©*Proc. Indian natn. Sci Acad.* **52**, A, No. 5, 1986, pp. 1261–1269.
Printed in India.

ELASTIC-PLASTIC AND CREEP TRANSITION OF AN ORTHOTROPIC CYLINDER UNDER INTERNAL PRESSURE

S K GUPTA *and* P C BHARDWAJ

Department of Mathematics, Himachal Pradesh University, Simla-5, India

(*Received 15 July 1985; After Revision 8 October 1985*)

Elastic-plastic and creep stresses for an orthotropic thick-walled cylinder under internal pressure have been derived. It has been shown that for vanishing anisotropy, the stresses are same as obtained by Nadai[15] for fully plastic state and by Bailey[1] and Odqvist[5] for creep state.

Key Words : Elastic-plastic and Creep Transitions; Orthotropic Cylinder; Internal Pressure; Anisotropy

INTRODUCTION

ELASTIC-PLASTIC and creep transition of a thick-walled cylinder under internal pressure has been discussed by many authors[1–5] for isotropic materials. These authors have analysed the problem after making some simplifying assumptions, like infinitesimal deformation, an yield criterion, a creep strain law and incompressibility of the material. Seth's transition theory[6,7] does not require any of these assumptions. He introduces the concept of generalised strain measure and then finds a solution of governing differential equations near the transition points. The incompressibility solution thus comes out as a particular case. It has been shown[8–13] that the asymptotic solution through principal stress-difference gives the plastic and creep stresses respectively.

In this paper, creep stresses for an orthotropic cylinder under internal pressure are obtained by using the concept of generalised strain measure and asymptotic solution through principal stress and stress-difference.

GOVERNING EQUATIONS

Consider a thick-walled circular cylinder of internal and external radii a and b respectively, subjected to internal pressure p. In cylindrical co-ordinates the components of displacement are given by[9,10]

$$u = r(1 - \beta), \quad v = 0, \quad w = dz, \qquad \ldots(1)$$

where β is a function of $r = (x^2 + y^2)^{1/2}$ and d is a constant.

The components of generalized strain are given by[6]

$$e_{rr} = \frac{1}{n^m}\{1 - (r\beta^1 + \beta)^n\}^m,$$

$$e_{\theta\theta} = \frac{1}{n^m}\{1 - \beta^n\}^m,$$

$$e_{zz} = \frac{1}{n^m}\{1 - (1 - d)^n\}^m$$

and $$e_{r\theta} = e_{\theta z} = e_{zr} = 0, \qquad \ldots(2)$$

where $$\beta' = \frac{d\beta}{dr}.$$

The stress-strain relations for an orthotropic materials are[14]

$$\tau_{rr} = c_{11}e_{rr} + c_{12}e_{\theta\theta} + c_{13}e_{zz},$$

$$\tau_{\theta\theta} = c_{21}e_{rr} + c_{22}e_{\theta\theta} + c_{23}e_{zz},$$

$$\tau_{zz} = c_{31}e_{rr} + c_{32}e_{\theta\theta} + c_{33}e_{zz},$$

$$\tau_{r\theta} = c_{44}e_{r\theta},$$

$$\tau_{\theta z} = c_{55}e_{\theta z}$$

and $$\tau_{zr} = c_{66}e_{zr}, \qquad \ldots(3)$$

where c_{11}, c_{12}, c_{13}, c_{21}, c_{22}, c_{23}, c_{31}, c_{32}, c_{33}, c_{44}, c_{55} and c_{66} are constants of materials.

