© *Proc. Indian natn. Sci. Acad.*, **52**, A, No. 3, 1986, pp. 573–583.
Printed in India.

# HALL EFFECTS ON HYDROMAGNETIC CHANNEL FLOW UNDER AN INCLINED MAGNETIC FIELD

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(*Received 30 March 1984; after revision 6 August 1985*)

The influence of Hall Currents on the generalised Couette flow of a viscous, conducting fluid under an inclined magnetic field is analysed. A closed form solution is obtained using Laplace transform technique. The effects resulting from the inclination of the magnetic field and the secondary flow induced by the Hall current are clearly brought out in the various profiles of the velocity distribution.

**Key Words : Hall Effect; Hydromagnetic Channel Flow; Inclined Magnetic Field; Laplace Transform**

## INTRODUCTION

IT is well known that a number of astronomical bodies viz., the Sun, the planets, the magnetic stars, pulsars etc., possess fluid interior and at least surface magnetic field. Hence, any flow phenomenon occurring in a celestial body takes place under the influence of external magnetic field. Magnetohydrodynamic Couette flow is first studied by Lehnert[1] assuming no applied electric field. The effect of magnetic field on the generalised Couette flow has been studied by Agarwal.[2] In the presence of a strong magnetic field, the effect of Hall currents is to be taken into account due to gyration and the drift of charged particles. Sato,[3] Yamanishi,[4] Shermon and Sutton,[5] have discussed the Hall effects on the steady hydromagnetic flow between two parallel plates.

In all the above investigations the applied magnetic field is always normal to the channel walls. Recently, Bharali and Borkakati[6] have investigated the steady hydromagnetic flow past an infinite porous plate, when a strong magnetic field is imposed at an angle $\alpha$ to the normal to the boundary plate. In this paper, an attempt has been made to analyse the generalised Couette flow of an electrically conducting viscous fluid in the presence of a uniform inclined magnetic field taking Hall effects into account. The solution is obtained analytically and the behaviour of the velocity and shear stresses is discussed numerically for different values of the governing parameters.

## FORMULATION AND SOLUTION

We consider an incompressible, viscous and electrically conducting fluid confined to a channel bounded by two horizontal flat plates at $y = \pm L$ under the influence of a strong uniform magnetic field of strength $H_0$ inclined at an angle $\alpha$

with the normal to the boundaries. Initially the whole system is at rest and at time $t > 0$, the upper plate moves with a uniform velocity $U$ in its own plane. Taking Hall effects into account, the unsteady hydromagnetic equations of motion in the non-dimensional form reduce[6] to :

$$\frac{\partial f}{\partial t} = P + R^{-1}\frac{\partial^2 f}{\partial y^2} - \left(\frac{M\lambda^2}{1 + im\lambda}\right) f \qquad \ldots(1)$$

and

$$E_v = \frac{m\sqrt{1-\lambda^2}}{1 + m^2\lambda^2}(u + m\,\lambda\, w), \qquad \ldots(2)$$

where $f = u + iw$,

$\lambda = \cos(\alpha)$,

$R = UL/\nu$ (Reynolds number),

$M = \dfrac{\sigma\mu_e^2 H_0^2 L}{\rho U}$ (Magnetic parameter)

and $m = \omega_e \tau_e$ (Hall parameter).

The boundary conditions in the non-dimensional form are

$$f = 1 \text{ on } y = 1 \qquad \ldots(3)$$
$$f = 0 \text{ on } y = -1$$

Using Laplace transform technique the general solution of (1) subject to the boundary conditions (3) is

$$f = u + iw = \frac{P}{A}\left(1 - \frac{\cosh(By)}{2\cosh(B)}\right) + \frac{\sinh B(y+1)}{\sinh(2B)} - \frac{P}{2}e^{-ARt}$$
$$- \sum_{n=0}^{\infty} \frac{\sinh(\gamma_n(y+1)/2)}{2 s_n \cosh(\gamma_n)} e^{-s_n t} + \frac{2P}{R}\sum_{n=0}^{\infty} \frac{\cosh(\gamma_n y/2)}{s_{n1}\gamma_{n1}\sinh(\gamma_{n1}/2)}$$
$$\times e^{-s_{n1}t}, \qquad \ldots(4)$$

where $\alpha^2 = R(s + A)$, $A = \dfrac{M\lambda^2}{1 + im\lambda}$, $B = \alpha_{s=0}$,

when $\alpha = \dfrac{in\pi}{2}$, $\alpha = \dfrac{i(2n+1)\pi}{2}$, $s = -s_n, -s_{n1}$ respectively.

