© *Proc. Indian natn. Sci. Acad.*, **52**, A, No. 3, 1986, pp. 592–597.
Printed in India.

VIBRATION AND BUCKLING OF AN ANISOTROPIC TRAPEZOIDAL PLATE ON NONLINEAR ELASTIC FOUNDATION AT LARGE AMPLITUDE

TARAPADA DAS

Department of Mathematics, Regional Engineering College, Durgapur 713 209, India

(*Received 2 July 1985*)

Nonlinear vibration and buckling have been studied for an anisotropic trapezoidal plate with base angle 45°, simply supported on all its edges and resting on a nonlinear elastic foundation using dynamic analogue of Berger's approximate field equations[1] for large deflection of elastic plates and subsequent use of Galerkin technique. Numerical results are presented in the form of graphs.

Key Words : Nonlinear Vibration; Buckling; Anisotropic Trapezoidal Plate; Berger's Field Equations; Galerkin Techniques

INTRODUCTION

BERGER'S[1] approximate field equations for the large deflections of elastic plates was extended to anisotropic plates by Iwinski and Nowinski.[2] Sircar[3] has solved the large amplitude vibration of an isosceles trapezoidal plate with base angle 45°, resting on Winkler type elastic foundation. The present investigation is an extension of the work of Sircar,[3,4] to include elastic foundation on twin problems of vibration and buckling of an anisotropic trapezoidal plate resting on nonlinear elastic foundation.

Numercial results in the form of graphs, showing the variation of relative time-period against relative amplitude and the variation of critical buckling load against relative amplitude, are presented.

VIBRATION PROBLEM

Consider a trapezoidal plate with base angle 45° and whose height is a and parallel sides are $2a$ and $4a$ respectively. The orientation of the x, y co-ordinate system is shown in Fig. 1.

Berger's field equations for the large amplitude free vibration of an anisotropic elastic plate resting on a nonlinear elastic foundation is[2,3]

$$w_{,xxxx} + \frac{2H}{D_x} w_{,xxyy} + (D_y/D_x)^2 w_{,yyyy} - \frac{12C}{h^2} f(t)(w_{,xx} + \sqrt{(D_y/D_x}w_{,yy}) + r_1 w + k_3 w^3 + \frac{\rho h}{D_x} \ddot{w} = 0 \quad \ldots(1)$$

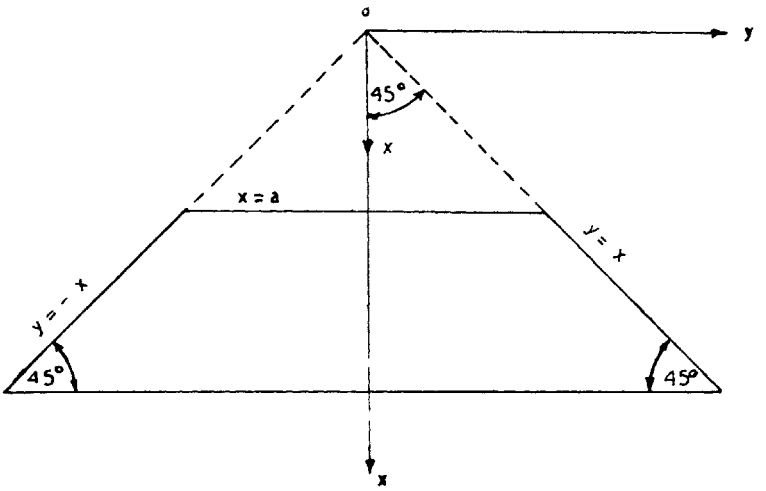

FIG 1 Geometry of the plate.

and

$$u_{,x} + \sqrt{D_y/D_x}v_{,y} + \frac{1}{2}\, w_{,x}^2 + \frac{1}{2}\sqrt{D_y/D_x}\, w_{,y}^2 = cf(t), \qquad \ldots(2)$$

where $w(x, y, t)$ is the transverse deflection of the plate at time $t\ (> 0)$, u, v are the in-plane displacement; ρ, h, k_1, k_3 are respectively the density, plate thickness and foundation parameters; c is the normalized constant of integration and $f(t)$ is an unknown function of the time and

$$D_x = \frac{E_x' h^3}{12}, \; D_y = \frac{E_y' h^3}{12}, \; D_1 = \frac{E'' h^3}{12}, \; D_{xy} = \frac{G h^3}{12}, \; H = D_1 + 2D_{xy};$$

$E_x' E_y' E''$ and G being elastic constants characterizing the elastic anisotropy.

