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COUPLED VIBRATIONS OF A ROTATING BEAM OF LINEARLY VARYING SEMI-CIRCULAR CROSS-SECTION SUBJECTED TO A STEADY EXPONENTIAL TEMPERATURE DISTRIBUTION

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A simple model of thermally induced vibrations of a rotating beam is presented for the use of research workers engaged in design of blades of helicopter rotors, wind turbines and aircraft propellers. The analysis presented here is to steady the effect of a temperature parameter on coupled vibrations of a beam of linearly varying semi-circular cross-section attached to a rotating disc. A method based on Rayleigh's quotient is used to obtain upper bounds of the frequencies corresponding to the first three modes of vibrations. The frequencies for various values of cross-section variation, hub-radius ratio and the temperature parameter are obtained for setting angle $\pi/2$.

Key Words : Thermal Vibrations; Blades of Helicopter Rotors; Rayleigh's Quotient; Hub-radius Ratio

NOTATION

x radial co-ordinate

W deflection of elastic axis in y-direction

L length of the beam in radial direction

E modulus of elasticity

G shear modulus of rigidity

ν Poisson's ratio

I_x moment of inertia of the cross-section at the point x about the axis perpendicular to the plane of bending

J_x a constant depending upon the cross-section of the beam

δ_x distance between the shear centre axis and the centroidal axis at the point x

I_{θ_x} mass moment of inertia about the shear centre per unit length at the point x

A_x area of cross-section of the beam at the point x

t time

M moment of centrifugal force at point x

χ angle made by Y-axis and axis of minor moment of inertia

C_x warping rigidity

ξ dimensionless variable ($\xi = x/L$)

ρ mass density per unit volume

θ angle of twist

T temperature excess above a given reference

α γT_o, a parameter known as temperature parameter

ω frequency of vibration

Introduction

In recent years, interest in the effect of temperature on solid bodies has highly increased because of rapid developments in space technology, high speed atmospheric flights and in nuclear energy applications. Further, in mechanical systems where certain parts of the machine have to operate under elevated temperatures its effect is far from negligible on vibrations of its various components. The reason for all these is that during heating up period of structures exposed to high intensity heat fluxes, the material properties undergo significant changes. Several authors [1-4] have studied coupled vibrations of uniform and non-uniform beams. It is well known[5] that in the presence of a constant thermal gradient, the elastic coefficients of homogeneous materials becomes functions of the space variable. Fauconneau and Marangoni[6] have investigated the effect of the non-homogeneity caused by a thermal gradient on the natural frequencies of simply supported plates of uniform thickness. Recently, Tomar and Gupta[7] have studied the effect of thermal gradient on frequencies of orthotropic elliptic plate of non-uniform thickness and temperature.

The analysis presented in this paper considers vibrations of a beam that could represent turbine blade of simple geometry. The beam is attached to a disc of radius r_0 and the disc rotates with angular velocity Ω.[8] The cross-section of the beam is linearly varying and the shear centre of each cross-section does not coincide with the centre of gravity, consequently the torsional and bending vibrations are 'coupled'. Further the beam is subjected to one dimensional temperature distribution along the length. As the cross-section of the beam varies A_x, I_x, J_x and I_{θ_x} are all functions of x.

Analysis and Equations of Motion

The governing differential equations for coupled vibrations of a rotating beam of linearly varying cross-section, according to Hallauer[1] and Tomar[8] are :

$$\frac{\partial^2}{\partial x^2}\left(E_x I_x \frac{\partial^2 W}{\partial x^2}\right) + \rho A_x \frac{\partial^2 W}{\partial t^2} - \rho A_x \delta_x \frac{\partial^2 \theta}{\partial t^2} = \frac{\partial^2 M}{\partial x^2} \qquad \ldots(1a)$$

and

$$\frac{\partial}{\partial x}\left(G_x J_x \frac{\partial \theta}{\partial x}\right) - \frac{\partial^2}{\partial x^2}\left(C_x \frac{\partial^2 \theta}{\partial x^2}\right) = -\rho A_x \delta_x \frac{\partial^2 W}{\partial t^2} + \rho A_x \delta^2{}_x \frac{\partial^2 \theta}{\partial t^2} + I_{\theta_x} \frac{\partial^2 \theta}{\partial t^2} \qquad \ldots(1b)$$

