, a Hubbard gap opens, and the system is an antiferromagnetic insulator (AFI) at low temperatures and an insulator with magnetic moments (I) at high temperatures. The transitions are generally second order, and some of the boundaries, viz., $M' \to I$ are notional, being based on a feature in an approximate conduc-

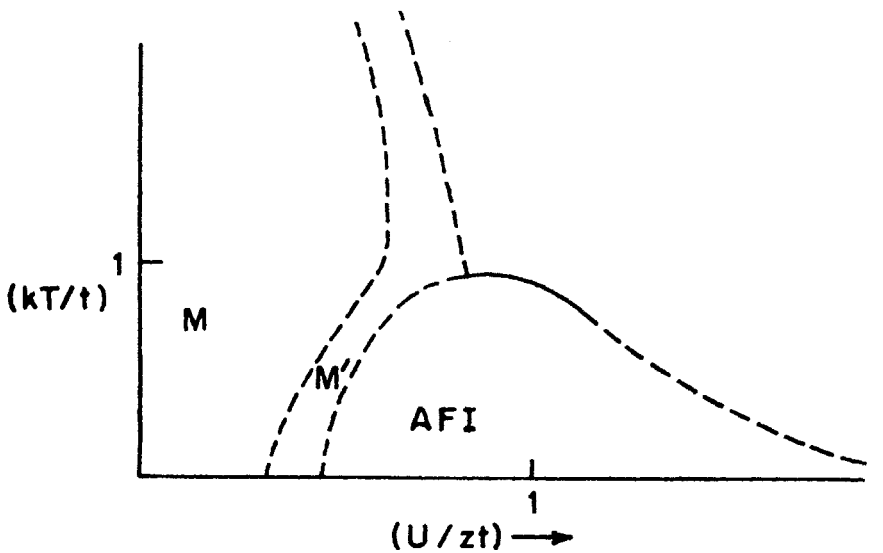

FIG 1 A schematic phase diagram for the Hubbard model in the temperature (k_BT/t) and correlation (U/zt) plane, after Economou.[8] Phases shown are metallic (M), metallic with moments (M'), antiferromagnetic insulator (AFI), and insulating (I). Transitions are continuous except where shown by full lines.

ctivity calculation. A region of first order transitions (shown by full lines) may also be an artefact of the approximations used.

The one body approximation is a symmetry breaking mean field theory. It breaks the symmetry between up and down spins even in the paramagnetic state, e.g. M' and I, where moments are present but not ordered. The mean field is the local magnetization. The approximation is a good first order theory if the mean field is large $m \simeq 1$) and transitions between the two broken symmetry solutions at each site (i.e. $\pm m$) i.e. spin fluctuations occur only on a long time scale. Deep inside the insulating regime, it is indeed true that m is close to unity, spin fluctuations are slow (rate $\Gamma \simeq zt^2/U$), and that charge or valence fluctuations are strongly suppressed because of strong correlation effects. However, in the metal insulator transition regime, charge fluctuations, spin fluctuations, temperature and mean field all have a comparable energy scale, so that the Hartree Fock approximation is specially bad.

A different kind of approach which emphasizes local correlation effects is due to Gutzwiller.[9] Brinkman and Rice[10] pointed out its relevance for the metal insulator transition due to correlation. The idea is to write down an approximate ground state wave function incorporating correlation effects; these discourage double occupancy of any site. The wave function is

$$\psi_G = \prod_i [1 - (1 - g)\, n_{i\uparrow} n_{i\downarrow}]\, \psi_F \qquad \ldots(3)$$

Here ψ_F is the uncorrelated or free Fermi or MO ground state which is exact for $g = 1$. For $g = 0$, it is clear that the ground state has no doubly occupied sites. The parameter g is thus a measure of correlation and is determined variationally. An approximate calculation shows that $g \to 0$ as $U \to U_c = -8 \sum_{k(\text{occ.})} t_k$. As the Hubbard correlation increases towards U_c, electrons get locked to their 'respective' sites, i.e. there is both spin and charge localization. This is reflected in both the spin susceptibility and effective mass diverging as $(U_c - U)^{-1}$. One thus has a strongly correlated heavy Fermi system on the verge of transition to a correlation localized or Mott insulating phase. The approximation thus provides an attractive picture of the situation near the transition. However, the insulating phase, and ordering therein are not described, nor are the $T \neq 0$ properties. Spatial correlations are completely ignored.

