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AN EXACT SOLVABLE MODEL OF MHD SHOCK WAVES IN VISCOUS, HEAT CONDUCTING AND FROZEN-IN-FIELD FLUID

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The magnetohydrodynamic equations are used to describe the formation and structure of shock waves in a viscous and thermal conducting fluid of infinite electrical conductivity. A phenomenological model for temperature dependent Prandtl number is preassumed. The fundamental resulting non-linear differential equation is solved exactly, and the shock-wave's structure is determined. The flow quantities and characteristics are found to depend, in non-analytical manner, on two phenomenological parameters, which are related to the model under consideration. As a consequence, special cases of interest are to be treated separately. All flow and shock characteristics are expressed in terms of these parameters, which satisfy two inequalities to ensure physical results. Numerical results are given and discussed.

Key Words : MHD Shock Waves; Viscous, Heat Conducting and Frozen-in-Field Fluid; Mach, Alfven & Prandtl Numbers

INTRODUCTION

THE structure of magnetohydrodynamics (MHD) shock waves has been investigated for various cases by using perturbational, or numerical methods.[1-5] As a consequence, the relation between the resulting shock and the different dissipation mechanisms is partially missed.[6,7] Also, some of the non-linear effects may disappear. The question arises whether it is possible to find, by analytical methods, exact solutions of the non-linear equations describing the shock wave phenomenon. Recently,[8] exact MHD shock solutions were found for finite constant transport coefficients.

In this paper, a phenomenological model for temperature dependent Prandtl number is preassumed for a fluid with infinite electrical conductivity—a case of interest in laboratory, geo- and astrophysical applications, where the magnetic Reynolds number is much greater than 1. The fundamental resulting non-linear equations are solved exactly and the shock profile is determined. Numerical results are given and discussed.

BASIC EQUATIONS AND SHOCK CONDITIONS

We consider the non relativistic channel flow of a frozen-in-field fluid (or plasma) with finite viscosity η and thermal conductivity λ, which are functions of tempera-

ture T. Let the fluid be moving with velocity u in x-direction; ρ and P its density and pressure. Let the magnetic field B be in y-direction, the electric field E and current density J in z-direction. All flow quantities are assumed to depend on the variable $X = x - Ut$, where U is the shock speed, i.e. we choose shock fixed coordinate system. Let $v = U - u$ be the shock velocity relative to the fluid. Subscripts 1 and 2 will refer to up- and downstream respectively. Define non-dimensional quantities in the following way :

$$v = v_1\bar{v},\ \rho = \rho_1\bar{\rho},\ P = P_1M_1^2\bar{P},\ B = B_1\bar{B},\ X = L\bar{X},$$

where L is a typical length and M the Mach number of the shock relative to the fluid. After performing one integration with respect to $\bar{X}$, the MHD equations[9] yield :

$$\bar{\rho}\bar{v} = \bar{B}\bar{v} = 1, \qquad \ldots(1)$$

$$\bar{P} = \bar{\rho}\bar{T}, \qquad \ldots(2)$$

$$\bar{v} + \frac{\bar{P}}{\gamma} + \tfrac{1}{2}a_4\bar{B}^2 + \frac{4}{(3R)}\frac{\mathrm{d}\bar{v}}{\mathrm{d}\bar{X}} = a_2, \qquad \ldots(3)$$

$$\tfrac{1}{2}\bar{v}^2 + \frac{\bar{P}\bar{v}}{(\gamma-1)} + a_4\bar{B} + \frac{4}{(3R)}\bar{v}\frac{\mathrm{d}\bar{v}}{\mathrm{d}\bar{X}} + \frac{1}{(\gamma-1)\,Pc}\frac{\mathrm{d}\bar{T}}{\mathrm{d}\bar{X}} = a_3, \qquad \ldots(4)$$

where γ is the ratio of specific heats; $R = \rho_1v_1L/\eta(\bar{T}\bar{X})$ $Pc = \rho_1v_1LC_p/\lambda(\bar{T}(\bar{X}))$ are local Reynolds and Peclet numbers of the flow relative to the shock; a_2, a_3 and a_4 are constants of integration (motion), which can be expressed in terms of the up- or down-stream Mach and Alfvén numbers M and M_A from shock conditions. The latter corresponds to the vanishing of gradients as $\bar{X} \to \pm\infty$, and the asymptotic conditions :

$$\bar{v} \to 1,\ \bar{v}_2 \text{ as } \bar{X} \to \pm\infty$$

and similarly for other flow quantities. Accordingly, we get :

$$a_2 = 1 + 1\Big/\frac{1}{(2M_{A1}^2)} + \frac{1}{(\gamma M_A^2)} = \bar{v}_2\left[1 + 1\Big/\frac{1}{(2M_{A1}^2)} + \frac{1}{(\gamma M_2^2)}\right],$$

$$a_3 = \tfrac{1}{2} + \frac{1}{M_{A1}^2} + \frac{1}{(\gamma-1)\,M_1^2} = \bar{v}_2^2\left[\tfrac{1}{2} + \frac{1}{M_{A2}^2} + \frac{1}{(\gamma-1)\,M_2^2}\right]$$

and $$a_4 = \frac{1}{M_{A1}^2} = \frac{v_2^3}{M_{A2}^2}. \qquad \ldots(5)$$

Relations (5) are the generalized MHD Rankine-Hugoniot jump conditions.[10] They imply that only two of the constants, e.g. a_2 and a_4, are independent. From (1) it is clear that :

$$\bar{B} = \bar{\rho} = 1/\bar{v}, \qquad \ldots(6)$$

which is a consequence of the frozen-in-field phenomenon arising from infinite electrical conductivity. In eq. (4), the first three terms correspond to specific enthalpies, while the latter two terms to dissipative work. The role of the driving force $\mathbf{J} \times \mathbf{B}$, as it appears in (3) and (4), is to increase the enthalpy and accelerates the fluid.[6,8]