In the absence of the pressure gradient ($P$) the results are in agreement with that of Mathur[7] for $\lambda = 1$ and $m = 0$. The transient terms in (4) decay in a time to order

$$\max\left\{\frac{1}{A}, \frac{1}{A + (2n+1)^2\pi^2}, \frac{1}{(A + n^2\pi^2)}\right\},$$

which decreases with an increase in either Hall parameter ($m$) or inclination ($\alpha$) of the magnetic field.

After the decay of the transient terms, (4) reduces to

$$f = \frac{P}{A}\left(1 - \frac{\cosh\,(By)}{\cosh\,(B)} + \frac{\sinh\,B(y+1)}{\sinh\,(2B)}\right) \qquad \ldots(5)$$

which in the absence of pressure gradient $(P)$ coincides with the result of MHD Couette flow[3] for $m = 0$, $\lambda = 1$ and agrees with that of steady MHD generalised Couette flow[1] for $m = 0$ and $\lambda = 1$.

The solution (5) indicates the existence of a boundary layer on either boundaries whose thickness is given by

$$\mathrm{Re}\left(\frac{1}{B}\right) + \left\{\frac{a_1}{a_1^2 + b_1^2}\right\},$$

where $$a_1 = \sqrt{R}\left(\frac{a + \sqrt{a^2 + b^2}}{2}\right)^{1/2}$$

$$b_1 = \left(\frac{\sqrt{a^2 + b^2} - a}{2}\right)^{1/2}\sqrt{R},$$

$$a = \frac{M\lambda^2}{1 + m^2\lambda^2}$$

and $$b = \frac{mM\lambda^3}{1 + m^2\lambda^2}.$$

The flux $Q_x$ and $Q_z$ of the fluid across the boundary in the $X$ and $Z$ directions are given by

$$Q_x + iQ_z = \int_{-1}^{+1} f\,dy = \frac{2P}{A}\left(1 - \frac{\tanh\,B}{B}\right) + \frac{\coth\,(2B) - \mathrm{cosech}\,(2B)}{B}$$

The non-dimensional shear strees components $\tau_x$ and $\tau_y$ in the steady state are given by

$$(\tau_x + i\tau_y)_{y=1} = B\left[\coth\,(2B) - \frac{P}{A}\tanh\,(B)\right]$$

$$(\tau_x + i\tau_z)_{y=-1} = B\left[\mathrm{cosech}\,(2B) + \frac{P}{A}\tanh\,(B)\right]$$

## Discussion

The velocity components $u$, $w$ and the electric field component $E_y$ are drawn in Figs. 1–6 for different values of $M$, $m$ and $\lambda$ in the generalised Couette flow. The primary velocity profiles are drawn in Fig. 1 for different $M$ and $m$. It is found that for any value of $m$ and $M$, $u$ increases from zero on the lower boundary to attain its maximum value on the uniformly moving upper boundary. It can be observed that $u$ steeply rises to its maximum value in the upper half of the channel,

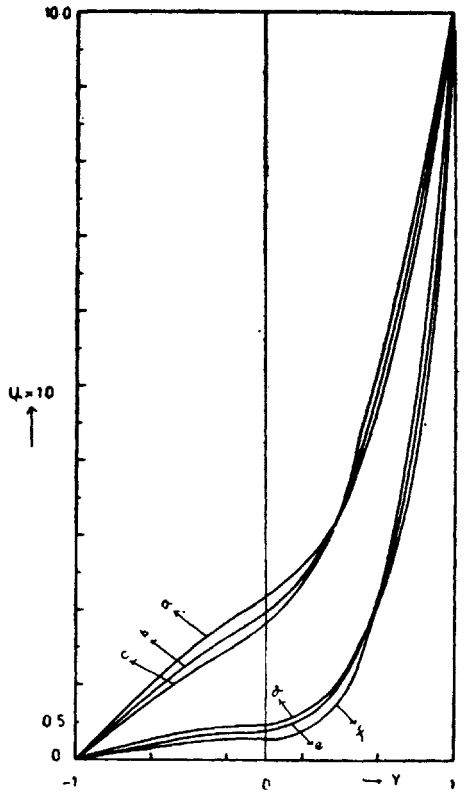


FIG 1 Profiles for the primary velocity $u$ with $\lambda = 0.5$.