For a plate with two way reinforcements along the axis of co-ordinates, set $H = (D_x D_y)^{1/2}$ and use the following transformations in equation (2.1)

$$x_1 = x, \; y_1 = y(D_x/D_y)^{1/4} \qquad \ldots(3)$$

Equation (1) yields the following equation,

$$\nabla^2 [\nabla^2 - \alpha^2 f(t)]\, w + k_1 w + k_3 w^3 + \frac{\rho h}{D_x}\ddot{w} = 0 \qquad \ldots(4)$$

in which $\nabla^2 \equiv \dfrac{\partial^2}{\partial x_1^2} + \dfrac{\partial^2}{\partial y_1^2}$ and $\alpha^2 = \dfrac{12c}{h^2}$

For fundamental mode of vibration take,

$$w = A\left(\sin\frac{2\pi x}{a} \sin\frac{\pi y}{a} - \sin\frac{\pi x}{a} \sin\frac{2\pi y}{a} \right) \theta(t), \qquad \ldots(5)$$

where A refers to amplitude.

Commensurate with the boundary conditions for simply supported edges given by

$$\left.\begin{aligned} &w = 0 \text{ for } x = a, 2a \text{ and } x + y = 0 \\ &w_{,xx} = 0 \text{ for } x = a, 2a \\ &\nabla^2 w - 2\left(\frac{1-\nu}{1+\nu}\right) w_{,xy} = 0 \quad \text{for} \quad x - y = 0 \\ &\nabla^2 w + 2\left(\frac{1-\nu}{1+\nu}\right) w_{,xy} = 0 \text{ for } x + y = 0 \end{aligned}\right\} \qquad \ldots(6)$$

$\theta(t)$ being a unknown function of time s.t. $\theta^2(t) = f(t)$.

Using (5) in (4) and applying the Galerkin method, one arrives at the following nonlinear differential equation in time variable

$$\ddot{\theta} + \Lambda_1\theta + \Lambda_2\theta^3 = 0, \qquad \ldots(7)$$

where

$$\left.\begin{aligned} \Lambda_1 &= \frac{D_x}{\rho h}\left(\frac{25\pi^4}{a^4} + k_1\right) \\ \Lambda_2 &= \frac{D_x}{\rho h}(A/h)^2\left\{\frac{75\pi^4(1+\sqrt{D_y/D_x})}{2a^4} + \frac{9}{20}k_3h^2\right\} \end{aligned}\right\} \qquad \ldots(8)$$

Using (3) in (2) and applying Galerkin Method with the choice of appropriate conditions for u and v with a view to their complete elimination during the process of integration over the area of the plate, one obtains

$$\alpha^2 = \frac{15\pi^2}{2a^2}(1 + \sqrt{D_y/D_x}\,(A/h)^2 \qquad \ldots(9)$$

The solution of (7), subject to the initial conditions

$$\theta(0) = 1, \dot{\theta}(0) = 0 \qquad \ldots(10)$$

is given by

$$\theta(t) = cn(\tilde{\omega}t, \mu), \qquad \ldots(11)$$

where

$$\tilde{\omega} = (\Lambda_1 + \Lambda_2)^{1/2}, \quad \mu = \frac{1}{\sqrt{2}}\left(1' + \frac{\Lambda_1}{\Lambda^2}\right)^{-1/2} \qquad \ldots(12)$$

and cn is the Jacobian elliptic function.

The nonlinear time-period T^* is given by

$$T^* = \frac{4\chi}{\tilde{\omega}}, \qquad \ldots(13)$$

where χ is the complete elliptic integral of the first kind.

The corresponding time-period T for the linear problem is given by

$$T = \frac{2\pi}{\sqrt{\Lambda_1}} \qquad \ldots(14)$$

The variation of T^*/T with relative amplitude A/h is depicted in graphical form in Fig. 2 for the case of an anisotropic plate. Comparison is also made for isotropic case, obtained from (13) by putting