From eqs (1)–(4), we get the following two equations :

$$\frac{4}{(3\bar{R})}\,\bar{v}^2\frac{d\bar{v}}{d\bar{X}} = a_2\bar{v}^2 - \bar{v}^3 - \tfrac{1}{2}a_4 - \frac{\bar{v}\bar{T}}{\gamma} \qquad \ldots(7)$$

$$\frac{3}{4Pr(\gamma-1)\,\bar{v}}\frac{d\bar{T}}{d\bar{v}} = \frac{\bar{v}^3 - 2a_2\bar{v}^2 + 2a_3\bar{v} - a_4 - (2/\gamma(\gamma-1))\,\bar{v}\bar{T}}{2a_2\bar{v} - 2\bar{v}^3 - a_4 - 2\bar{v}\bar{T}/\gamma}, \qquad \ldots(8)$$

where $Pr = Pc/R = Pr(\bar{T}(\bar{X}))$ is the local Prandtl number, which is in general a positive weekly increasing function of $\bar{T}$. Equations (7) and (8) are of singular type, and of more general form than those describing gas dynamical shocks.[8,11,12] They are to be solved under the shock conditions :

$$\frac{d\bar{v}}{d\bar{X}} \text{ and } \frac{d\bar{T}}{d\bar{X}} \longrightarrow 0 \text{ as } \bar{X} \longrightarrow \pm\infty. \qquad \ldots(9)$$

Solutions and Structure of Shock Waves

While solutions of equation (8) may be obtained in general by numerical or graphical integration,[10] our intention here is to investigate the possibility of finding analytically exact solutions of this equation. Before proceeding, it is worth mentioning the following two cases :

(*i*) *Infinite Prandtl Number (neglecting Thermal Conductivity)* :

From (8), we have :

$$\frac{2}{\gamma(\gamma-1)}\,\bar{v}\bar{T} = \bar{v}^3 - 2a_2\bar{v}^2 + 2a_3\bar{v} - a_4. \qquad \ldots(10)$$

Substituting in (7), we get :

$$\frac{4}{(3\bar{R})}\,\bar{v}^2\frac{d\bar{v}}{d\bar{X}} = \tfrac{1}{2}(\gamma+1)\,G(\bar{v}), \qquad \ldots(11)$$

where :

$$G(\bar{v}) = -\bar{v}^3 + \frac{2\gamma}{(\gamma+1)}\,a_2\bar{v}^2 - \frac{2(\gamma-1)}{(\gamma+1)}\,a_3\bar{v} - \frac{2-\gamma}{(\gamma+1)}\,a_4$$

$$= (1-\bar{v})\,(\bar{v}-\bar{v}_2)\,(\bar{v}+\bar{V}_3), \qquad \ldots(12)$$

$1, \bar{v}_2, -\bar{V}_3$ are the roots of $G(\bar{v}) = 0$.

Equation (11) gives shock wave solution for different models of viscosity.[13]

(ii) Zero Prandtl Number (neglecting Viscosity):

Equation (7) or (8) yields :

$$\frac{\bar{v}\bar{T}}{\gamma} = -\bar{v}^3 + a_2\bar{v}^2 - \tfrac{1}{2}a_4. \qquad \ldots(13)$$

The velocity profile should be determined from (4) which become :

$$\frac{1}{Pc}\frac{\psi d\bar{v}}{d\bar{X}} = -\left(1 + \frac{1}{\gamma}\right)\bar{v}G(\bar{v}), \qquad \ldots(14)$$

where $$\psi = -4\bar{v}^3 + 2a_2\bar{v}^2 + a_4. \qquad \ldots(15)$$

Equation (14) may have either shock solution or inner discontinuity,[12,14] depending on the function ψ. It should be noted, by the way, that in the case of constant Prandtl number, eq. (8) may be related to Abel's equation by setting : $1/H = \bar{T}/\gamma - a_2\bar{v} + \bar{v}^2 + \frac{1}{2}a_4/\bar{v}$. However, analytical solutions of Abel's equation can be found only for particular cases.[15]

Next we assume the following model for Prandtl number :

$$Pr = x(1 - \mu_3\bar{T})/(1 - \mu_1\bar{T} + \mu_2\bar{T}^2), \qquad \ldots(16)$$

where x is a normalization constant; μ_1, μ_2 and μ_3 are positive phenomenological constants, chosen such that Pr should be positive weakly increasing function of $\bar{T}$. Substituting (16) in eq. (8), and assuming a polynomial solution for $\bar{T}$, whose coefficients are to be determined by comparing the coefficients of the different powers of $\bar{v}$. After cumbersome manipulations, it can be verified that the solution of eq. (8) under conditions (9) is given by :

$$\bar{T}(\bar{v}) = \frac{2(\gamma - 1)}{3}\chi\beta(\alpha^2 - 1)[(\alpha + 1)\beta - \bar{v}]; \; v_2 \leqslant v \leqslant 1, \qquad \ldots(17)$$

$$\mu_1 = \frac{2\alpha - 1}{(\gamma - 1)\chi\beta^2(\alpha^2 - 1)^2}, \; \mu_2 = \frac{3}{4(\gamma - 1)^2\chi^2\beta^4(\alpha^2 - 1)^3}$$

and $$\mu_3 = \frac{1}{2(\gamma - 1)\chi\beta^2(\alpha^2 - 1)}, \qquad \ldots(18)$$

where the parameter α, β and χ are related by the following compatibility equation :

$$\frac{(2 - \gamma)}{(\gamma + 1)}(2 - \alpha)\beta^3\left[8(2 - \alpha)^2 + \frac{4(\gamma + 1)}{3\gamma}\chi(\alpha^2 - 1)\left\{\frac{5\gamma - 1}{(\gamma + 1)} - \alpha\right\}\right]$$

$$+ \frac{(\gamma - 1)}{2(\gamma + 1)}\beta^2\left[24(2 - \alpha)^2 + \frac{4(5\gamma - 1)}{3\gamma}\chi(\alpha^2 - 1)\left\{\frac{13\gamma + 1}{5\gamma - 1} - \alpha\right\}\right]$$

$$- \frac{\gamma}{(\gamma + 1)}\beta\left[6(2 - \alpha) + \frac{4(\gamma + 1)}{3\gamma}\chi(\alpha^2 - 1)\right] + 1 = 0 \qquad \ldots(19)$$

The expression for a_2 and a_4 in terms of these parameters are :

$$a_2 = 3\beta\left[(2-\alpha) + \frac{2(\gamma+1)}{9\gamma}\chi(\alpha^2-1)\right] \qquad \ldots(20)$$

$$a_4 = (2-\alpha)\beta^3\left[8(2-\alpha)^2 + \frac{4(\gamma+1)}{3\gamma}\chi(\alpha^2-1)\left\{\frac{5\gamma-1}{(\gamma+1)} - \alpha\right\}\right] \qquad \ldots(21)$$

