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THERMO-PIEZO STRESSES IN A PLATE

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This paper presents the response of a piezoelectric plate of finite thickness, free at both ends, to the imposition of prescribed thermal and polarisation fields. Small-time solutions for stress, electric intensity and potential difference across the plate have been derived by using Laplace-transform method. These expressions neatly breakup into two principal parts induced independently by the applied fields. The existence of a constraint nullifying thermal effects on stress field is predicted.

Key Words: Piezoelectric Plate; Thermal and Polarisation Fields; Small-Time Solution; Stress

1. INTRODUCTION

THERMAL stresses play an important and often a primary role in numerous problems in high temperature technology e.g. in aircraft and machine structures, rocket powered high speed flight etc. Moreover, for ceramics and cermals, we seek materials to withstand the mutation of an abnormal temperature process. Some problems on thermo-elastic stresses in bodies in the form of plates have recently been solved by Misra and Achari,[1] Wu and Taucher[2] and others.

The piezoelectric problems constitute an important branch of study in view of their applications in ultrasonics and acoustic engineering, electronics and manifold branches of physics. The piezoelectric devices operate satisfactorily when the ambient temperature is low. But at the elevated temperature, the performance and operational charateristics of these devices change to a great extent. These problems provide the situation when mechanical and electric fields interact with each other. Certainly, the studies become more interesting when this interaction is coupled with a thermal field. A few problems on thermo-piezoelectricity have recently been solved by Knap,[3] Kundu,[4] and Aronov.[5]

The present communication analyses the behaviour of a piezoelectric plate of finite thickness, free at both ends, when subjected to an electric polarisation gradient decaying exponentially with time along the thickness direction and exposed to a uniform heat input at one face, the other face being insulated. This study does not appear to have engaged the attention of researchers so far. Explicit expressions for thermo-piezo stress, electric intensity and potential difference across the faces of the plate, valid for small interval of time, have been obtained by using the method of Laplace transformation. It is found that these expressions beautifully separate out into two main parts induced independently by the applied fields. It is also revealed that there exists a critical relationship among certain constants for which the thermally induced component of stress vanishes even in presence of heat input. The other two quantities do not display such behaviour.

GOVERNING EQUATIONS

We consider a piezoelectric plate infiinite in extent in y and Z-directions and bounded by planes $x = 0$ and $x = L$. The plate is free at both ends and it is subjected to a time decaying electric polarisation gradient $\partial p_1/\partial x$ along x-axis (thickness direction). The surface $x = 0$ is kept perfectly insulated while surface $x = L$ is exposed to a uniform heat input q for time $t > 0$. The initial temperature of the slab is zero. Let ρ be the material density of the plate, c its specific heat, K the thermal conductivity, h the thermal diffusivity, C_{11} the elastic stress coefficient, K_{11} the electromechanical coupling factor, p_1 the pyroelectric contant, λ_1 the temperature coefficient, ξ the displacement, S_1 the normal strain, T_1 and E_1 respectively the stress and electric intensity developed at any point in the plate, α the decay constant, θ the temperature and e_{11} the piezoelectric stress coefficient. The piezoelectric equations due to Mindlin in the present case, are

$$T_1 = C_{11}S_1 + e_{11}E_1 + \lambda_1\theta \qquad \ldots 1(a)$$

and

$$P_1 = e_{11}S_1 + K_{11}E_1 + p_1\theta \qquad \ldots 1(b)$$

where

$$S_1 = \frac{\partial \xi}{\partial x}, \quad -\frac{\partial P_1}{\partial x} = P_0 e^{-\alpha t}, \ \alpha > 0, \ P_0 \text{ a constant} \qquad \ldots(2)$$

The equation of mechanical motion is given by

$$\frac{\partial T_1}{\partial x} = \rho \frac{\partial^2 \xi}{\partial t^2} \qquad \ldots(3)$$