| | *a* | *b* | *c* | *d* | *e* | *f* |
|---|---|---|---|---|---|---|
| *M* | 5 | 5 | 5 | 10 | 10 | 10 |
| *m* | 0.5 | 1 | 1.5 | 0.5 | 1 | 1.5 |

its growth near the boundary being much rapid in comparison with its growth in the remaining region. For large values of $M$, the growth in $u$ in the lower half of the channel is almost negligible in comparison to its behaviour for small values of $M$. Also, an increase in Hall parameter $m$ still contributes to its retardation in the lower half. This behaviour continues to a certain extent in the upper half of the channel near the mid plane. However, in the vicinity of the upper boundary the behaviour of $u$ with respect to an increase in $m$ totally gets reversed for all values of $M$. It is found that an increase in $m$ contributes to a small increment in $u$ near the upper plate. In general, for all $m$ and $M$, $u$ rapidly increases to reach its maximum on the upper boundary. Also for fixed $m$ and $\lambda$, $u$ decreases with $M$ and the depreciation is appreciably large in the mid region of the channel.[2] $u$ increases with an increase in $\alpha$ both in the lower and upper half of the regions and the rate of its growth in the upper half is much large in compared to its growth in the lower half (Fig. 2).[6] As the inclination $\alpha$ increases the growth rate in the upper half of the region tends to be almost uniform and for $\alpha \lesssim 60°$ the velocity monotonically

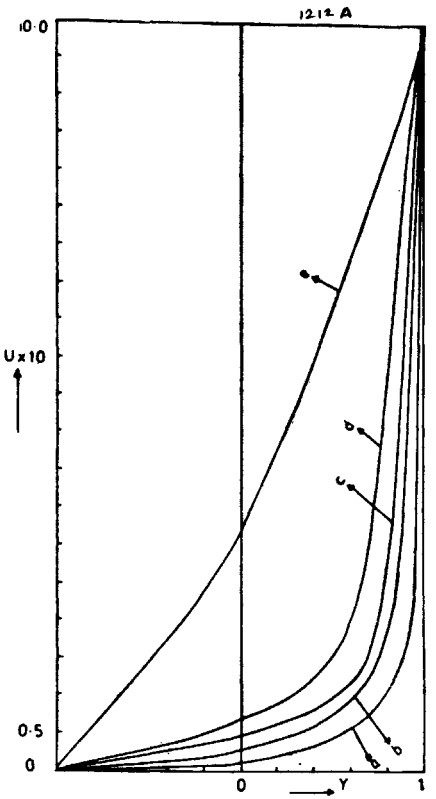


FIG 2 Profiles for $u$ with $M = 10$, $m = 0.5$.

*a* *b* *c* *d* *e*

cos 0 cos 30 cos 45 cos 60 cos 80

increases from zero on the lower plate to attain its maximum on the upper boundary. For small inclination a slight depreciation is noticed near the upper boundary, the corresponding velocity profiles is marked with a dip in that region. This depression has a tendency to move towards the mid plane for an increase in the inclination of the magnetic field until it disappears at $\alpha \sim 60°$. The presence of the Hall current gives rise to a secondary velocity ($w$) which is towards the mid plane in the lower region (Fig. 3). This secondary velocity vanishes in a narrow region close to the mid plane in the upper half of the channel and gets reversed in the remaining part of the upper half. In general, the secondary flow is towards the mid plane throughout the channel. It attains its maximum in the lower half in a narrow region near the mid plane for small values of $M$ while for $M \sim 10$, it attains its maximum near the moving plate. This secondary velocity increases with $m$ for fixed values of $M$ whereas it decreases with an increase in $M$ for a fixed $m$. The decay of the velocity with increased strength of the magnetic field is much rapid in comparison to its growth with increased Hall parameter $w$ is drawn for different $\alpha$ in Fig. 4. It is found that the secondary flow reverses its direction as