From (5) and (16), the upstream Mach, Alfvén and Prandtl numbers are given by the following formulae :

$$M_{A1}^2 = \frac{1}{a_4}\,;\ \gamma M_1^2 = \frac{\gamma}{\bar{T}_1} = \frac{1}{(a_2 - 1 - \frac{1}{2}a_4)}\,;$$

$$Pr_1 = \frac{\chi\beta(\alpha^2-1)\{1+\beta(2-\alpha)\}}{\{1-2\beta(2-\alpha)\}}. \qquad \ldots(22)$$

The formulae for pressure $\bar{P}$ and current density $\bar{J}$ are :

$$\bar{P} = \bar{T}/\bar{v}; \qquad \ldots(23)$$

and $$\bar{J} = -(3/4)\,R\Phi/\bar{v}^4. \qquad \ldots(24)$$

Substituting (17) in (7), the velocity profile is determined by solving the equation :

$$\frac{4}{(3R)}\,\bar{v}^2\,\frac{d\bar{v}}{d\bar{X}} = \Phi(\bar{v}), \qquad \ldots(25)$$

where $\Phi(\bar{v})$ is a third degree polynomial given by :

$$\Phi(\bar{v}) = -\bar{v}^3 + \left[a_2 + \frac{2(\gamma-1)}{3\gamma}\chi\beta(\alpha^2-1)\right]\bar{v} - \frac{2(\gamma-1)}{3\gamma}\chi\beta^2(\alpha+1)(\alpha^2-1)\,\bar{v} - \tfrac{1}{2}a_4$$

$$= (1-\bar{v})(\bar{v}-\bar{v}_2)(\bar{v}+\bar{v}_3), \qquad \ldots(26)$$

1, $\bar{v}_2$ and $-\bar{v}_3$ are the roots of $\Phi(\bar{v}) = 0$. Unlike cases (*i*) and (*ii*), $\bar{v}_2$ depends on the parameters α, β — and corresponding χ — so that the integral curves in $(\bar{v}, \bar{T})$ plane, given by (17), are straight lines emerging from the common point $(1, \bar{T}_1)$. To ensure physical results and shock formation, the following inequalities must hold :

$$\tfrac{3}{2} < \alpha < 2;\ \frac{2}{(1+\alpha)} < 2\beta < \frac{\bar{v}_2}{(2-\alpha)}. \qquad \ldots(27)$$

These ranges of values of the parameters cover a wide range of Mach and Alfvén numbers, which are in accordance with the evolution conditions for shock waves.[4]

Also, it can be shown that the total entropy increases along the flow passage, while the specific entropy has a maximum at $\bar{v}_S$ given by :

$$\bar{v}_S = \left(1 - \frac{1}{\gamma}\right)(\alpha + 1)\,\beta. \qquad \ldots(28)$$

It is interesting to note that for $\alpha = 2$, it follows from (21) that $a_4 = 0$, which is the case of gas dynamical shock waves with the following model of Prandtl number :[8]

$$Pr = \frac{\chi}{(1 - \mu_3 \bar{T})}.$$

From (24) and (25), it is clear that the velocity profile and current density depend upon the model of viscosity. In principle, equation (25) can be integrated in closed form for different models.[13] One form of these models is the following :

$$\bar{\eta} = \frac{\eta}{\eta_1} = \frac{1 - \mu_1 \bar{T}_1 + \mu_2 \bar{T}_1^2}{1 - \mu_1 \bar{T} + \mu_2 \bar{T}^2} \left(\frac{\bar{T}}{\bar{T}_1}\right)^{n/2}, \qquad \ldots(29)$$

where n is an integer. The thermal conductivity is, then :

$$\bar{\lambda} = \frac{\lambda}{\lambda_1} = \frac{1 - \mu_3 \bar{T}_1}{1 - \mu_3 \bar{T}} \left(\frac{\bar{T}}{\bar{T}_1}\right)^{n/2}. \qquad \ldots(30)$$

For simplicity, we consider the case $n = 1$. Substituting for $R = R_1/\bar{\eta}$ in equation (25) and integrating, we get :

$$\left(\frac{3R_1}{4}\right)\bar{X} = \frac{1}{(1 - \bar{v}_2)}\left[\frac{\bar{v}_2(1 - \bar{v}_4)\,(\bar{T}_2/\bar{T}_1)^{1/2}}{(\bar{v}_2 - \bar{v}_3)\,(\bar{v}_2 - \bar{v}_4)} \ln \frac{\bar{T}_2^{1/2} - \bar{T}^{1/2}}{\bar{T}_2^{1/2} + \bar{T}^{1/2}} - \frac{1}{(1 - \bar{v}_3)} \ln \frac{\bar{T}^{1/2} - \bar{T}_1^{1/2}}{\bar{T}^{1/2} + \bar{T}_1^{1/2}}\right] - \frac{1}{(\bar{v}_4 - \bar{v}_3)}\left[\frac{\bar{v}_4(\bar{T}_4/\bar{T}_1)^{1/2}}{(\bar{v}_2 - \bar{v}_4)} \ln \frac{\bar{T}_4^{1/2} - \bar{T}^{1/2}}{\bar{T}_4^{1/2} + \bar{T}^{1/2}} - \frac{\bar{v}_3(1 - \bar{v}_4)\,(\bar{T}_3/\bar{T}_1)^{1/2}}{(1 - \bar{v}_3)\,(\bar{v}_2 - \bar{v}_3)} \ln \frac{\bar{T}_3^{1/2} - \bar{T}^{1/2}}{\bar{T}_3^{1/2} + \bar{T}^{1/2}}\right], \qquad \ldots(31)$$

where $\bar{v}_4 = 2\beta(2 - \alpha)$ and $\bar{T}_4 = \bar{T}(\bar{v}_4)$.