where

$$\frac{\partial^2 M}{\partial x^2} = \rho A_0 \Omega^2 \left[\left[r_0 (L - x) + \frac{1}{2}(L^2 - x^2) - \frac{\Lambda r_0}{2L}(L^2 - x^2) - \frac{\Lambda}{3L}(L^3 - x^3) \right] \frac{\partial^2 W}{\partial x^2} - \left[r_0 + x - \frac{\Lambda r_0}{L} x - \frac{\Lambda}{L} x^2 \right] \frac{\partial W}{\partial x} + \left(1 - \Lambda \frac{x}{L}\right) W \cdot \sin^2 \chi \right] \quad \ldots(2)$$

and using

$$\left.\begin{aligned} \Lambda &= \left(1 - \frac{A_L}{A_0}\right), \\ A_x &= A_0 \left(1 - \Lambda \frac{x}{L}\right), \\ I_x &= I_0 \left(1 - \Lambda \frac{x}{L}\right)^3, \\ \delta_x &= \delta_0 \left(1 - \Lambda \frac{x}{L}\right), \\ I_{\theta_x} &= I_{\theta_0} \left(1 - \Lambda \frac{x}{L}\right)^3, \\ J_x &= J_0 \left(1 - \Lambda \frac{x}{L}\right), \\ C_x &= E_x C_0 \left(1 - \Lambda \frac{x}{L}\right)^5. \end{aligned}\right\} \quad \ldots(3)$$

It is assumed that the beam is subjected to a steady one-dimensional temperature distribution along the length i.e. in x-direction

$$T = T_0 \left(\frac{e}{e - 1} - \frac{e^{\xi}}{e - 1} \right), \quad \ldots(4)$$

where T denotes the temperature excess above the reference temperature at any point at a distance $\xi = x/L$ and T_0 denotes the temperature excess above the reference temperature at the end $x = L$ or $\xi = 1$.

The temperature dependence of a modulus of elasticity for most of engineering material is given by

$$E(T) = E_1(1 - \gamma T), \quad \ldots(5)$$

where E_1 is the value of the modulus at reference temperature i.e. at $T = 0$ along x-direction.

Taking as the reference temperature, the temperature at the end of the beam i.e. at $\xi = 1$ the modulus variation becomes :

$$E(\xi) = E_1\left[1 - \alpha\left(\frac{e}{e-1} - \frac{e^{\xi}}{e-1}\right)\right], \qquad \ldots(6)$$

where $\alpha = \gamma T_0 (0 \leqslant \alpha < 1)$.

Now equations (1) using (2), (3) and (6) are put in terms of the dimensionless variable $\xi = x/L$, one gets,

$$\begin{aligned} &DW_{,\xi\xi\xi\xi} + 2D_{,\xi}W_{,\xi\xi\xi} + D_{,\xi\xi}W_{,\xi\xi} + \rho A_0(1 - \Lambda\xi)W_{,tt} \\ &\quad - \rho A_0\delta_0(1 - \Lambda\xi)^2\theta_{,tt} \\ &= \rho A_0\Omega^2\left[\left[\frac{r_0}{L}(1-\xi) + \frac{1}{2}(1-\xi^2) - \frac{\Lambda r_0}{2L}(1-\xi^2)\right.\right. \\ &\quad \left.\left. - \frac{\Lambda}{3}(1-\xi^3)\right]W_{,\xi\xi} - \left[\frac{r_0}{L} + \xi - \frac{\Lambda r_0}{L}\xi - \Lambda\xi^2\right]\right. \\ &\quad \left. W_{,\xi} + (1 - \Lambda\xi)W\sin^2\chi\right] \qquad \ldots(7a) \end{aligned}$$