Metal insulator transitions in transition metal oxides, with V_2O_3 being the most extensively studied example, do have features of the sort discused above. The phase diagram of V_2O_3 doped with Cr and Ti (supposed to uniformly dilate or contract the lattice) is shown in Fig. 2. There is a metallic phase, an antiferromagnetic insulating phase and disordered insulating phase. The transitions are all first order except that the M-I transition ends in a critical point. The broad similarity between Figs. 1 and 2 is apparent. The observed transitions are probably first order because of electron lattice coupling. However, the magnetic susceptibility, electronic heat capacity, and T^2 coefficient of resistivity are all enhanced by a factor to 20 to 50 with respect to typical transition metal values. This is suggestive of

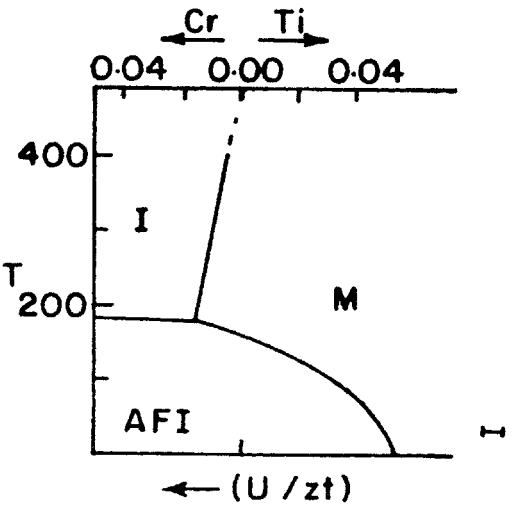

FIG 2 Phase diagram for V_2O_3 alloyed with Cr and Ti. Metallic, insulating and antiferromagnetic insulating phases are shown. Full and broken lines indicate first order and continuous transitions respectively.

Brinkman-Rice type of critical correlation effects preceding a metal insulator transition. The fact that the resistivity of the metal near the metal insulator transition is very high implies also that strong disorder effects are present.

The main conclusion is that even for this simple model, the nature of the metal insulator transition is not understood in detail.

Shortcoming of the Model

The Hubbard model does not describe realistically any known physical system. Some of the features which could be relevant and have been left out are the following:

Orbital Effects: Metal insulator transitions occur most commonly in systems based on transition metal atoms which have large orbital degeneracies N. Atomic and solid state features such as Hund's rule coupling, configurational splitting, spin-orbit coupling, crystal field lifting of orbital degeneracy, are all neglected. These degrees of freedom may have qualitative effects, e.g. orbital superlattice formation.[11] An important quantitative consequence could be that intersite energies are disfavoured by a $(1/N)$ factor relative to intra or single site energies.[12,13]

Another recently discussed effect is due to the presence of other bands, e.g. states of s or p local symmetry. An example is NiO, an insulator due to correlation effects. The lowest excitation gap (e.g. the conductivity gap) is not $U_{dd} \simeq 10$eV, but is about 2eV, this being the energy required to remove an electron from the filled oxygen p band and to put it into a Ni d-orbital. Such a system is most naturally described as a charge transfer insulator. A one site theory of energy level spectra

of such systems has been discussed recently by Sawatzky and Allen,[14] comparison being made with photoemission experiments. Hüfner[15] has recently suggested, on the basis of such data and charge transfer spectral energies, that in the 3*d* series, the first half atoms (e.g. Ti, V) are likely to form Mott insulators with U_{dd} as the relevant electronic gap (Mott insulators) while those in the second half (e.g. Fe, Ni) are likely to form charge transfer insulators. It should be noted however that electron correlation is the basic cause of the insulating phases in either case. The second class of systems are best described as Anderson lattices, i.e. as lattice generalizations of a model for a magnetic impurity in a metal involving correlated *d* or *f* states, conduction band *s* or *p* states, and hybridization or admixture.