The corresponding boundary value problem for heat flow is

$$\begin{aligned} h\frac{\partial^2\theta}{\partial x^2} &= \frac{\partial\theta}{\partial t}, && 0 < x < L, \ t > 0 \\ \frac{\partial\theta}{\partial x} &= 0, && x = 0, \quad t > 0 \\ K\frac{\partial\theta}{\partial x} &= q, && x = L, \quad t > 0 \\ \theta &= 0, && 0 < x < L, \quad t = 0 \end{aligned} \qquad \ldots(4)$$

Combining (1) and (3), we derive

$$\frac{\partial^2\xi}{\partial x^2} + \frac{\rho}{\left(\frac{e_{11}^2}{k_{11}} - C_{11}\right)}\frac{\partial^2\xi}{\partial t^2} = \frac{e_{11}}{K_{11}\left(\frac{e_{11}^2}{K_{11}} - C_{11}\right)}\frac{\partial P_1}{\partial x} + \frac{\left(\lambda_1 - \frac{p_1 e_{11}}{K_{11}}\right)}{\left(\frac{e_{11}^2}{K_{11}} - C_{11}\right)}\frac{\partial\theta}{\partial x} \qquad \ldots(5)$$

The stress T_1 satisfies the folowing boundary conditions

$$T_1 = 0 \text{ at } x = 0, \quad x = L, \ t > 0 \qquad \ldots(6)$$

3. Solution

Solving (4), we obtain[7]

$$\theta(x, t) = \frac{q}{KL}\left[ht + \frac{3x^2 - L^2}{6} - \frac{2L^2}{\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}e^{-\beta t}\cos\frac{n\pi x}{L}\right]$$

with $\beta = \dfrac{n^2\pi^2 b}{L^2}$. ...(7)

Taking Laplace transformation (of parameter p) of equations (5), (2) and of $\dfrac{\partial\theta}{\partial x}$ obtained from (7) and combining we derive

$$\frac{\partial^2\bar{\xi}}{\partial x^2} = \frac{p^2}{v^2}\bar{\xi} + \frac{\lambda P_0}{p+\alpha} - \frac{\mu q}{KL}\left[\frac{x}{p} + \frac{2L}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{n(p+\beta)}\sin\left(\frac{n\pi x}{L}\right)\right], \quad ...(8)$$

where

$$v^2 = \frac{\left(C_{11} - \dfrac{e_{11}^2}{K_{11}}\right)}{\rho}, \quad \lambda = \frac{e_{11}}{K_{11}\left(C_{11} - \dfrac{e_{11}^2}{K_{11}}\right)}, \quad \mu = \frac{\left(\lambda_1 - \dfrac{p_1 e_{11}}{K_{11}}\right)}{\left(C_{11} - \dfrac{e_{11}^2}{K_{11}}\right)} \quad ...(9)$$

and where bar (—) over a quantity denotes its laplace transform. Solving (8) under the following transformed boundry conditions

$$\bar{T}_1 = 0 \text{ at } x = 0, \quad x = L \quad ...(10)$$

proceeding as per Redwood[8] and taking inverse we get

$$\xi = \frac{\mu v^3 q}{KL}$$

$$\frac{1}{6}\left[\sum_{m=1}^{\infty}\left\{\left(t - \frac{(2mL - x)}{v}\right)^3 + \left(t - \frac{(2mL + x)}{v}\right)^3 - \left(t - \frac{(\overline{2m-1}\,L - x)}{v}\right)^3 - \left(t - \frac{(\overline{2m-1}\,L + x)}{v}\right)^3\right\} + \left(t - \frac{x}{v}\right)^3\right] + 2\sum_{n=1}^{\infty}(-1)^n\left[\frac{1}{\beta y}\left\{\sum_{m=1}^{\infty}\right.\right.$$

$$\cdot\left(U\left(t - \frac{(2mL - x)}{v}\right) + U\left(t - \frac{(2mL + x)}{v}\right) - \cos n\pi\right.$$

$$\left.\left.\left.\times\left(U\left(t - \frac{(\overline{2m-1}\,L - x)}{v}\right) + U\left(t - \frac{(\overline{2m-1}\,L + x)}{v}\right)\right)\right)\right.$$