$$\begin{aligned} &P\theta_{,\xi\xi} + P_{,\xi}\theta_{,\xi} - [Z\theta_{,\xi\xi\xi\xi} + 2Z_{,\xi}\theta_{,\xi\xi\xi} + Z_{,\xi\xi}\theta_{,\xi\xi}] \\ &= -\rho A_0\delta_0(1 - \Lambda\xi)^2 W_{,tt} + \rho A_0\delta_0^2(1 - \Lambda\xi)^3\theta_{,tt} + I_{\theta_0}(1 - \Lambda\xi)^3\theta_{,tt} \end{aligned} \qquad \ldots(7b)$$

where

$$\left.\begin{aligned} D &= E_1 I_0(1 - \Lambda\xi)^3\left[1 - \alpha\left(\frac{e}{e-1} - \frac{e^{\xi}}{e-1}\right)\right] \\ P &= \frac{E_1 J_0(1 - \Lambda\xi)}{2(1+\nu)}\left[1 - \alpha\left(\frac{e}{e-1} - \frac{e^{\xi}}{e-1}\right)\right] \\ Z &= C_0 E_1(1 - \Lambda\xi)^5\left[1 - \alpha\left(\frac{e}{e-1} - \frac{e^{\xi}}{e-1}\right)\right] \end{aligned}\right\} \qquad \ldots(8)$$

and a comma followed by a suffix denotes partial differentiation with respect to that variable.

If the cross-section of the blade is taken as semi-circle of radius a_0 at $x = 0$ and thickness as t_1, then

$$\begin{aligned} &\delta_0 = 2a_0/\pi,\ A_0 = \pi a_0 t_1,\ J_0 = \frac{1}{3}\pi a_0 t_1^3,\ C_0 = a_0^5 t_1\left(\frac{\pi^3}{12} - \frac{8}{\pi}\right), \\ &I_0 = \pi t_1 a_0^3(1 + 16/\pi^2),\ I_{\theta_0} = \pi\rho t_1 a_0^3\left(1 + \frac{16}{\pi^2}\right) \end{aligned} \qquad \ldots(9)$$

Determination of Frequency Parameter

The solutions of equations (7) are taken in the form

$$\left.\begin{aligned} W(\xi, t) &= Af(\xi)\,e^{i\omega t} \\ \theta(\xi, t) &= B\phi(\xi)\,e^{i\omega t} \end{aligned}\right\} \qquad \ldots(10)$$

where A and B are constants which are not independent and $f(\xi)$ and $\phi(\xi)$ are functions of ξ only.

The functions $f(\xi)$ and $\phi(\xi)$ satisfy all the boundary conditions of the beam which are as follows :

$$\left.\begin{aligned} W(0) = W_{,\xi}(0) = \theta(0) = \theta_{,\xi\xi}(0) = 0 \\ W_{,\xi\xi}(1) = W_{,\xi\xi\xi}(1) = \theta_{,\xi}(1) = \theta_{,\xi\xi\xi}(1) = 0 \end{aligned}\right\} \quad ...(11)$$