It is easy to show that the pressure, density and magnetic field have the same thickness l_1 of transition layer, whilst the velocity and temperature have another thickness l_2. Let V_1 be the root of the equation :

$$\left(\frac{1}{R}\frac{dR}{d\bar{v}} - \frac{4}{\bar{v}}\right) + \frac{d\Phi}{d\bar{v}} = 0, \qquad \ldots(32)$$

corresponding to maximum $|\bar{p}'|$; similarly, V_2 is the root of the equation :

$$\left(\frac{1}{RR}\frac{dR}{d\bar{v}} - \frac{2}{\bar{v}}\right) + \frac{d\Phi}{d\bar{v}} = 0, \qquad \ldots(33)$$

corresponding to maximum $|\bar{v}'|$. Then, it follows that :

$$\frac{l_1}{L} = \frac{4V_1^4(1-\bar{v}_2)\,\bar{\eta}(V_1)}{3R_1\bar{v}_2\Phi(V_1)}, \quad \frac{l_2}{L} = \frac{4V_2^2(1-\bar{v}_2)\,\bar{\eta}(V_2)}{3R_1\Phi(V_2)}. \qquad \ldots(34)$$

The strength of the shock St is given by :

$$St = M_1^2\bar{P}_2 - 1 = \frac{(\alpha+1)\,\beta(1-\bar{v}_2)}{\bar{v}_2\{(\alpha+1)\,\beta - 1\}}. \qquad \ldots(35)$$

Remark : If m_{A1} is the Alfvén number of the fluid ahead of the shock, it can be shown that

$$\frac{U}{u_1} = 1 + \frac{1}{(a_4 m_{A1}^2)^{1/2}}, \quad \frac{u_2}{u_1} = 1 + \frac{(1-\bar{v}_2)}{(a_4 m_{A1}^2)^{1/2}}. \qquad \ldots(36)$$

Formulae (36) imply that the shock is supersonic viewed from ahead and subsonic from behind, which is the evolution condition for MHD shock waves. The electric field is found to be :

$$\bar{E} = \frac{E}{u_1 B_1} = \frac{U}{u_1} - 1 - \left(\frac{U}{u_1}\right)\bar{B}. \qquad \ldots(37)$$

For hydromagnetic bore wave[16] which is of great interest in astrophysical applications, formulae (36) lead to :

$$\frac{U}{u_2} = \frac{1}{(1-\bar{v}_2)}. \qquad \ldots(38)$$

Results and Discussion

The obtained solution describes fully the formation and structure of shock waves in a viscous, thermal conducting and frozen-in-field fluid under the prescribed model of temperature-dependent Prandtl number. The compatibility equation (19), relating the phenomenological constants α, β and χ, is a necessary condition for the evolution of shocks. Together with (27), it restricts the possible values of the upstream Prandtl number Pr_1. Values of these constants and the corresponding values of the shock characteristics are listed in Tables (I) to (III).

The velocity, current density profiles and the variations of density, magnetic field and pressure are illustrated in Figs. 1–4 for different values of α and Pr_1 and for $\gamma = 4/3$, 7/5 and 5/3. Curves (1)–(4) in each figure correspond to values in the tables denoted by "*".

The nonanalytical dependence of the flow quantities and characteristics on α and Pr_1 is a pure nonlinear effect, so that special cases of interest cannot be deduced from this solution, and are to be treated separately. The basic features of the solution are :

(1) For smaller values of α, M_1 is greater than M_{A1}, i.e., the sound speed is initially less than the Alfvén speed, and conversely.

TABLE I

Flow characteristics for $\gamma = 4/3$

α	β	Pr_1	M_1	M_{A1}	$\bar{p}_2$	St	$R_1 l_1/L$	$R_1 l_2/L$
	.42	.863	7.628	3.844	3.731	22.926	34.455	17.093
*	.47	.877	5.495	3.435	3.333	10.96	26.08	14.537
1.7	.67	.94	3.445	2.409	2.336	2.981	19.327	13.816
	.87	1.015	2.944	1.854	1.795	1.383	19.453	15.88
*	1.07	1.116	2.807	1.505	1.456	0.695	23.455	20.457
	.414	.87	7.234	4.419	4.032	25.14	35.067	16.88
	.464	.887	5.176	3.94	3.597	12.048	26.267	14.304
1.75	.664	.964	3.142	2.743	2.5	3.312	18.76	13.18
	.864	1.063	2.566	2.098	1.901	1.555	18.04	14.8
	1.064	1.206	2.289	1.69	1.517	0.788	20.352	18.187
	.401	.878	6.585	6.27	4.717	29.887	37.068	16.599
	.451	.897	4.661	5.556	4.184	14.32	27.355	14.015
1.85	.651	.988	2.696	3.791	2.809	3.918	18.121	12.684
	.851	1.106	2.079	2.843	2.062	1.808	16.597	13.705
	1.051	1.27	1.733	2.24	1.582	0.873	18.441	16.979
	.389	.878	6.072	11.91	5.525	35.143	39.771	16.571
*	.439	.898	4.261	10.47	4.808	16.734	28.635	13.949
1.95	.639	.989	2.381	6.948	3.086	4.44	18.147	12.536
	.839	1.104	1.773	5.082	2.165	1.957	16.379	13.716
*	1.039	1.251	1.43	3.911	1.59	0.875	18.724	17.541

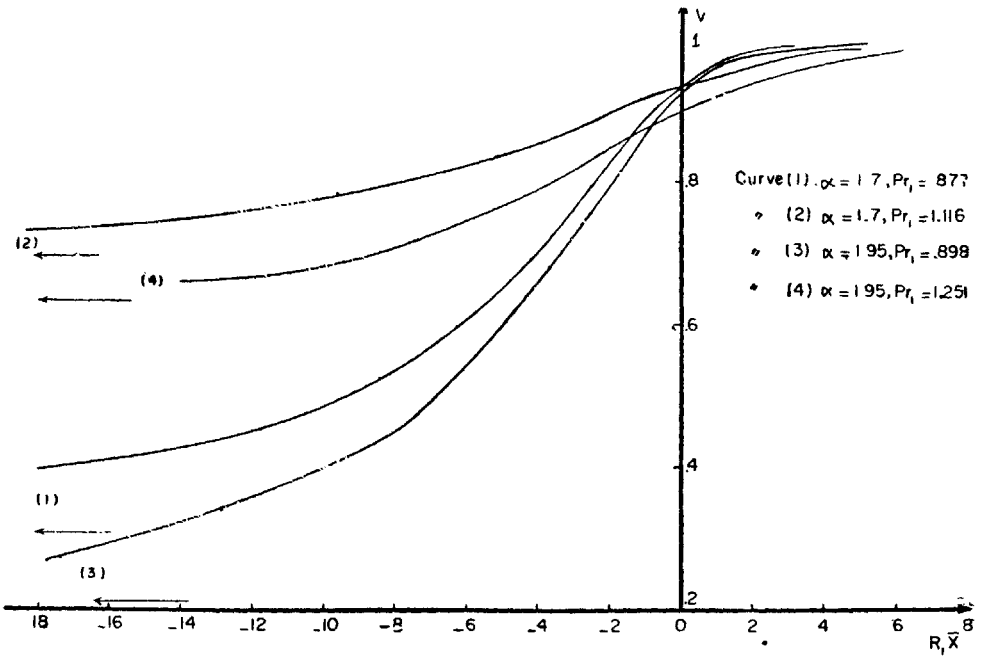

FIG 1*a* Velocity profile for $\gamma = 4/3$ and different α and Pr_1 : (1) $\alpha = 1.7$, $Pr_1 = 0.877$; (2) $\alpha = 1.7$, $Pr_1 = 1.116$; (3) $\alpha = 1.95$, $Pr_1 = 0.898$; (4) $\alpha = 1.95$, $Pr_1 = 1.251$.