$$\frac{D_y}{D_x} = 1.$$

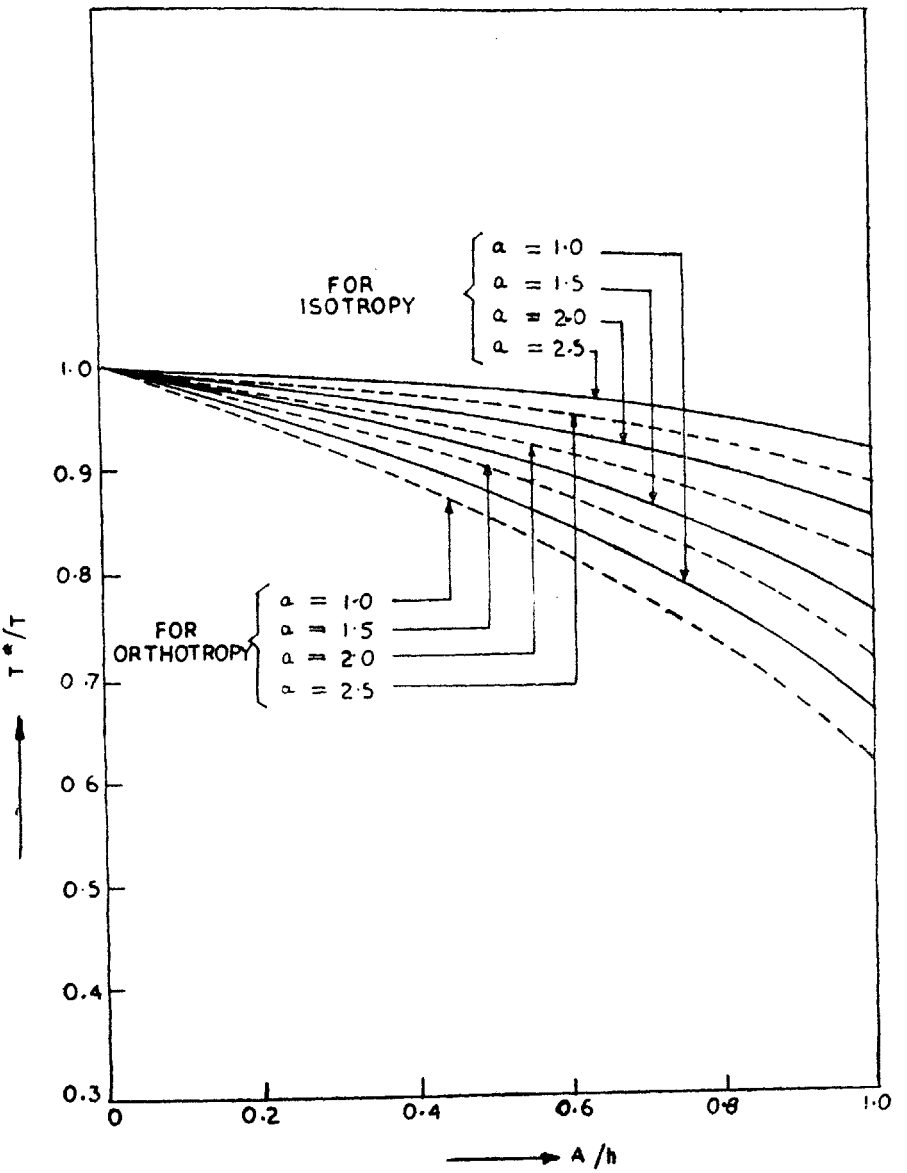

FIG 2 Variation of time period (T^*/T) *vs* relative amplitude (A/h).

Buckling Problem

If the plate is subjected to compressive force N, the nonlinear static equations governing the elastic stability of the anisotropic elastic plates resting on the nonlinear elastic foundation are given by[4,5]

$$D_x w_{,xxxx} + 2(D_1 + 2D_{xy})\, w_{,xxyy} + D_y w_{,yyyy} + N(w_{,xx} + w_{,yy}) - \frac{\alpha^2 h^2}{12} E'(w_{,xx} + w_{,yy}) + k_1 w + k_3 w^3 = 0 \qquad \ldots(15)$$

$$u_{,x} + \sqrt{D_y/D_x}\, v_{,y} + \frac{1}{2} w_{,x}^2 + \frac{1}{2}\sqrt{D_y/D_x}\, w_{,y}^2 = C \qquad \ldots(16)$$

To solve the equation (15) take

$$w = \sum_{n=1,3,5\ldots}^{\infty} A_n \left(\sin\frac{2n\pi x}{a} \sin\frac{n\pi y}{a} - \sin\frac{2n\pi y}{a} \sin\frac{n\pi x}{a} \right) \qquad \ldots(17)$$

commensurate with the boundary conditions.

Using eq. (17) in eq. (15), we take resort to Galerkin technique to obtain the expression for the buckling load N as follows

$$N = \frac{2h(E''^2 - E'_x E'_y)}{\frac{15}{4} n^2\pi^2 \{2h(E''^2 - E'_x E'_y) + (E'_x + E'_y)(2E'' - E'_x - E'_y)\}} \times \left[\frac{51}{8} \frac{n^4\pi^4}{a^2}(D_x + D_y) + \frac{6n^4\pi^4}{a^2}(D_1 + 2D_{xy}) + \frac{3}{4} k_1 a^2 + \frac{63}{64} k_3 a^2 A_n^2 \right] \qquad \ldots(18)$$

The value of α is computed from (16) using the inplane relations between stress and strain[6]

$$\frac{\alpha^2 h^2}{12} = \left(\frac{E'_x + E'_y - 2E''}{E''^2 - E'_x E'_y} \right)(N/h) \qquad \ldots(19)$$

The critical buckling load N_{cr} of the plate is obtained by putting $n = 1$ in (18)

$$N_{cr} = \frac{2h(E''^2 - E'_x E'_y)}{\frac{15}{4}\pi^2 \{2h(E''^2 - E'_n E'_y) + (E'_x + E'_y)(2E'' - E'_n - E'_y)} \times \left[\frac{51}{8} \frac{\pi^4}{a^2}(D_x + D_y) + \frac{6\pi^4}{a^2}(D_1 + 2D_{xy}) + \frac{3}{4} k_1 a^2 + \frac{63}{64} k_3 a^2 A_1^2 \right]. \qquad \ldots(20)$$

The variation of critical buckling load N_{cr} shown against various values of A/h in Fig. 3.