$$+ U\left(t - \frac{x}{v}\right)\Big\} - \frac{1}{\beta(\beta^2 + y)}\left\{\sum_{m=1}^{\infty}\left(\exp\left(-\beta\left(t - \frac{(2mL + x)}{v}\right)\right)\right.\right.$$

$$\left.\left. + \exp\left(-\beta\left(t - \frac{(2mL - x)}{v}\right)\right)\right) + \exp\left(-\beta\left(t - \frac{x}{v}\right)\right)\right\} - \frac{\beta}{y(\beta^2 + y)}$$

$$\times\left\{2\cos\sqrt{y}\left(\frac{x}{v}\right)\sum_{m=1}^{\infty}\left(\cos\sqrt{y}\left(t - \frac{2mL}{v}\right) - \cos n\pi\cos\sqrt{y}\right.\right.$$

$$\left.\left.\times\left(t - \frac{\overline{2m-1}\,L}{v}\right)\right) + \cos\sqrt{y}\left(t - \frac{x}{v}\right)\right\} - \frac{1}{\sqrt{y}(\beta^2 + y)}$$

$$\times\left\{2\cos\sqrt{y}\left(\frac{x}{v}\right)\sum_{m=1}^{\infty}\left(\sin\sqrt{y}\left(t - \frac{2mL}{v}\right) - \cos n\pi\sin\sqrt{y}\left(t - \frac{\overline{2m-1}\,L}{v}\right)\right.\right.$$

$$\left.\left. + \sin\sqrt{y}\left(t - \frac{x}{v}\right)\right\}\right]\Big] + \frac{\mu v q}{KL}\left[\frac{h}{2}\left\{\sum_{m=1}^{\infty}\left(\left(t - \frac{(2mL - x)}{v}\right)^2\right.\right.\right.$$

$$+ \left(t - \frac{(2mL + x)}{v}\right)^2 - \left(t - \frac{\overline{(2m-1}\,L - x)}{2}\right)^2 - \left.\left.\left(t - \frac{\overline{2m-1}\,L + x}{v}\right)^2\right)\right.$$

$$\left. + \left(t - \frac{x}{v}\right)^2\right\} - \frac{L^2}{6}\left\{6\sum_{m=1}^{\infty}\left(t - \frac{2mL}{v}\right) + \left(t + \frac{2L - x}{v}\right)\right\} - \frac{2L^2}{\pi^2}$$

$$\times\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\left\{\frac{1}{\beta}\left\{\sum_{m=1}^{\infty}\left(U\left(t - \frac{(2mL - x)}{v}\right) + U\left(t - \frac{(2mL + x)}{v}\right)\right.\right.\right.$$

$$\left.\left.\left. - \cos n\pi\left(U\left(t - \frac{\overline{(2m-1}\,L - x)}{v}\right) + U\left(t - \frac{\overline{(2m-1}\,L + x)}{v}\right)\right)\right)\right.$$

$$\left. + U\left(t - \frac{x}{v}\right)\right\} - \frac{1}{\beta}\left\{\sum_{m=1}^{\infty}\left(\exp\left(-\beta\left(t - \frac{(2mL + x)}{v}\right)\right)\right.\right.$$

$$+ \exp\left(-\beta\left(t - \frac{(2mL - x)}{v}\right)\right) - \cos n\pi\left(\exp\left(-\beta\left(t - \left(\frac{\overline{(2m-1}\,L + x)}{v}\right)\right)\right)\right.$$

$$\left.\left. + \exp\left(-\beta\left(t - \frac{\overline{(2m-1}\,L - x)}{v}\right)\right)\right) + \exp\left(-\beta\left(t - \frac{x}{v}\right)\right)\right\}\Big\}\Big]$$

$$+\frac{\nu\lambda P_0 L}{\alpha}\left[\sum_{m=1}^{\infty}\left\{U\left(t-\frac{(2\overline{m-1}\,L-x)}{\nu}\right)+U\left(t-\frac{(2\overline{m-1}\,L+x)}{\nu}\right)\right.\right.$$