TABLE II

Flow characteristics for $\gamma = 7/5$

α	β	Pr_1	M_1	M_{A1}	$\bar{\rho}_2$	St	$R_1 l_1/L$	$R_1 l_2/L$
	.427	.877	7.274	3.276	3.205	18.885	32.204	17.607
	.477	.894	5.268	2.932	2.874	8.941	24.881	15.145
1.65	.677	.963	3.394	2.066	2.024	2.31	19.679	15.093
	.877	1.048	3.024	1.594	1.56	0.983	21.899	18.771
	.977	1.104	3.013	1.43	1.399	0.65	25.069	22.755
	.42	.887	6.866	3.697	3.448	20.576	32.676	17.244
*	.47	.907	4.935	3.302	3.077	9.772	24.925	14.729
1.7	.67	.997	3.055	2.31	2.146	2.563	18.747	14.212
	.87	1.121	2.558	1.771	1.637	1.11	19.696	16.949
*	1.07	1.333	2.345	1.426	1.305	0.468	25.779	24.96
	.401	.901	5.934	5.923	4.219	25.808	34.589	16.843
	.451	.925	4.191	5.241	3.717	12.243	25.552	14.291
1.85	.651	1.039	2.399	3.549	2.457	3.167	17.485	13.267
	.851	1.198	1.824	2.633	1.77	1.31	17.281	15.425
	1.051	1.434	1.489	2.041	1.312	0.47	24.072	23.507
	.389	.9	5.474	11.092	4.785	29.455	36.277	16.768
*	.439	.924	3.835	9.733	4.167	13.865	26.505	14.192
1.95	.639	1.037	2.122	6.402	2.618	3.448	17.523	13.205
	.839	1.184	1.563	4.629	1.799	1.339	17.397	15.787
*	1.039	1.386	1.24	3.509	1.279	0.414	27.947	26.667

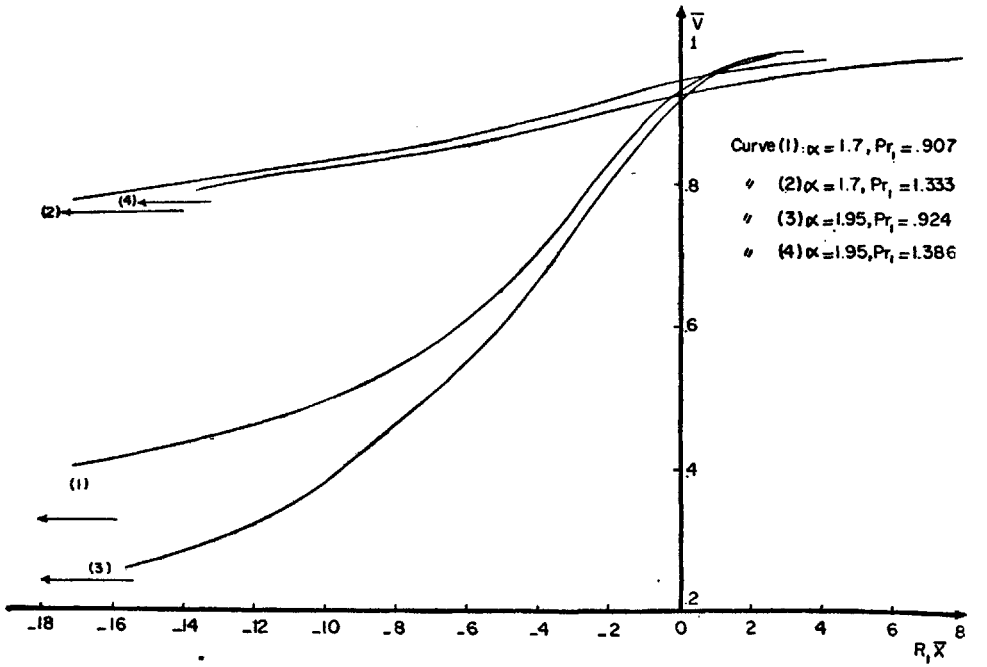

FIG 1*b* Velocity profile for $\gamma = 7/5$ and different α and Pr_1 : (1) $\alpha = 1.7$, $Pr_1 = 0.907$; (2) $\alpha = 1.7$, $Pr_1 = 1.333$: (3) $\alpha = 1.95$, $Pr_1 = 0.924$; (4) $\alpha = 1.95$, $Pr_1 = 1.386$.