Substitution of (8) to (10) in (7) gives

$$\begin{aligned} &\frac{t_1 a_0^3}{L^4}\left(1 + \frac{16}{\pi^2}\right)\left[\left\{1 - \alpha\left(\frac{e - e^{\xi}}{e - 1}\right)\right\}(1 - \Lambda\xi)^2 f_{,\xi\xi\xi\xi}\right. \\ &\quad + 2\left[\frac{\alpha e^{\xi}}{e-1}(1 - \Lambda\xi)^3 - 3\Lambda\left\{1 - \alpha\left(\frac{e - e^{\xi}}{e - 1}\right)\right\}(1 - \Lambda\xi)^2\right] f_{,\xi\xi\xi} \\ &\quad + \left[\frac{\alpha e^{\xi}}{e-1}(1 - \Lambda\xi)^3 - \frac{6\Lambda\,\alpha e^{\xi}}{e-1}(1 - \Lambda\xi)^2\right. \\ &\quad \left. + 6\Lambda^2\left\{1 - \alpha\left(\frac{e - e^{\xi}}{e - 1}\right)\right\}(1 - \Lambda\xi)\right] f_{,\xi\xi}\Big] \\ &\qquad A - \lambda\left[(1 - \Lambda\xi) f A - \frac{2a_0}{\pi}(1 - \Lambda\xi)^2 \phi B\right] \\ &= \frac{\rho a_0 t_1 \Omega^2}{E_1}\left[\left(\frac{r_0}{L}(1 - \xi) + \frac{1}{2}(1 - \xi^2) - \frac{\Lambda r_0}{2L}(1 - \xi^2)\right.\right. \\ &\quad \left.\left. - \frac{\Lambda}{3}(1 - \xi^3)\right] f_{,\xi\xi} A - \left(\frac{r_0}{L} + \xi - \frac{\Lambda r_0}{L}\xi - \Lambda\xi^2\right] f_{,\xi} A \right. \\ &\quad \left. + (1 - \Lambda\xi) f \sin^2\chi A\right] \qquad ...(12a) \\ &\left[\frac{a_0 t_1^3}{6L^2(1+\nu)}\left[\left\{1 - \alpha\left(\frac{e - e^{\xi}}{e - 1}\right)\right\}(1 - \Lambda\xi)\phi_{,\xi\xi\xi} + \left[\frac{\alpha e^{\xi}}{e-1}(1 - \Lambda\xi)\right.\right.\right. \\ &\qquad \left.\left.\left. - \Lambda\left\{1 - \alpha\left(\frac{e - e^{\xi}}{e - 1}\right)\right\}\right]\phi_{,\xi\xi}\right] \\ &\quad - \frac{a_0^5 t_1}{L^4}\left[\frac{\pi^2}{12} - \frac{8}{\pi^2}\right]\left[(1 - \Lambda\xi)^5\left\{1 - \alpha\left(\frac{e - e^{\xi}}{e - 1}\right)\right\}\phi_{,\xi\xi\xi\xi}\right. \\ &\quad + 2[(-5\Lambda)(1 - \Lambda\xi)^4\left\{1 - \alpha\left(\frac{e - e^{\xi}}{e - 1}\right)\right\} \\ &\quad \left. + (1 - \Lambda\xi)^5\frac{\alpha e^{\xi}}{e-1}\right]\phi_{,\xi\xi\xi} + \left[20\Lambda^2(1 - \Lambda\xi)^3\left\{1 - \alpha\left(\frac{e - e^{\xi}}{e - 1}\right)\right\}\right. \\ &\quad \left. - 10\Lambda(1 - \Lambda\xi)^4\frac{\alpha e^{\xi}}{e-1} + (1 - \Lambda\xi)^5\frac{\alpha e^{\xi}}{e-1}\right]\phi_{,\xi\xi}\Big]\Big] B \\ &= \lambda\left[\frac{2a_0}{\pi}(1 - \Lambda\xi)^2 f A - \left[\left(\frac{2a_0}{\pi}\right)^2 + a_0^2\left(1 + \frac{16}{\pi^2}\right)\right](1 - \Lambda\xi)^3\phi B\right], \\ &\qquad ...(12b) \end{aligned}$$

where

$$\lambda = \frac{\rho a_0 t_1 \omega^2}{E_1} \qquad \ldots(13)$$

is a frequency parameter.

For an approximate determination of frequency parameters, $f(\xi)$ and $\phi(\xi)$ are chosen respectively as the shape functions for the different modes of bending and torsional vibrations of an uncoupled uniform cantilever beam. These shape functions satisfy the boundary conditions (11) and are

$$f(\xi) = \cosh \lambda_n \xi - \cos \lambda_n \xi - \sigma_n(\sinh \lambda_n \xi - \sin \lambda_n \xi) \qquad \ldots(14)$$

and

$$\phi(\xi) = \sin\left(\frac{2n+1}{2}\right)\pi\xi \quad n = 0, 1, 2, \ldots\ldots,$$

where λ_n and σ_n are constants corresponding to the n^{th} mode of vibrations. The values of these constants for first three modes of vibrations are taken as given in Bishop and Johnson.[9] Equations (12) can be solved for λ but the result is a function of ξ. Since f and ϕ are not the exact shape functions. This difficulty can be overcome,[10] by multiplying the equations (12*a*) and (12*b*) by f and ϕ respectively, and integrating with respect to ξ from 0 to 1. The method results in the familiar Rayleigh quotient,[11] when applied to uncoupled problems, and is extension of Rayleigh's method to coupled problem. The following equations are obtained