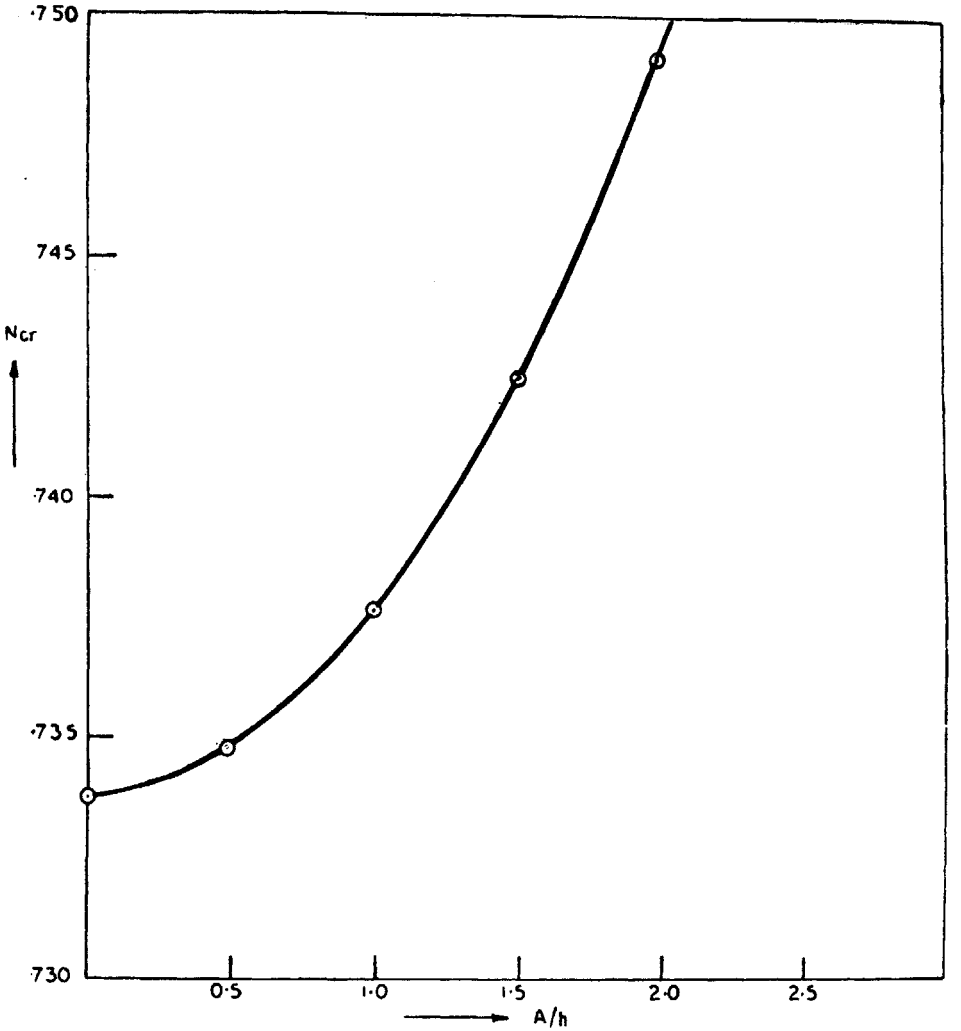

FIG 3 Variation of critical buckling load (N_{CT}) *vs* relative Amplitude (A/h).

ACKNOWLEDGEMENTS

Thanks are due to University Grants Commission for financial support. The outhor expresses his sincere thanks to Dr R Sircar for supervision and guidance during the work.

REFERENCES

1. H M Berger *J appl Mech ASME* **22** (1955) 465–472
2. T Iwinski and J Nowinski *Arch Mech Stos* **9** (1957) 503–600
3. R Sircar *Bull Acad L Polon Sci* **XXII**(1) (1974) 197–203
4. *Ibid ZAMM* **64** (198S) 64–66
5. T Das and R Sircar (*Communicated*)
6. S Timoshenko and W Krieger *Theory of Plates and Shells* Mc Graw Hill Co N Y (1959)