$$\left.\left.-\exp\left(-\alpha\left(t-\frac{(2\overline{m-1}\,L+x)}{\nu}\right)\right)-\exp\left(-\alpha\left(t-\frac{(2\overline{m-1}\,L-x)}{\nu}\right)\right)\right\}\right]$$

$$+\frac{\nu^2 P_0\lambda}{\alpha^2}\left[1-\alpha t-e^{-\alpha t}\right]+\frac{\mu\nu^2 q}{KL}\left[\frac{xt^2}{2}+\frac{2L}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\right.$$

$$\left.\cdot\frac{\sin\left(\frac{n\pi x}{L}\right)}{(\beta^2+y)}\left\{e^{-\beta t}-\cos\sqrt{y}\,t+\frac{\beta}{\sqrt{y}}\sin\sqrt{y}\,t\right\}\right],\quad \text{for } t>\frac{2mL+x}{\nu},$$

where

$$y=\frac{n^2\pi^2\nu^2}{L^2} \qquad \ldots(11)$$

and where $U_{(t-a)}$ is a Heaviside step function defined by

$$U_{(t-a)}=1,\quad t>a$$
$$=0,\quad t<a$$

Calculation of Stress

Eliminating E_1 from equations (1) and using (2), (7) and (11) we derive

$$T_1(x,t)=\frac{q}{KL}\left(\lambda_1-\frac{p_1 e_{11}}{K_{11}}\right)\left[\left[\frac{1}{2}\left\{x(2\nu t-\overline{x+4L})\right\}-2\nu^2\sum_{m=1}^{\infty}\frac{(-1)^n}{\beta^2+y}\right.\right.$$

$$\times\left\{\left\{\sum_{m=1}^{\infty}\left(\exp\left(-\beta\left(t-\frac{(2mL+x)}{\nu}\right)\right)\right.\right.\right.$$

$$-\exp\left(-\beta\left(t-\frac{(2mL-x)}{\nu}\right)\right)-\cos\pi$$

$$\times\left(\exp\left(-\beta\left(t-\frac{(2\overline{m-1}\,L+x)}{\nu}\right)\right)\right.$$

$$\left.\left.\left.\exp\left(-\beta\left(t-\frac{(2\overline{m-1}\,L-x)}{\nu}\right)\right)\right)\right)+\exp\left(-\beta\left(t-\frac{x}{\nu}\right)\right.$$

$$\left.\left.-\cos\left(\frac{n\pi x}{L}\right)e^{-\beta t}\right\}-\frac{\beta}{\sqrt{y}}\left\{2\sin\sqrt{y}\left(\frac{x}{\nu}\right)\sum_{m=1}^{\infty}\left(\cos\sqrt{y}\left(t-\frac{2mL}{\nu}\right)\right.\right.\right.$$

$$- \cos n\pi \cos \sqrt{y}\left(t - \frac{\overline{2m-1}\,L}{v}\right)\Big) - \sin \sqrt{y}\left(t - \frac{x}{v}\right)$$

$$+ \sin \sqrt{y}\, t \cos\left(\frac{n\pi x}{L}\right)\Big\} - \Big\{2 \sin \sqrt{y}\left(\frac{x}{v}\right)$$

$$\sum_{m=1}^{\infty}\left(\sin \sqrt{y}\left(t - \frac{2mL}{v}\right) - \cos n\pi \sin \sqrt{y}\left(t - \frac{\overline{2m-1}L}{v}\right)\right)$$

$$+ \cos \sqrt{y}\left(t - \frac{x}{v}\right) - \cos \sqrt{y}\, t \cos\left(\frac{n\pi x}{L}\right)\Big\}\Big\}\Big] + \Big[\frac{hx}{v} + \frac{x^2}{2}$$

$$+ \frac{2L^2}{\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\Big\{\sum_{m=1}^{\infty}\left(\exp\left(-\beta\left(t - \frac{(2mL+x)}{v}\right)\right)\right.$$

$$- \exp\left(-\beta\left(t - \frac{(2mL-x)}{v}\right)\right)$$

$$- \cos n\pi\left(\exp\left(-\beta\left(t - \frac{\overline{(2m-1}\,L+x)}{v}\right)\right)\right)$$