$$\left.\begin{aligned} (a_1 - a_2 - \lambda a_3) A + \lambda a_4 B = 0 \\ -\lambda a_4 A + (a_5 - a_6 + \lambda a_7) B = 0 \end{aligned}\right\}, \qquad \ldots(15)$$

where

$$a_1 = \frac{t_1 a_0^3}{L^4}\left(1 + \frac{16}{\pi^2}\right)\int_0^1 \left[\left\{1 - \alpha\left(\frac{e - e^\xi}{e-1}\right)\right\}(1 - \Lambda\xi)^3 f_{,\xi\xi\xi\xi}\right.$$

$$+ 2\left[\frac{\alpha e^\xi}{e-1}(1 - \Lambda\xi)^3 - 3\Lambda\left\{1 - \alpha\left(\frac{e - e^\xi}{e-1}\right)\right\}(1 - \Lambda\xi)^2\right] f_{,\xi\xi\xi}$$

$$+ \left[\frac{\alpha e^\xi}{e-1}(1 - \Lambda\xi)^3 - 6\Lambda\frac{\alpha e^\xi}{e-1}(1 - \Lambda\xi)^2 + 6\Lambda^2\right.$$

$$\left.\left\{1 - \alpha\left(\frac{e - e^\xi}{e-1}\right)\right\}(1 - \Lambda\xi)\right] f_{,\xi\xi}] f \, d\xi,$$

$$a_2 = \frac{\rho a_0 t_1 \Omega^2}{E_1}\int_0^1 \left[\left[\frac{r_0}{L}(1 - \xi) + \frac{1}{2}(1 - \xi^2) - \frac{\Lambda r_0}{2L}(1 - \xi^2)\right.\right.$$

$$\left. - \frac{\Lambda}{3}(1 - \xi^3)\right] f_{,\xi\xi} - \left(\frac{r_0}{L} + \xi - \frac{\Lambda r_0}{L}\xi - \Lambda\xi^2\right) f_{,\xi}$$

$$\left. + (1 - \Lambda\xi)\sin^2\chi f\right] f d\xi,$$

$$a_3 = \int_0^1 (1 - \Lambda\xi) f^2 d\xi,$$

$$a_4 = \frac{2a_0}{\pi} \int_0^1 (1 - \Lambda\xi)^2 \phi f \, d\xi,$$

$$a_5 = \frac{a_0 t_1^3}{6L^2(1+\nu)_0} \int_0^1 \left[\left[1 - \alpha\left(\frac{e - e^{\xi}}{e-1}\right)\right](1 - \Lambda\xi)\phi_{,\xi\xi} + \left[\frac{\alpha e^{\xi}}{e-1}(1 - \Lambda\xi) - \Lambda\left\{1 - \alpha\left(\frac{e - e^{\xi}}{e-1}\right)\right\}\right]\phi_{,\xi}\right]\phi \, d\xi,$$

$$a_6 = \frac{a_0^5 t_1}{L^4}\left(\frac{\pi^2}{12} - \frac{8}{\pi^2}\right) \int_0^1 \left[(1 - \Lambda\xi)^5 \left\{1 - \alpha\left(\frac{e - e^{\xi}}{e-1}\right)\right\}\phi_{,\xi\xi\xi\xi} - 2\left[5\Lambda(1 - \Lambda\xi)^4\left\{1 - \alpha\left(\frac{e - e^{\xi}}{e-1}\right)\right\} - (1 - \Lambda\xi)^5 \frac{\alpha e^{\xi}}{e-1}\right]\phi_{,\xi\xi\xi} + \left[20\Lambda^2(1 - \Lambda\xi)^3\left\{1 - \alpha\left(\frac{e - e^{\xi}}{e-1}\right)\right\} - 10\Lambda(1 - \Lambda\xi)^4 \frac{\alpha e^{\xi}}{e-1} + (1 - \Lambda\xi)^5 \frac{\alpha e^{\xi}}{e-1}\right]\phi_{,\xi\xi}\right]\phi \, d\xi$$

and $$a_7 = \left[\left(\frac{2a_0}{\pi}\right)^2 + a_0^2\left(1 + \frac{16}{\pi^2}\right)\right] \int_0^1 (1 - \Lambda\xi)^3 \phi^2 \, d\xi. \qquad \ldots(16)$$