Coulomb Interaction : In the Hubbard model, electrons interact only when on the same site. In fact, the coulomb interaction is unscreened and of long range for the insulator. Mott[3] pointed out that consequently the 'electron' 'hole' excitations (with respect to the lower Hubbard band filled ground state) attract and form bound states. The system thus continues to be insulating unless the number of electrons and holes is large enough so that the screened interaction is too weak to bind. At this point there is a discontinuous jump in the carrier density, so that the insulator metal transition is first order. Thus the coulomb interaction because of its long range can change the character of the transition. The argument relies on the 'electrons' and 'holes' being well defined (long lived) excitations. For $U \simeq U_c$ (zt) the Hubbard gap is small, and this assumption may be incorrect. The question is not settled, especially since the hydrogenic system Si : P has a *continuous* metal insulator transition.

Electron Lattice Interaction : The electronic energy depends strongly on ionic positions for *d* electron as well oxygen and chalcogen states. Especially when the electronic state tends to localize, this electron lattice interaction is effectively stronger, and almost inevatibly causes lattice distortions. The interplay between correlation and electron lattice distortion has not been explored seriously except for one dimensional systems. We briefly mention here two ideas developed for the situation where electron lattice interaction only is present. It is generally believed[16] that when electron acoustic phonon (lattice distortion) coupling λ exceeds a certain strength the local distortion produced by the electron traps it (small polaron). Thus with increasing λ, there is a very rapid increase of electron effective mass around a particular value λ_c. This is obviously relevant to metal insulator transitions in two ways. Such as a very narrow band system can become insulating with even a modest correlation U. Secondly, correlation can help in selftrapping. The other idea concerns the possibility of a bipolaron formation. Two close polarons may be more stable due to the extra lattice distortion caused lowering of energy than two polarons far away which repel each other because of the coulomb interaction.[17] Interestingly, there also seems to be a regime where a small bipolaron is stable whereas two small polarons are not. In a lattice, such a bipolaron formation could be the driving force for a characteristic metal insulator transition to a molecular crystal like phase which is a diamagnetic dimerized solid. The transition is most likely discontinuous.

Disorder : The high electrical resistivities of the metallic phase near the transition and their anomalous dependence on temperature imply the existence of strong disorder. The theory of strongly correlated and disordered systems is not well developed, but there has recently been considerable progress in the theory of disordered electronic systems close to localization (Anderson localization).[18] We now discuss this route to the metal insulator transition.

III Anderson Localization

Consider for concreteness a model similar to that used above, i.e. eq. (1). However, one assumes that there is no correlation between electrons, but that the energies ϵ_i at the sites i are random. The randomness can be characterized by a spread of width W. Anderson[5] showed that when this exceeds a critical value, an electron locates itself around an appropriate potential fluctuation so that the state is spatially localized rather than extended as it is for weak disorder. The critical value W_c, i.e. the ratio (W/zt) depends on the energy of the electron, and the dimensionality as well as the connectivity of the lattice. The consequences of this localization were first spelt out by Mott.[19,20] Since the localization criterion depends on electronic energy, some states are extended and others localized for a given disorder. The sharp energy separating these is the mobility edge. A system is metallic or insulating depending on whether states near the Fermi energy are extended or localized. Now, with increasing disorder or decreasing Fermi energy or a combination of the two, the mobility edge energy crosses the Fermi energy and the system goes insulating. This type of metal insulator transition (Anderson transition) has been observed in a large number of systems, e.g., atomically disordered metal semiconductor composites ($Nb_x Si_{1-x}$, $Au_x Ge_{1-x}$ . . . .), granular metal systems, metal inert gas mixtures . . . . . Our present understanding of the transition is summarized below.