$$- \exp\left(-\beta\left(t - \frac{\overline{(2m-1}\,L-x)}{v}\right)\right) + \exp\left(-\beta\left(t - \frac{x}{v}\right)\right)$$

$$- e^{-\beta t}\cos\left(\frac{n\pi x}{L}\right)\Big\}\Big]\Big] - \frac{e_{11}P_0}{K_{11}}\left[\frac{1}{\left(c_{11} - \frac{e_{11}^2}{K_{11}}\right)}\Big[\sum_{m=1}^{\infty}\right.$$

$$\times\Big\{\exp\left(-\alpha\left(t - \frac{\overline{(2m-1}\,L+x)}{v}\right)\right)$$

$$- \exp\left(-\alpha\left(t - \frac{\overline{(2m-1}\,L-x)}{v}\right)\right)\Big\}\Big] + xe^{-\alpha t}\Big] \qquad \ldots(12)$$

Electric Intensity

Combining (1) (b), (2), (7) and (11) we derive

$$E_1(x,t) = \frac{q}{\overline{KK_{11}}L}\left[\frac{\left(\lambda_1 - \frac{p_1 e_{11}}{K_{11}}\right)e_{11}}{\rho}\Big[-\frac{x}{v}\left\{2t - \frac{x+4L}{v}\right\}\right.$$

$$+ 2\sum_{n=1}^{\infty}\frac{(-1)^n}{\beta^2 + y}\Big\{\Big\{\sum_{m=1}^{\infty}\left(\exp\left(-\beta\left(t - \frac{(2mL+x)}{v}\right)\right)\right.$$

$$- \exp\left(-\beta\left(t - \frac{(2mL - x)}{v}\right)\right) - \cos n\pi$$

$$\times \left(\exp\left(-\beta\left(t - \frac{(\overline{2m-1}\,L + x)}{v}\right)\right)\right.$$

$$\left. - \exp\left(-\beta\left(t - \frac{(\overline{2m-1}\,L - x)}{v}\right)\right)\right)\Big\} - \exp\left(-\beta\left(t - \frac{x}{v}\right)\right)$$

$$+ e^{-\beta t} \cos\left(\frac{n\pi x}{L}\right)\Big\} - \frac{\beta}{\sqrt{y}}\left\{2 \sin\sqrt{y}\left(\frac{x}{y}\right)\sum_{m=1}^{\infty}\left(\cos\sqrt{y}\right.\right.$$

$$\times\left(t - \frac{2mL}{v}\right) - \cos n\pi \cos\sqrt{y}\left(t - \frac{\overline{2m-1}\,L}{v}\right)\Big) - \sin\sqrt{y}$$

$$\times\left(t - \frac{x}{y}\right) + \sin\sqrt{y}\, t \cos\left(\frac{n\pi x}{L}\right)\Big\} - \Big\{2 \sin\sqrt{y}\left(\frac{x}{v}\right)\sum_{m=1}^{\infty}$$

$$\times\left(\sin\sqrt{y}\left(t - \frac{2mL}{v}\right) - \cos n\pi \sin\sqrt{y}\left(t - \frac{\overline{2m-1}\,L}{v}\right)\right)$$

$$+ \cos\sqrt{y}\left(t - \frac{x}{v}\right) - \cos\sqrt{y}\, t \cos\left(\frac{n\pi x}{L}\right)\Big\}\Big\}\Big] + \Big[\, h\left\{p_1 t - \mu e_{11}\right.$$

$$\times\left(t - \frac{x}{v}\right)\Big\} + \frac{1}{6}\{\mu e_{11}L^2 - p_1(L^2 - 3x^2)\} + \frac{2}{\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}$$

$$\times\left\{\mu e_{11}\left\{\sum_{m=1}^{\infty}\left(\exp\left(-\beta\left(t - \frac{(2mL + x)}{z}\right)\right)\right.\right.\right.$$

$$- \exp\left(-\beta\left(t - \frac{(2mL - x)}{v}\right)\right) - \cos n\pi$$

$$\times\left(\exp\left(-\beta\left(t - \frac{(\overline{2m-1}\,L + x)}{v}\right)\right)\right.$$