For a non-trivial solution A and B must not both vanish, consequently the determinant of the coefficients of equations (15) must be zero.

$$\begin{vmatrix} a_1 - a_2 - \lambda a_3 & \lambda a_4 \\ -\lambda a_4 & a_5 - a_6 + \lambda a_7 \end{vmatrix} = 0$$

i.e. $$Q\lambda^2 - R\lambda + S = 0, \qquad \ldots(17)$$

where

$$Q = a_4^2 - a_3 a_7,$$

$$R = (a_5 - a_6)\, a_3 - (a_1 - a_2)\, a_7$$

and $$S = (a_1 - a_2)\,(a_5 - a_6).$$

The solution of equation (17) is

$$\lambda = \frac{R \pm \sqrt{R^2 - 4QS}}{2Q} \qquad \ldots(18)$$

The right hand side of equation (18) is positive since it may be shown that $R^2 - 4QS > 0$ for real value of λ and value of λ is always positive. The smallest of the two values of λ given by equation (18) gives the frequency for coupled torsional vibrations of the particular mode. The larger of the two values of λ is the frequency of the next higher mode of coupled vibrations.

Results and Conclusion

The equations of motion for coupled vibrations of a rotating beam with linearly varying semi-circular cross-section having exponentially temperature distribution along the length of the beam are obtained. The method based on Rayleigh's quotient has been used to solve the equations of motion of the system under consideration to gives upper bounds of the frequencies correponding to first three modes of coupled vibrations. The accuracy and the convergence of the method has already been investigated by Tomar[8] specially where δ_x is small. The physical constants are taken as follows:

$$L = .1524\text{m}, E_1 = 2.0685 \times 10^{11}\ N/\text{m}^2, t_1 = .001727\text{m},$$

$$a_0 = .0127\text{m}, \rho = 7.8576 \times 10^3\ \text{kg/m}^3, \nu = 0.25.$$

The frequency parameters for various values of α, Λ and r_0/L have been tabulated in tables 1, 2 and 3 and plotted in Figs. 1 and 2, for first three modes of vibrations.

Table I

Frequency parameter λ/λ_0 for $r_0/L = 2.0$ and $\Lambda = 0.75$

α	First mode	Second mode	Third mode
0.0	5.09147337	21.19464381	76.02817894
0.1	4.67691228	19.68186187	70.32282770
0.2	4.26234873	18.16908315	64.61750433
0.3	3.84778068	16.65629675	58.91219290
0.4	3.43320642	15.14350128	53.20684284
0.5	3.01862238	13.63068988	47.50151473
0.6	2.60402250	12.11785936	41.79616496
0.7	2.18939564	10.60499509	36.09079327
0.8	1.77471465	9.09207414	30.38539025
0.9	1.35989790	7.57903577	24.67991685

Table II

Frequency parameter λ/λ_0 for $r_0/L = 2.0$ and $\alpha = 0.4$

Λ	First mode	Second mode	Third mode
0.00	0.66055975	10.75935463	49.11120707
0.25	1.10107991	11.96949725	50.76804686
0.50	1.87137931	13.48015453	51.09351929
0.75	3.43320642	15.14350128	53.20684284
1.00	6·26755917	16.19872992	53.76212567