The equations of equilibrium are satisfied except

$$\frac{\partial\tau_{rr}}{\partial r} + \frac{\tau_{rr} - \tau_{\theta\theta}}{r} = 0. \qquad \ldots(4)$$

Using equation (2) and (3) in equation (4), we have a nonlinear differential equation satisfied by β as

$$\frac{\beta dp}{d\beta} = \frac{[(c_{11} - c_{21})\{1 - \beta^n(p+1)^n\}^m + (c_{12} - c_{22})\{1 - \beta^n\}^m + (c_{13} - c_{23})\{1 - (1 - d)^n\}^m]}{mnc_{11}\beta^n p(p+1)^{n-1}\{1 - \beta^n(p+1)^n\}^{m-1}}$$

$$- \frac{\{1 - \beta^n\}^{m-1} c_{12}}{c_{11}(p+1)^{n-1}\{1 - \beta^n(p+1)^n\}^{m-1}} - (p+1), \qquad \ldots(5)$$

where

$$r\beta' = p\beta.$$

For $m = 1$ which holds good for secondary stage of creep, equation (5) reduces to :

$$\frac{\beta dp}{d\beta} = \frac{[(c_{11} - c_{21})\{1 - \beta^n(p+1)^n\} + (c_{12} - c_{22})\{1 - \beta^n\} + (c_{13} - c_{23})\{1 - (1 - d)^n\}]}{nc_{11}\beta^n p(p+1)^{n-1}}$$

$$- \frac{c_{12}}{c_{11}(p+1)^{m-1}} - (p+1). \qquad \ldots(6)$$

The transition points of β in (5) are $p \to 0$, $p \to -1$ and $p \to \pm \infty$. $p \to 0$ gives nothing of importance.

The boundary conditions are

$$\tau_{rr} = -p \text{ at } r = a. \qquad \ldots(7)$$

$$\tau_{rr} = 0 \quad \text{at } r = b. \qquad \ldots(8)$$

Solution through the Principal Stress

We define the transition function R as

$$R = 1 - \frac{n\tau_{rr}}{c_{11} + c_{12} + c_{13}} \qquad \ldots(9)$$

Using equation (3) in equation (9) and taking logrithmic differentiation of equation (9) with respect to r, we get for $m = 1$,

$$\frac{\mathrm{d}}{\mathrm{d}r} \log R = \frac{n\beta^n p}{r} \cdot \frac{[(p+1)^n + \beta(\mathrm{d}p/\mathrm{d}\beta)\,(p+1)^{n-1} + c_{12}/c_{11}]}{[\beta^n(p+1)^n + \beta^n(c_{12}/c_{11}) + (c_{13}/c_{11})\,(1-\mathrm{d})^n]} \qquad \ldots(10)$$

Using (6) in (10), we have

$$\frac{\mathrm{d}}{\mathrm{d}r} \log R = \frac{1}{rc_{11}} \cdot \frac{\begin{matrix}[(c_{11} - c_{21})\,\{1 - \beta^n(p+1)^n\} + (c_{12} - c_{22})\,\{1 - \beta^n\} \\ + (c_{13} - c_{23})\,\{1 - (1-\mathrm{d})^n\}]\end{matrix}}{[\beta^n(p+1)^n + \beta^n(c_{12}/c_{11}) + (c_{13}/c_{11})\,(1-\mathrm{d})^n} \qquad \ldots(11)$$

Asymptotic value of β in eqn. (11) as $p \to \pm \infty$ is,

$$\frac{\mathrm{d}}{\mathrm{d}r} \log R = -\frac{1}{r} \cdot \frac{(c_{11} - c_{21})}{c_{11}} \,. \qquad \ldots(12)$$

Integrating eqn. (12), we get

$$R = A_0 r \exp\left(\frac{-(c_{11} - c_{21})}{c_{11}}\right), \qquad \ldots(13)$$

where A_0 is a constant of integration.

Using eqn. (9) in eqn. (13), we get

$$\tau_{rr} = \frac{c_{11} + c_{12} + c_{13}}{n} \cdot \left[1 - A_0 r \exp\left(\frac{-(c_{11} - c_{21})}{c_{11}}\right)\right]. \qquad \ldots(14)$$

The value of material constant in the transition range is given by :[9,10]

$$y = \frac{1}{n} \cdot \frac{[c_{11}c_{22}c_{33} - c_{11}c_{23}^2 - c_{12}^2 c_{33} + 2c_{12}^2 c_{23} - c_{12}^2 c_{22}]}{[c_{22}c_{33} - c_{23}^2]}, \qquad \ldots(15)$$

where y is the yield stress in tension.