If the single particle energy ϵ_i is the same at every site i, one has an electronically periodic solid, and the eigenstates are Bloch waves. For weak disorder (i.e. $W \ll zt$), the Bloch waves are scattered occasionally. The residual or $T = 0$ conductivity σ is given in terms of the mean free path l by

$$\sigma = \left(\frac{ne^2}{\hbar k_F^2}\right)(k_F l), \qquad \ldots(3)$$

where n is the electron density and k_F is the Fermi wavevector. As disorder increases, the mean free path decreases. This mode of description fails when mean free path becomes of the order of or smaller than the interatomic spacing, i.e. for $k_F l \sim 1$. This is the minimum possible conductivity of a metal, a concept due to Mott.[20] The minimum metallic conductivity of a metal in three dimensions is $\sigma_{\min}^{3d} = (e^2/3\hbar\pi^2)\, k_F$ and in two dimensions it is $\sigma_{\min}^{2d} = (e^2/2\hbar\pi^2)$. We note that these values, involving quantum diffusion depend on Planck's constant h, and that in two dimensions $\sigma_{\min}$ depends only on fundamental constants. The conductivity of a metal drops abruptly from $\sigma_{\min}$ to zero at the metal insulator transition. There

is a very large amount of experimental avidence for a change of transport regime in disordered systems near σ_{min}. For example, the temperature coefficient of resistivity changes sign when $\sigma \sim \sigma_{min}$. In many cases the boundary between metal and insulator does seem to be at σ_{min}.[20] Howerver, careful recent experiments at low temperature very close to critical disorder strongly suggest that zero temperature conductivity actually goes continuously to zero at the localization transition (see below).

On the localized side of the transition, as the metallic phase is approached, the localization length ξ increases and diverges, e.g. $\xi \sim (W - W_c)^{-1}$ where W_c is the critical disorder. The polarizability goes as ξ^2, and ignoring local field corrections in this random system, the static dielectric constant ϵ also diverges as ξ^2 of $(W - W_c)$.[-2] In this picture, the localization transition is asymmetric, discontinuous from the metallic side, and being continuous from the insulating side. The asymmetry is due to neglect of interference effects in the former regime. Localization is due to destructive interference of waves scattered repeatedly back and forth in the random medium. Just above critical disorder, the interference a effect localizes electronic states over a diverging length scale ξ. However, just below critical disorder it is assumed that this interference has no effect on physical properties so that the conductivity σ has the same value at all length scales larger than the mean free path l.

In the scaling theory of localization[21] account is taken of scale dependent interference effects. The localizing process is singular backscattering. The amplitude for an electron to go from one point to another is a weighted sum over the partial amplitudes for various paths. On these, consider the ones with a loop. This loop can be transversed in opposite senses (in the random medium) and the partial amplitudes are the same. Thus the probability (amplitude^2) for return to the origin of the loop is enhanced because of backscattering interference. Neighbouring paths mute this effect, but it is still crucial in three dimensions. Based on this idea, the zero temperature conductivity near W_c is seen to be

$$\sigma\,(T = 0)\ (e^2/\hbar\pi^2)\left(\frac{1}{\xi_>}\right) = \left(\frac{e^2}{\hbar\pi^2\, l}\right)\left(\frac{W_c - W}{W}\right). \qquad \ldots(4)$$

The first term in the last equation is the Mott σ_{min} and the second term is the reduction due to quantum interference. The conductivity thus goes to zero smoothly, being characterized by a length $\xi_>$ which diverges the same way as the localization length ξ on the other side. This is just like any continuous phase transition with a single diverging correlation length. A detailed analogy with a phase transition is possible using a field theory mapping of the problem due to Wegner.[22] The diffusion constant is analogous to the spin wave stiffness of a ferromagnet. In the localized regime, the effective spin wave stiffness at long length scales is zero.

In addition to the above, a number of detailed predictions for transport properties, e.g. temperature-dependence of resistivity and for magnetoresistance are possible. They are all based on the idea that backsattering interference is cut

off by temperature (inelastic scattering leads to incoherence) and by a magnetic field (which introduces a phase difference between the two oppositely traversed loop paths). The former implies that quantum interference effects decrease with increasing temperature. This means that resistivity can decrease with increasing temperature even in the metallic regime (i.e. one with $\sigma\,(T = 0) \neq 0$). Also, a $\sigma_{\min}$ like behaviour can be inferred from higher temperature observations. The mangnetoresistance is negative and has a characteristic $\sqrt{H}$ dependence in high fields.

A number of experiments have confirmed these preditions in great detail (including those for two dimensional systems).[18] It is generally believed, since in a number of cases $\sigma < \sigma_{\min}$ has been observed with the dependence of eq. (4), that the localization transition is continuous (see however the detailed analysis of Möbius for a contrary view[23]). We now turn to a discussion of what is expected when disorder and interaction are both present.