TABLE III

Flow characteristics for $\gamma = 5/3$

α	β	Pr_1	M_1	M_{A1}	$\bar{p}_2$	St	$R_1 l_1/L$	$R_1 l_2/L$
	.435	.957	5.705	2.667	2.475	12.803	28.884	18.743
	.485	.989	4.13	2.389	2.212	5.877	23.027	16.493
1.6	.585	1.066	3.068	1.974	1.825	2.406	20.441	16.648
	.685	1.167	2.638	1.678	1.543	1.239	21.303	19.001
	.785	1.319	2.406	1.454	1.326	0.641	25.893	23.941
	.427	.968	5.363	2.944	2.591	13.621	28.623	18.315
	.477	1.005	3.848	2.63	2.31	6.256	22.645	16.051
1.65	.577	1.092	2.801	2.161	1.887	2.56	19.608	16.100
	.677	1.208	2.348	1.825	1.58	1.31	20.131	17.924
	.777	1.377	2.074	1.569	1.34	0.662	24.277	22.729
	.42	.976	5.07	3.279	2.71	14.387	28.608	18.055
*	.47	1.016	3.612	2.918	2.404	6.598	22.417	15.731
1.7	.57	1.109	2.586	2.382	1.942	2.685	19.083	15.508
*	.67	1.233	2.128	1.999	1.608	1.356	19.479	17.379
	.72	1.311	1.972	1.843	1.468	0.965	20.681	19.245
	.401	.983	4.399	5.031	3.049	16.39	28.968	17.746
	.451	1.025	3.083	4.572	2.646	7.428	22.05	15.415
1.85	.551	1.123	2.135	3.531	2.062	2.921	18.411	15.092
	.651	1.247	1.697	2.899	1.642	1.391	19.023	17.263
	.751	1.407	1.419	2.421	1.321	0.602	24.74	24.361
	.389	.978	4.067	9.141	3.247	17.504	29.456	17.709
*	.439	1.019	2.828	7.965	2.786	7.835	22.291	15.455
1.95	.539	1.112	1.929	6.255	2.11	2.987	18.519	15.28
*	.639	1.226	1.512	5.063	1.639	1.36	19.641	17.993
	.739	1.367	1.25	4.178	1.29	0.536	27.849	27.027

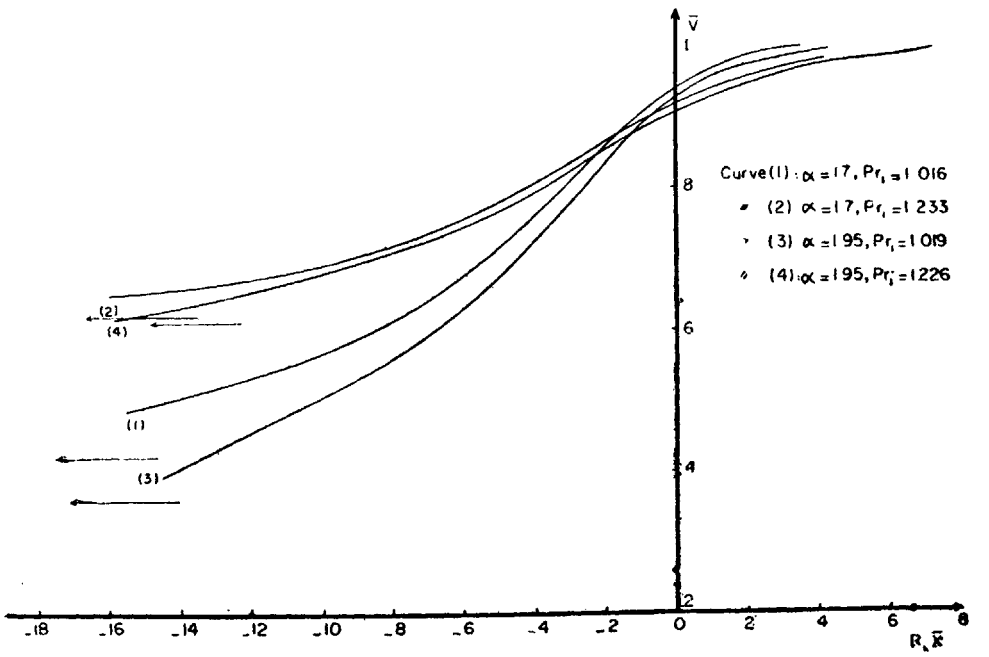

FIG 1c Velocity profile for $\gamma = 5/3$ and different α and Pr_1 : (1) $\alpha = 1.7, Pr_1 = 1.016$; (2) $\alpha = 1.7, Pr_1 = 1.233$; (3) $\alpha = 1.95, Pr_1 = 1.019$; (4) $\alpha = 1.95, Pr_1 = 1.226$.

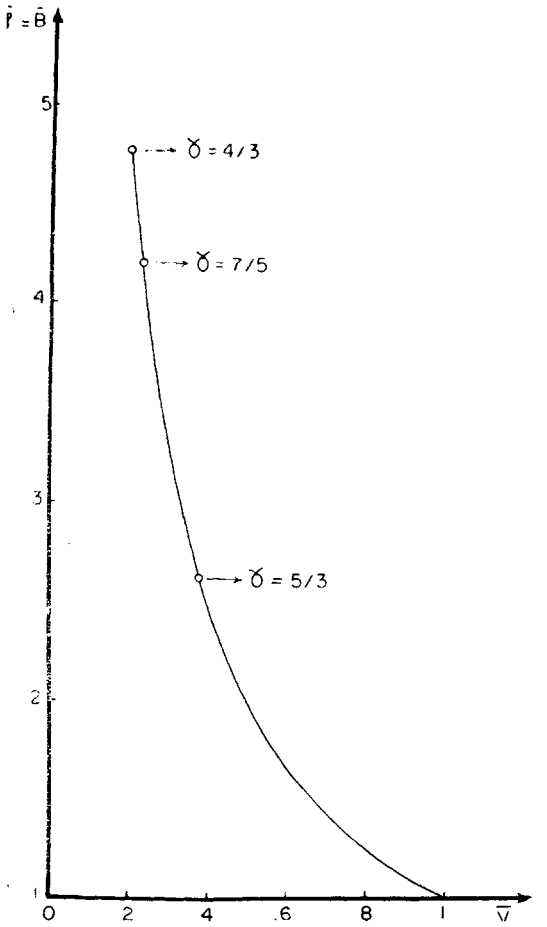

FIG 2 Variation of density and magnetic field with relative velocity of different γ.

(2) For the same α and γ and increasing Pr_1 : M_1, M_{A1}, St and the compression ratio $\bar{\rho}_2$ decrease; whilst β increases.

(3) For the same γ and increasing α : M_{A1}, St, $\bar{\rho}_2$ and Pr_1 increase; whilst M_1 and β decrease.

(4) For the same α and β and increasing γ : Pr_1 increases; whilst M_1, M_{A1}, $\bar{\rho}_2$ and St decrease.

(5) The thickness l_1 is greater than l_2. For larger or smaller Pr_1, both of them increase or decrease as St decreases. In fact, the variation of the transport coefficients is relatively small for larger Pr_1 and α; and conversely. This, in turn, affects the changes of thicknesses with strength.