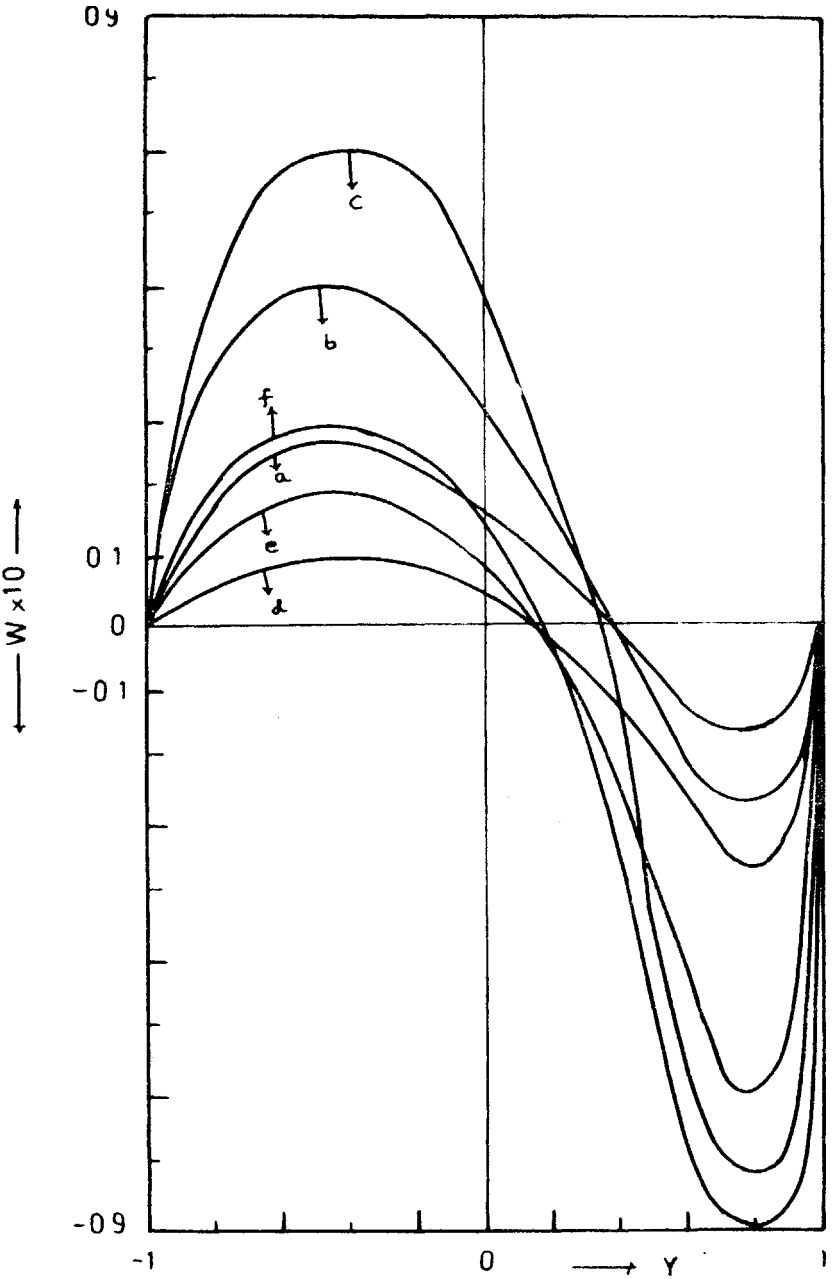


FIG 3 Profiles for the secondary velocity ($w$) with $\lambda = 0.5$. (For legend *see* Fig. 1.)

we move from the lower to the upper channel and the reversal takes place invariably in the upper half for all magnetic fields whose inclination is less than or approximately equal to 60°. It is to be noted that this reversal which is confined to a narrow