IV Disordered Interacting Systems

Most theories of such systems are perturbative, i.e. they describe the weakly disordered, weakly interacting metal, and seek to extrapolate from this limit. Perhaps the only exception is the theory of Efros and Shklovskii[18] for the coulomb interaction effects in the highly localized regime (i.e. extremely strong disorder). Altshuler and Aronov [24,25] were the first to realize that in a disordered system interaction effects are characteristically different. The extension of these to the metal insulator transition regime, using scaling ideas, is due to McMillan[26] and a number of others.[18] There are no theories yet which treat interaction or correlation effects exactly or accurately enough to lead to the Mott transition, *and* include disorder effects.

In a disordered medium, electrons move diffusively, i.e. separation increases as the square root of time rather than as time. This effectively enhances the interaction and makes it retarded. For example, the probability for an electron to be at r at a time t starting from $r = 0$ (at $t = 0$) is $P(r, t) \sim (1/Dt)^{3/2} \exp(-r^2/Dt)$. Thus the probability that two initially close electrons stay close ($r \sim 0$) is proportional to $(1/Dt)^{3/2}$. Two such electrons with energy difference E stay coherent for a time $t \sim (h/E)$, so that the probability that they stay near each other and interfere is

$$P(E) \simeq \int_0^{(h/E)} P(t)\, dt \simeq E^{1/2}/D^{3/2}. \qquad \ldots(5)$$

This leads to several characteristic and novel effects, some of which are listed below :—

(*i*) There is square root cusp $\delta N(E)$ in the density of states $N(E)$ as a function of energy E (with respect to the Fermi energy). The cusp is generally a dip, and is given by

$$\frac{\delta N(E)}{N(0)} = \frac{V(0)\, N(0)}{4\pi^2 (\mathrm{d}n/\mathrm{d}\mu)\, D} \left(\frac{E}{D}\right)^{1/2},$$

where $V(0)$ is the zero range interaction strength, and $(dn/d\mu)$, is the compressibility. The effect obviously becomes stronger with increasing disorder (decreasing D).

(ii) There is a $\sqrt{T}$ term in resistivity, the sign depending on details of interaction strengths in various partial wave channels. It is generally negative.

(iii) There can be a positive magnetoresistance due to spin splitting, with a $\sqrt{H}$ field dependence for $\mu H > k_B T$.

(iv) There is a temperature-dependent spin susceptibility enhancement.

(v) Inelastic decay rates, e.g. due to electron electron interaction are enhanced. For example in three dimensions, the rate is $\Gamma(E, T) \sim \{\max(E, T)\}^{3/2}$ in contrast to the quadratic dependence for pure system. Consequently, inelastic mean free paths are much shorter in disordered systems than in clean systems.

Many of these effects have been quantitatively confirmed in metallic glasses, disordered metals, doped semiconductors etc.

Interaction effects clearly become more pronounced as disoder increases. Also, as the density of states depletion becomes larger, screening becomes poorer, and the interaction strength increases. Thus there is a nonlinear feedback effect leading to an interaction and disorder dominated metal insulator transition. A schematic scaling model for such a transition was first presented by McMillan[26] and more carefully and extensively by Finkelshtein[27] and Castellani *et al.*[28] A number of scenarios have been worked out ; we summarise two below :—

One is interested in the effective interaction between electrons at long length and time scales, given their values at short scales l (mean free path) and τ (collision time) respectively. This effective coupling is influenced by diffusive electronic motion and by interaction between various modes (density fluctuations, spin fluctuations, electron pairs). It is obtained by progressively integrating out effects on length and time scales longer than l and τ respectively. The modes can be characterized by total spin, which is conserved. The spin singlet ($S = 0$) electron hole mode corresponds to an orbitally symmetric, mainly isotropic (s-wave) scattering and describes pure density fluctuations in the latter case. Similarly, the $S = 1$ or spin triplet mode with p-wave orbital character, describes spin fluctuations. We ignore higher l partial wave interaction. In the particle particle channel, only the $L = 0$, $S = 0$ or Cooper channel is considered significant. The model is thus characterized by three parameters corresponding to bare interaction strength in these channel, and by a diffusion constant D. The scaling theory with these four scale dependent parameters has not yet been solved. We mention two special cases, considered to leading order in D^{-1}.