$$\left.\left.\left. - \exp\left(-\beta\left(t - \frac{(\overline{2m-1}\,L - x)}{v}\right)\right)\right)\right)$$

$$\left.\left. + \exp\left(-\beta\left(t - \frac{x}{v}\right) - p_1\, e^{-\beta t} \cos\left(\frac{n\pi x}{L}\right)\right\}\right\}\right] + \frac{P_0}{K_{11}}$$

$$\times\left[\lambda L e_{11}\left[\sum_{m=1}^{\infty}\left\{\exp\left(-\alpha\left(t-\frac{(\overline{2m-1}\,L+x)}{v}\right)\right)\right.\right.\right.$$

$$\left.\left.\left.-\exp\left(-\alpha\left(t-\frac{(\overline{2m-1}\,L-x)}{v}\right)\right)\right\}\right]-xe^{-\alpha t}\right] \qquad \ldots(13)$$

Potential Difference

The potential difference across the plate is given by

$$V=-\int_{x=0}^{x=L}E\,\mathrm{d}x \qquad \ldots(14)$$

Substituting (13) in (14), we obtain

$$V_{(t)}=\frac{q}{KK_{11}L}\left[\mu e_{11}\left[\frac{L^2}{3}(3vt-7L)-2v^3\sum_{n=1}^{\infty}\frac{(-1)^n}{\beta^2+\gamma}\left\{\frac{1}{\beta}\left\{\sum_{m=1}^{\infty}\right.\right.\right.\right.$$

$$\times\left(\exp\left(-\beta\left(t-\frac{\overline{2m+1}\,L}{v}\right)\right)-2\exp\left(-\beta\left(t-\frac{2mL}{v}\right)\right)\right.$$

$$+\exp\left(-\beta\left(t-\frac{\overline{2m-1}\,L}{v}\right)\right)-\cos n\pi\left(\exp\left(-\beta\left(t-\frac{2mL}{v}\right)\right)\right.$$

$$-2\exp\left(-\beta\left(t-\frac{\overline{2m-1}\,L}{v}\right)\right)$$

$$\left.\left.+\exp\left(-\beta\left(t-\frac{2(m-1)\,L}{v}\right)\right)\right)-\exp\left(-\beta\left(t-\frac{L}{v}\right)\right)$$

$$\left.+e^{-\beta t}\right\}+\frac{\beta}{\sqrt{\gamma}}\left\{2\left(\cos\sqrt{\gamma}\,\frac{L}{v}-1\right)\right.$$

$$\times\sum_{m=1}^{\infty}\left(\cos\sqrt{\gamma}\left(t-\frac{2mL}{v}\right)-\cos n\pi\cos\sqrt{\gamma}\left(t-\frac{\overline{2m-1}\,L}{v}\right)\right)$$

$$\left.+\cos\sqrt{\gamma}\left(t-\frac{L}{v}\right)-\cos\sqrt{\gamma}\,t\right\}+\frac{1}{\sqrt{\gamma}}\left\{2\left(\cos\sqrt{\gamma}\,\frac{L}{v}-1\right)\right.$$

$$\times\sum_{m=1}^{\infty}\left(\sin\sqrt{\gamma}\left(t-\frac{2mL}{v}\right)-\cos n\pi\sin\sqrt{\gamma}\left(t-\frac{\overline{2m-1}\,L}{v}\right)\right)$$

$$\left.\left.\left.+\sin\sqrt{\gamma}\left(t-\frac{L}{v}\right)-\sin\sqrt{\gamma}\,t\right\}\right\}\right]+\left[hL\left\{p_1t-\mu e_{11}\right.\right.$$

$$\times\left(t-\frac{L}{2v}\right)\Big\}+\frac{\mu e_{11}L^3}{6}+\frac{2v}{\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\left\{\frac{\mu e_{11}}{\beta}\left\{\sum_{m=1}^{\infty}\right.\right.$$

$$\left(\exp\left(-\beta\left(t-\frac{\overline{2m+1}\,L}{v}\right)\right)-2\exp\left(-\beta\left(t-\frac{2mL}{v}\right)\right)\right.$$