Using eqn. (15) in eqn. (14), we get

$$\tau_{rr} = \frac{[c_{11} + c_{12} + c_{13}] \cdot y \cdot [c_{22}c_{33} - c_{23}^2] \{1 - A_0 r^{-(c_{11}+c_{21})/c_{11}}\}}{[c_{11}c_{22}c_{33} - c_{11}c_{23}^2 - c_{12}^2 c_{33} + 2c_{12}^2 c_{23} - c_{12}^2 c_{22}]} \quad \ldots(16)$$

Substituting eqn. (16) in eqn. (5), we get

$$\tau_{\theta\theta} = \frac{[c_{11} + c_{12} + c_{13}] \cdot y \cdot [c_{22}c_{33} - c_{23}^2] \cdot \{1 - (1 - (-c_{11} - c_{21})/c_{11}) A_0 r^{(-c_{11}-c_{21})/c_{11}}\}}{[c_{11}c_{22}c_{33} - c_{11}c_{23}^2 - c_{12}^2 c_{33} + 2c_{12}^2 c_{23} - c_{12}^2 c_{22}]} \quad \ldots(17)$$

Using eqn. (2) and (3), we get

$$\tau_{zz} = \frac{c_{32}}{c_{12} + c_{22}} \left[\tau_{rr} + \tau_{\theta\theta} \right] + \left\{ c_{31} - \frac{c_{32}(c_{11} + c_{21})}{c_{12} + c_{22}} \right\} + \left\{ c_{33} - \frac{c_{32}(c_{13} + c_{23})}{c_{12} + c_{22}} \right\} \cdot e_{zz}. \quad \ldots(18)$$

Using boundary conditions (7) and (8) in eqn. (16), we have

$$A_0 = b^{(c_{11} - c_{21})/c_{11}} \quad \ldots(19)$$

and

$$p = \frac{[c_{11} + c_{12} + c_{13}] \cdot [c_{22}c_{33} - c_{23}^2] \cdot [(b/a)^{-(c_{11}+c_{21})/c_{11}} - 1]}{[c_{11}c_{22}c_{33} - c_{11}c_{23}^2 - c_{12}^2 c_{33} + 2c_{12}^2 c_{23} - c_{12}^2 c_{22}]}. \quad \ldots(20)$$

Substituting eqn. (19) in eqn. (16) to (18), we get

$$\tau_{rr} = \frac{[c_{11} + c_{12} + c_{13}] \cdot y \cdot [c_{22}c_{33} - c_{23}^2] \cdot \{1 - (b/r)^{-(c_{11}-c_{21})/c_{11}}\}}{[c_{11}c_{22}c_{33} - c_{11}c_{23}^2 - c_{12}^2 c_{33} + 2c_{12}^2 c_{23} - c_{12}^2 c_{22}]},$$

$$\tau_{\theta\theta} = \frac{[c_{11} + c_{12} + c_{13}] \cdot y \cdot [c_{22}c_{33} - c_{23}^2] \{1 - (1 - c_{11} - c_{21}/c_{11}) (b/r)^{(c_{11}-c_{21})/c_{11}}\}}{[c_{11}c_{22}c_{33} - c_{11}c_{23}^2 - c_{12}^2 c_{23} + 2c_{12}^2 c_{23} - c_{12}^2 c_{22}]} \quad \ldots(21)$$

and

$$\tau_{zz} = \frac{c_{32}}{c_{12} + c_{22}} \cdot y \cdot \frac{[c_{11} + c_{12} + c_{13}] \cdot [c_{22}c_{33} - c_{23}^2] \{2 - 2(b/r)^{(c_{11}-c_{21})/c_{11}} + (c_{11} - c_{21})/c_{11} (b/r)^{(c_{11}-c_{21})/c_{11}}\}}{[c_{11}c_{22}c_{33} - c_{11}c_{23}^2 - c_{12}^2 c_{33} + 2c_{12}^2 c_{23} - c_{12}^2 c_{22}]} + \frac{a^2 p}{b^2 - a^2} \left[1 - \frac{2c_{32}}{c_{12} + c_{22}} \right].$$