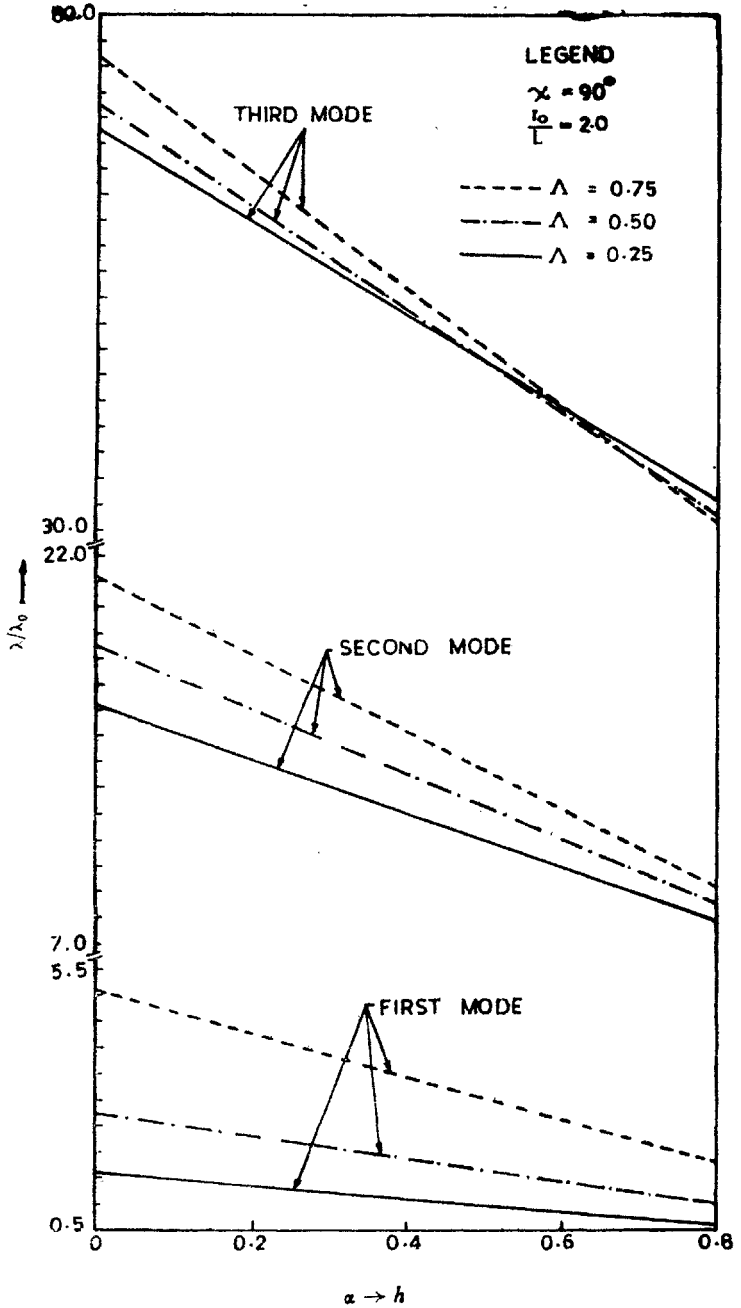

FIG 1 Variation of frequency parameter with temperature parameter for different values of cross-section variation.

The calculated value of frequency parameter λ_0 of unheated uniform rotating beam ($\alpha = 0.0$, $\Lambda = 0.0$, $r_0/L = 2.0$, $\Omega = 314\ S^{-1}$, $\chi = 90°$) for first mode is $2.05273698 \times 10^{-6}$.