(i) *Singlet Only* : This can be experimentally realized if there is considerable scattering by magnetic impurities. Such scattering leads to non-zero energy for long wavelength spin fluctuation and Cooper modes. One then has a system of *only* density fluctuations coupled by the coulomb interaction. Altshuler and Aronov[25] show that the metal insulator transition is continuous with $\sigma \to 0$ according

to the exponent unity. The dielectric constant diverges with the exponent (3/2), the density of states at the Fermi energy varies as exp $\{-1/8\ (\ln T)^2\}$ and the ac conductivity as $\omega^{2/5}$. The conductivity exponent is the same as for pure localization, though the other exponents are different.

(ii) *Singlet and Triplet* : Suppose particle scattering is neglected. The effective spin-spin coupling becomes very large on scaling, and indeed diverges at a finite length scale l_e. At this length scale, density diffusion is non-zero, but spin diffusion vanishes. The spin susceptility also diverges. One thus has a metal with clump of frozen magnetization on a length scale l_e, i.e. a local moment. Unfortunately, the perturbative scaling approach breaks down when coupling constants become large, and the above result for a $T = 0$ spin localization transition preceding a metal-insulator transition is only suggestive.

V. Metal Insulator Transition in Doped Semiconductors

We discuss this specific case because it has been studied experimentally in great detail, and because the system is relatively simple. As mentioned earlier, the uncompensated shallow donor/acceptor system is close to Mott's model for a lattice of hydrogen atoms except for considerable variation in intersite hopping energy. The earliest prediction was that because of electron hole attraction the transition would be first order, marked by a discontinuity in the carrier concentration. The observed transition is continuous. If the short range correlation or Hubbard U is the dominant interaction, then the transition (Hubbard-Mott) is continuous, from a localized spin insulator to a paramagnetic metal. With increasing compensation, correlation effects are reduced, and if randomness predominates, one expects an Anderson or localization transition. Mott suggested that this might continue to be true even for uncompensated systems. We thus have a variety of predictions depending on the dominant mechanism.

Experimentally, the conductivity of Si : P goes continuously to zero at the $T = 0$ metal insulator transition, which occurs for $n_P \simeq 3.8 \times 10^{18}$/c.c. The exponent is half for uncompensated systems[29] and seems to be unity for compensated ones[30,32] as also for a unmber of other systems.[33] The conductivity exponent is unity in localization theory as also in one version of disordered electron gas theory (density fluctuations only; see above). An exponent of half is possible for a noninteracting disordered electron gas if singular backscattering effects are suppressed by magnetic impurities or a magnetic field. The static dielectric constant of insulating Si : P diverges with an exponent of unity.[34] No theory yet predicts this exponent but the ratio of this and the conductivity exponent is two, the value expected in localization only theory, if only one scale parameter is relevant.

There is an increasing body of evidence pointing to strong spin correlation near the metal insulator transition in doped semiconductors. First, far away from the transition, characteristic resistivity and magnetoresistance anomalies have the right size, and sign. Secondly, spin lattice relaxation rates in the metal near the

metal insulator transition are singularly enhanced going as $T^{1/2}$ rather than as T for a Fermi system. This implies critical slowing down of spin diffusion, i.e. spin localization. Very recently, careful low temperature spin susceptibilty measurements[36] show a large rise at temperatures below about 50mk. A similar but more modest specific heat increases in metallic systems farther from the transition has been observed earlier.[37] All these observations remain basically unexplained, though an obvious suggestion is that a Brinkman Rice kind of enhancement, characteristically modified by disorder, is present near the transition.

The ac conductivity just on the insulating side has been measured recently.[34] It is linear in frequency, and the coefficient diverges as the dielectric constant ϵ. Bhatt and Ramakrishnan[38] argue that this is direct evidence for Efros Shklovskii type of long range coulomb effects. In their absence, the coefficient would diverge as $\epsilon^{5/2}$ rather than ϵ, and the linear term would be proportional to $k_B T$. From the temperature-dependence of conductivity just on the insulating side, in *compensated* Ge : Sb, Zabrodskii and Zinoveva[31] find similar effects, with a coulomb gap vanishing as one approaches the metallic phase.