$$+\exp\left(-\beta\left(t-\frac{\overline{2m-1}\,L}{v}\right)\right)-\cos n\pi$$

$$\times\left(\exp\left(-\beta\left(t-\frac{2mL}{v}\right)\right)-2\exp\left(-\beta\left(t-\frac{\overline{2m-1}\,L}{v}\right)\right)\right.$$

$$\left.\left.\left.+\exp\left(-\beta\left(t-\frac{2(m-1)\,L}{v}\right)\right)\right)\right)+\exp\left(-\beta\left(t-\frac{L}{v}\right)\right)$$

$$\left.\left.-e^{-\beta t}\right\}\right\}\Big]\Big]-\frac{P_0}{K_{11}}\left[\frac{\lambda L e_{11}v}{\alpha}\left[\sum_{m=1}^{\infty}\left\{\exp\left(-\alpha\left(t-\frac{2mL}{v}\right)\right)\right.\right.\right.$$

$$-2\exp\left(-\alpha\left(t-\frac{\overline{2m-1}\,L}{v}\right)\right)$$

$$\left.\left.\left.+\exp\left(-\alpha\left(t-\frac{2(m-1)\,L}{v}\right)\right)\right\}\right]-\frac{L^2}{2}e^{-\alpha t}\right] \qquad \ldots(15)$$

Discussion

(*A*) *Thermo-Piezo Stresses*

Expression (12) represents the dynamic stress at any point x at time t It contains terms which are linear, exponential and periodic functions of space-time coordinates and also contains terms which are quadratic function of spatial coordinate. It also shows that the value of stress at any time t depends on its value at time t as well as its values at the retarded times like $\left(t-\frac{2mL\pm x}{v}\right)$ where m takes positive integral values. In other words, the stress at any time depends on its present value as well as its past history specified by the retarded times. Alternatively, it may also be said that the stress at any point x depends not only on its value at x but also on the contributions made by stresses at various other points specified by $L-x$, $L+x$, $2L-x$, $2L+x$, ...etc . . in the semi-infinite solid. For reasonable accuracy, it is sufficient to consider only first few terms of the n-series as the magnitude of successive terms decrease with increasing n. Similarly, only first few terms in m-series need be retained so as not to violate the inequality $t>\frac{2mL+x}{v}$. Further, expression (12) remains finite, whether the decay constant α is large or small. The quantity v (*vide* expression 9) plays the role of velocity of propagation of the disturbance.

The expression for stress T_1 can be divided into two principal components. The first component is induced thermally and vanishes when there is no heat input. The second component arises due to polarisation field impressed on the plate and vanishes in its absence. It is interesting to observe that if

$$\lambda_1 k_{11} = p_1 e_{11} \qquad ...(16)$$

the thermal component of stress vanishes at all points for all time even in the presence of thermal influx but the other component arising due to polarisation field remains unaffected under this criterion. Thus expression (16) defines an important relation satisfied by the critical values of the constants involved. Moreover, when

$$C_{11} K_{11}^2 = e_{11}^2 \qquad ...(17)$$

the velocity v vanishes and the stress becomes indeterminate as the inequality $t > \frac{2mL + x}{v}$ is violated. When $c_{11}\ k_{11}\ e_{11}^2$, the velocity and stress fields are real. When $c_{11}\ K_{11} < e_{11}^2$, the velocity becomes imaginary and stress becomes complex whose real part alone is physically meaningful, When density of the meterial is very small, the velocity v becomes very large, Consequently, the stress also becomes very large making the plate unstable.

(B) Electric Intensity and Potential Field

The general behaviour of electric intensity and potenial field [*vide* expressions (13) and (15)] is similar to that of stress field except that the thermally induced components of these quantities do not vanish when $\lambda_1 K_{11} = p_1 e_{11}$ but indeed have a finite value. Thus, there is no evidence of any critical relation ship of the type (16) among the constants. Moreover, while the variation of electric intensity with decay constant α is similar to that of stress, the potential difference acquires large value when α is small.

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