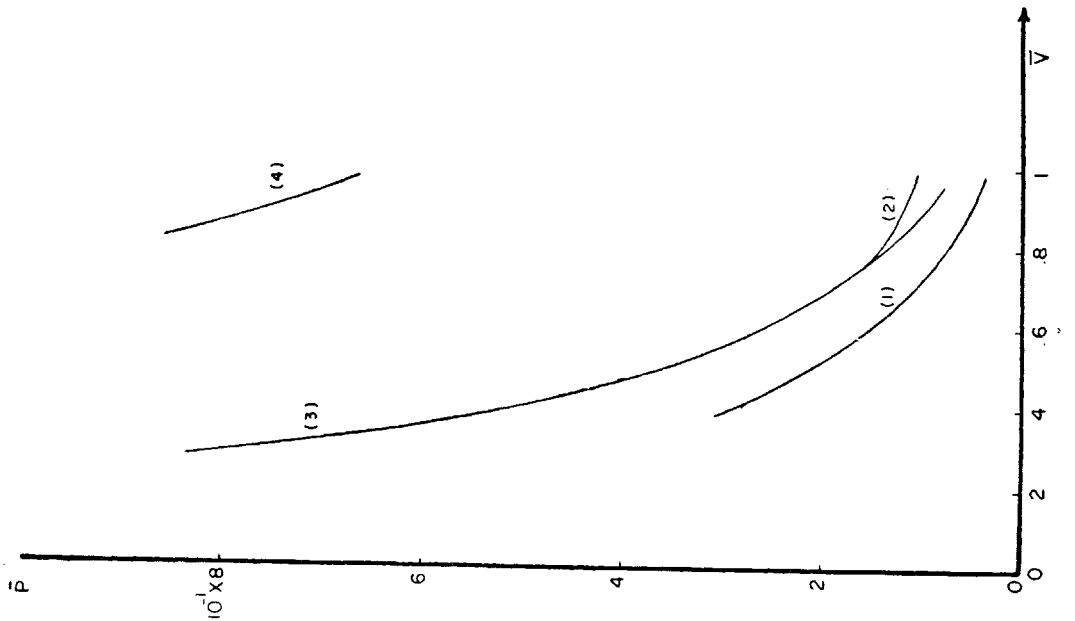

FIG 3(*b*) Variation of pressure for $\gamma = 7/5$.

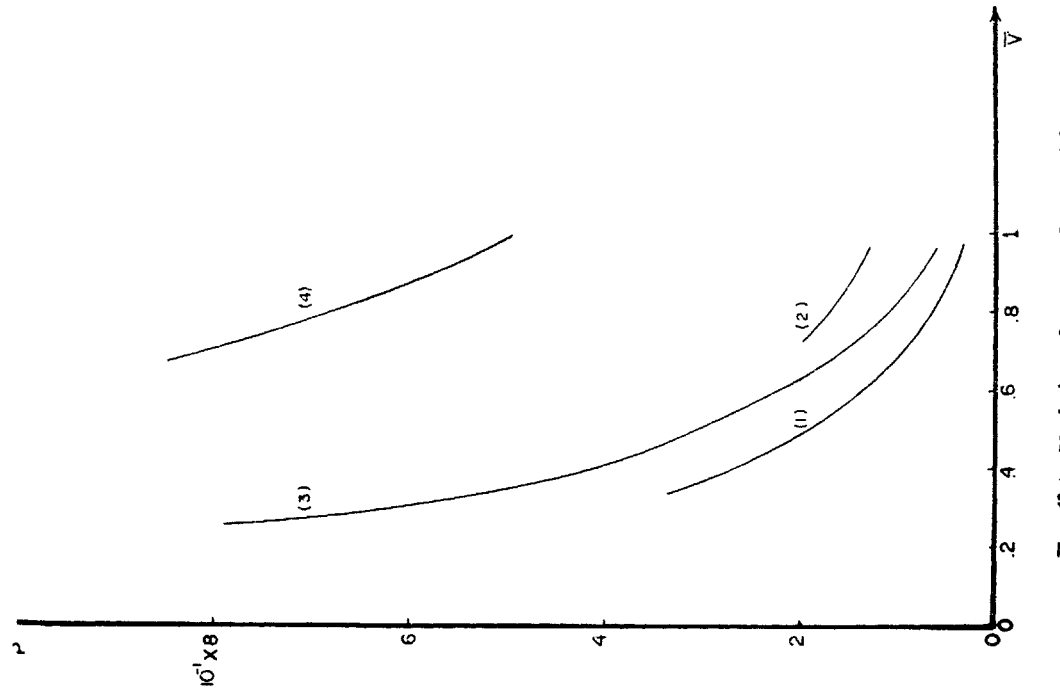

FIG (3*a*) Variation of pressure for $\gamma = 4/3$.

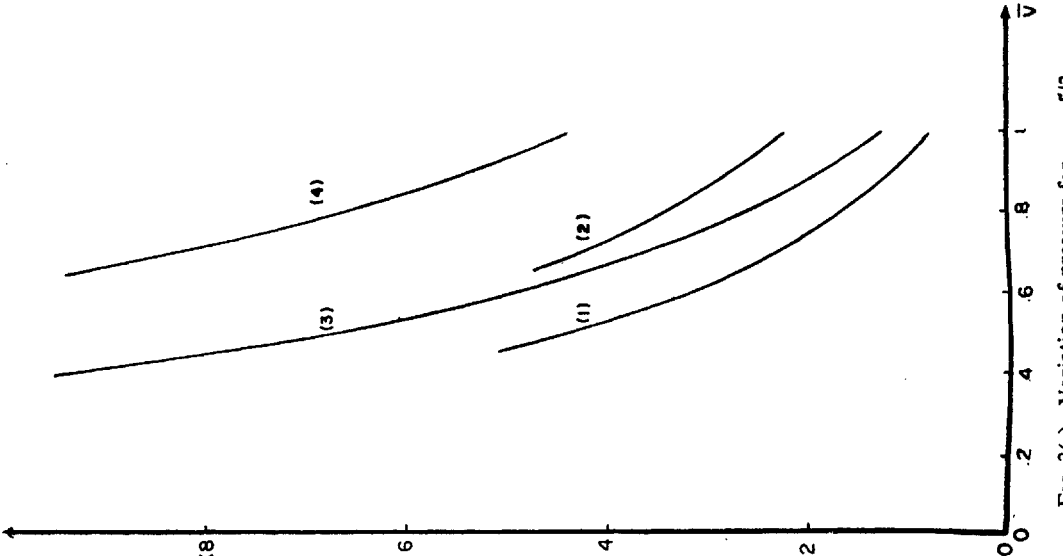

FIG 3(c) Variation of pressure for $\gamma = 5/3$.