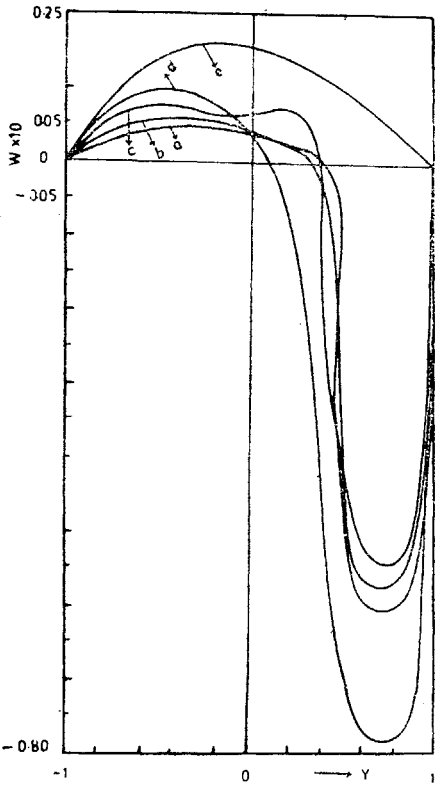


FIG 4 Profiles for $w$ with $M = 10$ and $m = 0.5$. (For legend *see* Fig. 2.)

layer near the upper plate in the case of an external magnetic field in the normal direction goes on increasing its domain away from the boundary as the inclination of the magnetic field increases. However, when this inclination exceeds certain limit ($\alpha \sim 80°$) the secondary velocity has no change in the direction throughout the region. It attains its maximum in the upper half of the region for all $\alpha$ less than or approximately equal to 60° and for $\alpha > 60°$ it attains in the lower half of the region.

Another important feature to be noted is the generation of electric field $E_y$ in the transverse $y$-direction to the plate in presence of an inclined magnetic field $\alpha \neq 0$. From the expression (2) it is clear that in the case of normal magnetic field ($\alpha = 0$) $\lambda = 1$ and hence, $E_y = 0$. The presence of this electric field contribute to an additional current in the $z$-direction parallel to the plate. The behaviour of this electric field with reference to variation in $m$, $M$ and $\lambda$ is plotted in figures 5 and 6. In Couette flow, $E_y$ increases from zero on the lower plate to a maximum on the upper plate monotonically for all $m$, $M$ and for $\alpha > 60°$. $E_y$ reduces at the corresponding points in the region for an increase in $m$ or $M$. However, this reduction is much large for an increase in $M$ in comparison to $m$ (Fig. 5). An observation of $E_y$ for variation in $\alpha$, reveals that for fixed $m$ and $M$ it increases