It thus seems that disorder, short range Hubbard corrrelation, long range Coulomb interaction, and compensation are *all* important near the transition in doped semiconductors. There is a large discrepancy yet between these experimental observations and available theoretical models. This discrepancy is a measure of our current understanding of metal insulator transitions.

VI. Some Comments

(i) *Criteria for the Metal Insulator Transition*

We consider here the question of whether agreement with a particular criterion is support for the related mechanism of metal insulator transition. The electron hole unbinding criterion of Mott implies that an insulator turns metallic when the carrier density n satisifies the equation

$$n^{1/3}\, a_H^* \simeq 0.25, \qquad \ldots(6)$$

where a_H^* is a characteristic atomic spacing, i.e. shallow state radius, interactomic spacing, or atomic orbital size. To begin with, we note that the Mott criterion is spectacularly successful, over a density range of nearly 10^8 in doped semiconductors etc. even though the transition is continuous, and *not* first order as predicted by the mechanism underlying eq. (6).

Several different mechanisms lead to criteria similar to eqn. (6). For example, the Hubbard-Mott transition takes place when $zt \simeq U$. It can be shown for hydrogenic systems that this, as well as the Anderson localization criterion $\xi t \simeq W$ (the randomness of the one electron potential) amount to criterion similar to eqn. (6). Another criterion, due to Herzfeld, views the insulator as a lattice of nonoverlapping polarizable atoms of polarizeability α, and considering local field corrections,

predicts for the dielectric constant $\epsilon_0 \Big/ \left(1 - \frac{4\pi n}{3}\alpha\right)$ where n is the atom density, clearly the dielectric constant blows up for $(4\pi\, n\, \alpha/3) = 1$ which *also* translates into a Mott like criterion. It is not clear whether this local field form survives when the electrons from neighbouring atoms overlap, and there is spatial randomness.

The reason different physical mechanisms lead to similar criteria is basically the following. The transition is due to a competition between potential and kinetic energies. The former localizes, the latter delocalizes. They depend differently on electron density, viz. as $n^{+1/3}$ and $n^{2/3}$ respectively, so that they are comparable when n satisfies the Mott criterion eq. (6). Thus the criterion is inevitable on physical and dimensional grounds, and does not support any particular mechanism.

(ii) *Electronic Incommensuration*

In the Hubbard model, only the commensurate electronic system has a metal insulator, transition; the incommensurate system is always metallic. However, if the system is random in addition, the Anderson localized insulating phase is a possibility. These and related questions have scarcely been explored seriously. For example, the metal insulator transition is V_2O_3 is affected much more by nonisoelectronic impurities than by isoelectronic impurities. The former favours the metallic phase strongly. In $LaNiO_3$ which could be described as a commensurate system with $U \simeq U_c$ (Brinkman-Rice correlated electron system) addition of a small amount of Cr or Mn ($\sim$ 2 per cent). Anderson localizes these almost spin localized states and leads to an insulating state.[30] A spectacular example of incommensuration effects is the drastic influence of oxygen vacancies on, the Verwey transition in single crystal magnetite Fe_3O_4.

A careful experimental study of this class of phenomena requires single crystals, electrochemical fine tuning of commensuration, and a wide range of experiments on transport and equilbrium properties. There are at present no theoretical results on highly correlated, disordered, incommensurately filled systems.

Incommensuration without disorder (especially weak incommensuration δ) in strongly correlated systems leads to characteristic behaviour associated with heavy fermions, e.g. low Fermi energy $\epsilon_F \sim \delta(zt)$, and presumably p wave superconductivity due to an electronic attraction mechanism. Such effects have so far been seen in actinide systems, e.g. UBe_{13}, UPt_3.[41] Here the f-shell is nearly intergral valent, hybridization with s-p band leading to the small deficit. An obviously interesting questing question is whether transition metal based systems with similar properties exist. The proximity of a metal insulator transition, magnetic ordering, and superconductivity for large U, small incommensurations and weak disorder makes this regime very interesting.

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