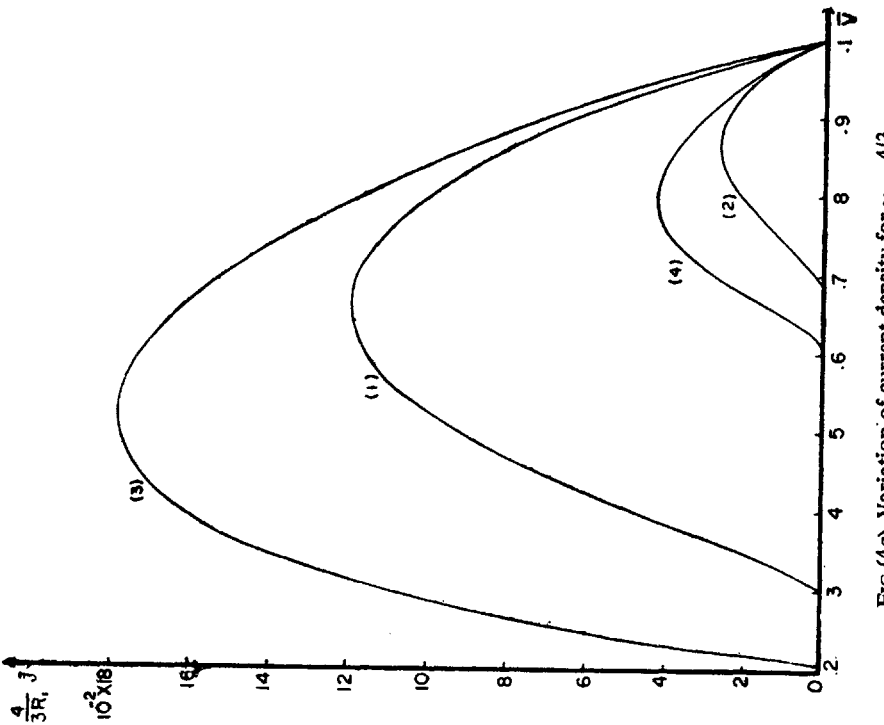

FIG (4a) Variation of current density for $\gamma = 4/3$.

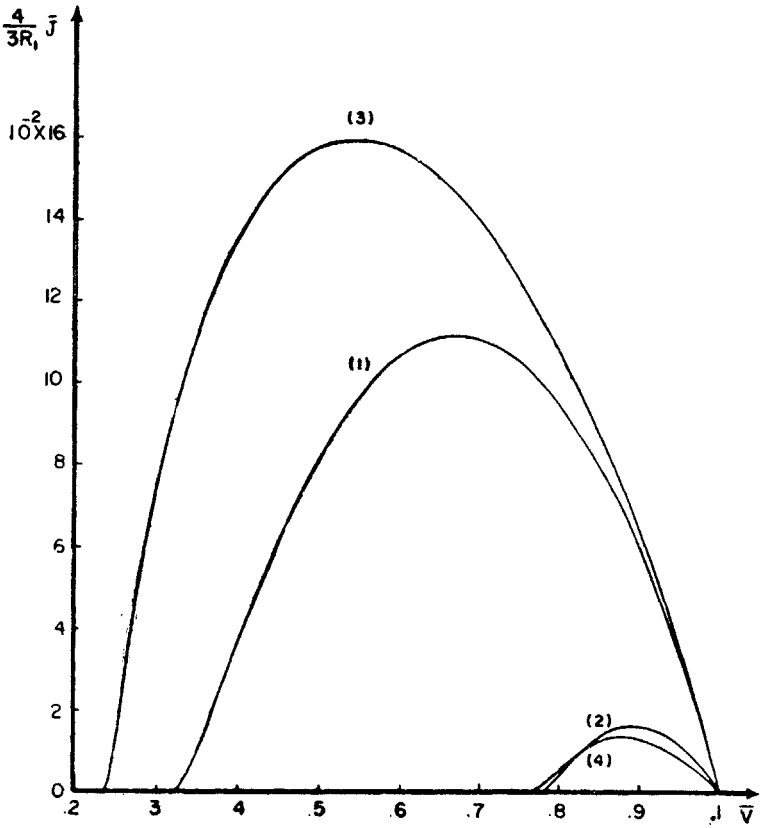

FIG. 4*b* Variation of current density for $\gamma = 7/5$.

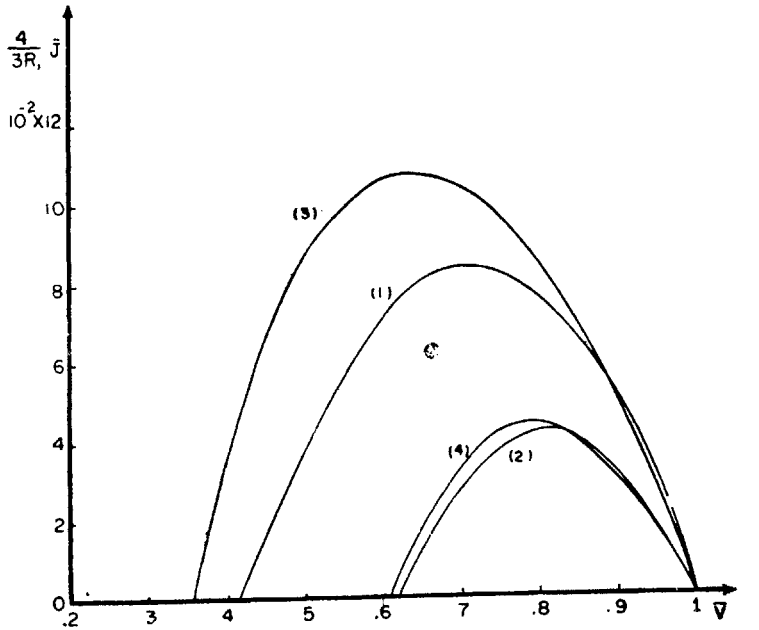

FIG. 4 Variation of current density for $\gamma = 5/3$.

(6) The negative current density and pressure $P/P_1 = M_1^2\bar{P}$ are higher for smaller Pr_1 and larger α.

The numerical calculations are performed using an IBM 370/3033 computer at King Saud University, Riyadh.

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