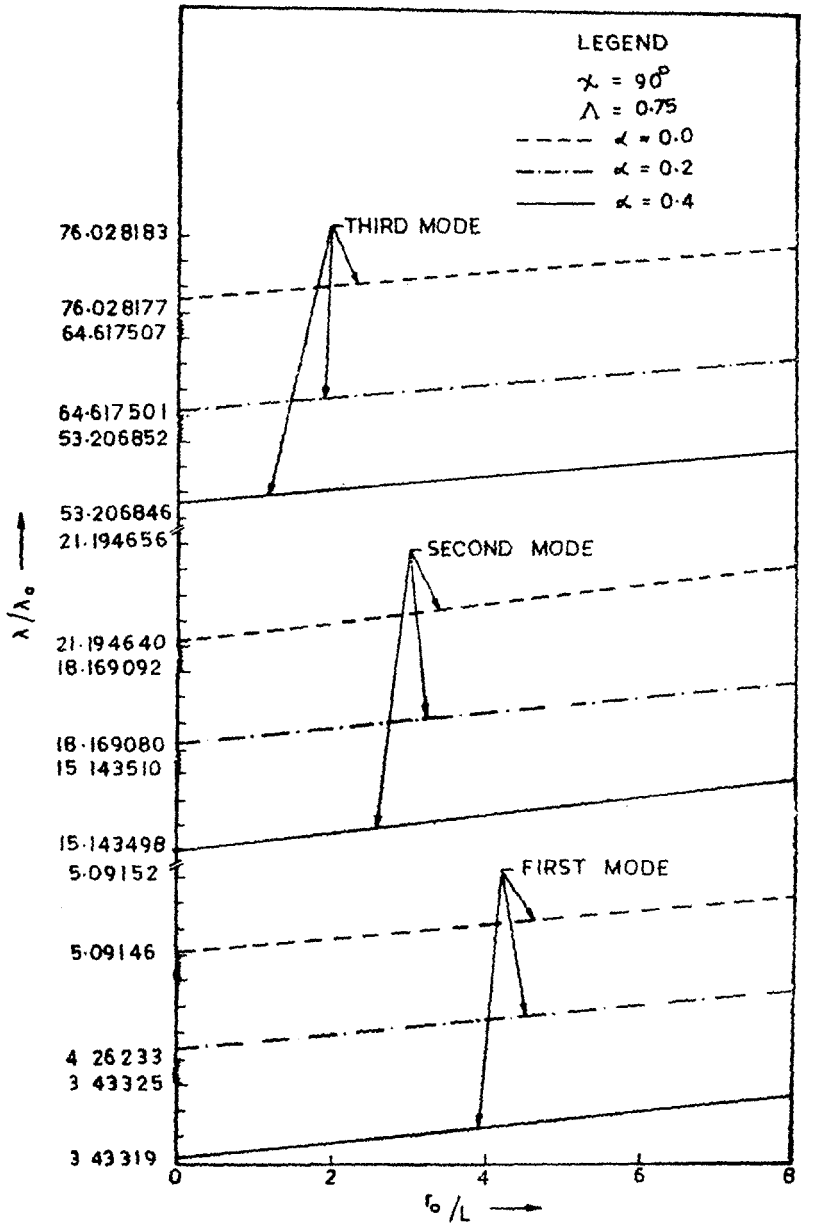

FIG 2 Variation of frequency parameter with hub-radius ratio for different values of temperature parameter.

The values of frequency parameter λ for $\alpha = 0.2$, $\Delta = 0.0$, $r_0/L = 2.0$ and $\Omega = 314\ S^{-1}$ have been found to decrease 16.9722 per cent in first mode, 12.9263 per cent in second mode and 13.1755 per cent in third mode of vibrations in comparison to unheated uniform rotating beam. It can be observed that the effect

TABLE III

Frequency parameter λ/λ_0 *for* $\alpha = 0.4$ *and* $\Lambda = 0.75$

r_0/L	First mode	Second mode	Third mode
0	3.43319392	15.14349822	53.20684732
2	3.43320642	15.14350128	53.20684825
4	3.43321849	15.14350421	53.20684937
6	3.43323135	15.14350697	53.20685058
8	3.43324369	15.14350974	53.20685176
10	3.43325616	15.14351263	53.20685283

of temperature parameter on frequency parameter for third mode of vibrations is higher than that of the second mode of vibrations and lower than that of the first mode of vibrations.

In Fig.1, the effect of temperature parameter on the frequency parameter corresponding to different values of cross-section variation for first three modes of vibrations have been shown. It has been observed that the frequencies decrease with the increase in temperature parameter α, while the frequencies increase with the increase in cross-section variation Λ, for first three modes of vibrations. Also, it shows that the nature of frequency curves changes in the third mode of vibrations as the numerical value of α exceeds 0.5. The reason for this may be that the realistic value of which α can take should be less than 0.5. This fact (i.e. $\alpha < 0.5$) is also observed by Tomar and Gupta[13] while applying the condition of convergence on the solution by Frobineous method.

The results presented in Fig. 2 shows the variation of frequency parameter with hub-radius ratio for different values of temperature parameter corresponding to the first three modes of vibrations. It has been concluded that the frequencies increase with increase in hub-radius ratio r_0/L for all the three modes of vibrations.

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