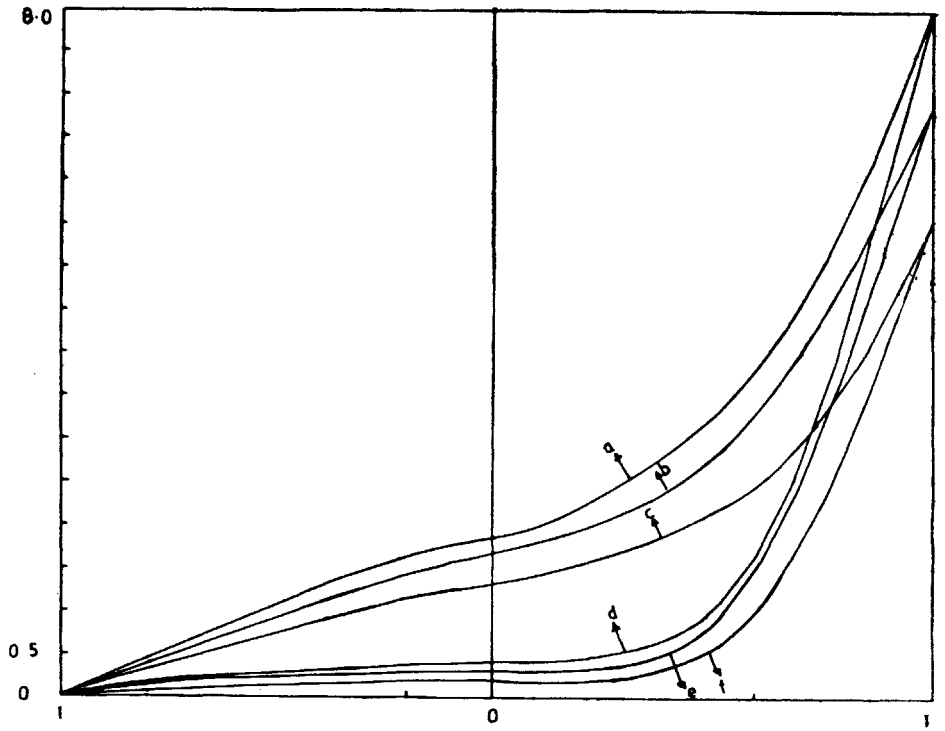


FIG 5 Profiles for the electric field $E_y$ with $\lambda = 0.5$. (For legend *see* Fig. 1.)

from zero on the lower boundary to attain the prescribed value on the upper boundary. There is a steep raise in $E_y$ near the moving upper boundary (Fig. 6).

The shear stresses components $\tau_x$ and $\tau_z$ along the boundary plates are calculated for different $m$, $M$ and $\lambda$. The shear stress components in the case of Couette flow are given in Tables I–IV. For small magnetic field $M \sim 1$ the shear stress $\tau_x$ at both the upper and lower plates decreases with an increase in $m$ and $\alpha$ and at

TABLE I

*Shear stress $\tau_x$ at the upper plate $y = 1$ for $m = 0.5/1.0$*

| $M/\lambda$ | cos 0° | cos 30° | cos 45° | cos 60° | cos 80° |
|---|---|---|---|---|---|
| 1 | 0.0019 | 0.133 | 0.268 | 0.396 | 0.489 |
| | 0.316 | 0.327 | 0.359 | 0.418 | 0.480 |
| 2 | −3.694 | −2.474 | −2.365 | −0.188 | −0.452 |
| | −0.695 | −0.646 | −0.422 | −0.009 | 0.453 |
| 3 | −14.529 | −10.390 | −5.897 | −1.898 | 0.375 |
| | −4.032 | −3.702 | −2.779 | −1.186 | 0.379 |
| 4 | −37.917 | −26.722 | −15.543 | −5.487 | 0.232 |
| | −11.481 | −20.379 | −7.841 | −3.694 | 0.242 |

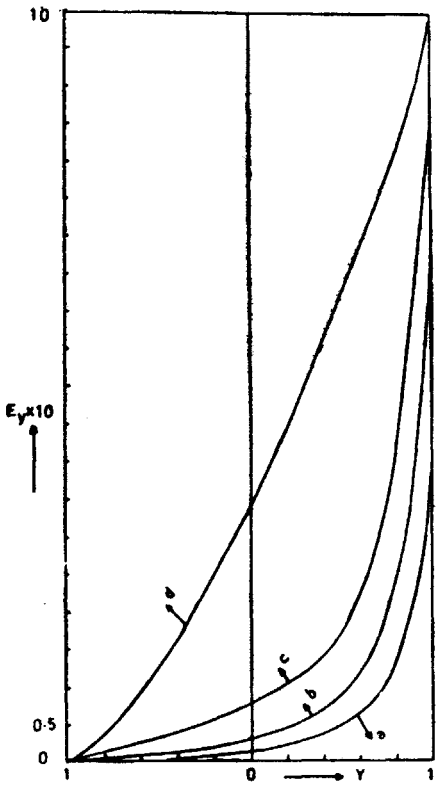


FIG 6 Profiles for $E_y$ wite $M = 10$ and $m = 0.5$ (For legend *see* Fig. 2.)

the moving upper plate it increases with an increase in $\alpha$ while at the stationary plate it decreases with increase in $\alpha$ for $M \sim 2$.[2] The behaviour of $\tau_x$ with an increase in $m$ is in contrast to the case of $M \sim 1$. It is found that $\tau_x$ decreases to a large extent for an increase in $m$ at either of the boundaries. $\tau_x$ at the upper plate

TABLE II

*Shear stress $\tau_x$ at the lower plate $Y = -1$ for $m = 0.5/1.0$*

| $M/\lambda$ | cos 0° | cos 30° | cos 45° | cos 60° | cos 80° |
|---|---|---|---|---|---|
| 1 | 1.917 | 1.607 | 1.263 | 0.901 | 0.551 |
| | 1.327 | 1.218 | 1.064 | 0.838 | 0.548 |
| 2 | 7.376 | 5.727 | 3.989 | 2.220 | 0.702 |
| | 3.796 | 3.465 | 2.870 | 1.915 | 0.697 |
| 3 | 20.445 | 15.264 | 9.058 | 4.844 | 0.961 |
| | 8.693 | 7.933 | 6.431 | 3.958 | 0.950 |
| 4 | 45.280 | 33.221 | 20.956 | 9.399 | 1.337 |
| | 17.696 | 16.02 | 12.713 | 7.376 | 1.317 |

TABLE III

*Shear stress $\tau_z$ at the upper plate $Y = 1$ for $m = 0.5/1.0$*

| $M/\lambda$ | cos 0° | cos 30° | cos 45° | cos 60° | cos 80° |
|---|---|---|---|---|---|
| 1 | −0.4112 | −0.245 | −0.117 | −0.0329 | −0.0009 |
| | −0.4007 | −0.274 | −0.153 | −0.0529 | −0.002 |
| 2 | −3.837 | −2.265 | −1.045 | −0.265 | −0.004 |
| | −3.691 | −2.467 | −1.328 | −0.411 | −0.009 |
| 3 | −13.877 | −8.271 | −3.657 | −0.978 | −0.0138 |
| | −13.657 | −9.158 | −4.943 | −1.515 | −0.0263 |
| 4 | −33.760 | −20.275 | −9.568 | −2.468 | −0.0321 |
| | −33.721 | −22.755 | −12.389 | −3.838 | −0.3621 |

TABLE IV

*Shear stress $\tau_z$ at the lower plate $Y = -1$ for $m = 0.5/1.0$*

| $M/\lambda$ | cos 0° | cos 30° | cos 45° | cos 60° | cos 80° |
|---|---|---|---|---|---|
| 1 | 0.787 | 0.519 | 0.286 | 0.103 | 0.004 |
| | 0.922 | 0.674 | 0.419 | 0.173 | 0.009 |
| 2 | 4.702 | 2.929 | 1.492 | 0.477 | 0.0177 |
| | 4.967 | 3.498 | 2.065 | 0.786 | 0.0347 |
| 3 | 15.181 | 9.280 | 4.551 | 1.829 | 0.0411 |
| | 15.589 | 10.736 | 6.009 | 2.146 | 0.0303 |
| 4 | 35.498 | 21.621 | 10.496 | 2.947 | 0.0764 |
| | 36.295 | 24.859 | 13.937 | 4.702 | 0.149 |

is negative for $M \sim 2$ and $\alpha < 60°$ and is positive at the lower plate and for other variations in the parameters. $\tau_z$ is negative at the upper plate and positive at the lower plate. When the magnetic field is normal to the plate, for an increase in $M$, $\tau_z$ increases while it decreases for an increase in $m$. Whereas for an inclined magnetic field a raise in $\tau_z$ is observed for an increase in either $m$ or $M$. At the lower plate, for all $\alpha$ including $\alpha = 0$ a rise in $\tau_z$ is observed corresponding to a raise in $m$ or $M$.

ACKNOWLEDGEMENTS

The authors are to Dr D V Krishna, Professor and Head of the Department of Mathematics for his encouragement in the preparation of this paper. One of us (R Siva Prasad) is thankful to C S I R, India for their financial support. Thanks are due to the referee for his kind suggestions.

## REFERENCES

1. E Lehnert *Arkiv Fysik* **5** (1952) 69
2. J P Agarwal *Appl Sci Res* **9B** (1962) 255
3. H Sato *J phys Soc Japan* **16** (1961) 1427
4. T Yamanishi *Preprint—Proc 17th Ann Meet Phys Soc Japan Osaka* **5** (1962) 29
5. A Sherman and G W Sutton *Magnetohydrodynamics* Evanston Illinois (1961) 173
6. A Bharali and A K Borkakati *J phys Soc Japan* **49**(5) (1980) 2091
7. A K Mathur *Indian J Phys* **46** (1972) 165