It is found that value of $\tau_{\theta\theta} - \tau_{rr}$ is maximum at $r = a$ which means that yielding of the cylinder will take place at the internal surface. Therefore, we have

$$| \tau_{\theta\theta} - \tau_{rr} | = \frac{[c_{11} + c_{12} + c_{13}] \cdot y \cdot [c_{11} - c_{21}] \cdot [c_{22}c_{33} - c_{23}^2] \cdot (b/a)^{(c_{11}-c_{21})/c_{11}}}{c_{11}[c_{11}c_{22}c_{33} - c_{11}c_{23}^2 - c_{12}^2 c_{33} + 2c_{12}^2 c_{23} - c_{12}^2 c_{22}]} \equiv y^* \quad \ldots(22)$$

Substituting the value of y in terms of y^* in eqn. (20) and (21) the transitional stresses are given by

$$\tau_{rr} = \frac{c_{11}}{c_{11} - c_{21}} \cdot \left(\frac{a}{b}\right)^{(c_{11}-c_{21})/c_{11}} \cdot y^* \cdot \left\{1 - \left(\frac{b}{r}\right)^{(c_{11}-c_{21})/c_{11}}\right\},$$

$$\tau_{\theta\theta} = \frac{c_{11}}{c_{11} - c_{21}} \cdot \left(\frac{a}{b}\right)^{(c_{11}-c_{21})/c_{11}} \cdot y^* \left\{1 - \left(1 - \frac{c_{11} - c_{21}}{c_{11}}\right) \times \left(\frac{b}{r}\right)^{(c_{11}-c_{21})/c_{11}}\right\},$$

$$\tau_{zz} = \frac{c_{32}}{c_{12} + c_{22}} \cdot y^* \cdot \frac{c_{11}}{c_{11} - c_{21}} \cdot \left(\frac{a}{b}\right)^{(c_{11}-c_{21})/c_{11}} \left\{2 - 2\left(\frac{b}{r}\right)^{(c_{11}-c_{21})/c_{11}} + \frac{c_{11} - c_{21}}{c_{11}}\left(\frac{b}{r}\right)^{(c_{11}-c_{21})/c_{11}}\right\} + \frac{a^2 p}{b^2 - a^2}\left[1 - \frac{2c_{32}}{c_{12} + c_{22}}\right] \quad \ldots(23)$$

and $$p = \frac{c_{11}}{c_{11} - c_{21}} \cdot \left(\frac{a}{b}\right)^{(c_{11}-c_{21})/c_{11}} \cdot y^* \cdot \left\{\left(\frac{b}{a}\right)^{(c_{11}-c_{21})/c_{11}} - 1\right\}.$$

For fully plastic case we have the following relations between the constants,[9,10] $c_{11} = c_{13} = -c_{12}$, $c_{23} = c_{21} = -c_{22}$, the eqn. (23) then becomes,

$$\tau_{rr} = \frac{c_{12}}{c_{12} - c_{22}} \cdot y^* \cdot \left(\frac{a}{b}\right)^{(c_{12}-c_{22})/c_{12}} \cdot \left\{1 - \left(\frac{b}{r}\right)^{(c_{12}-c_{22})/c_{12}}\right\},$$

$$\tau_{\theta\theta} = \frac{c_{12}}{c_{12} - c_{22}} \cdot y^* \cdot \left(\frac{a}{b}\right)^{(c_{12}-c_{22})/c_{12}} \left\{1 - \left(1 - \frac{c_{12} - c_{22}}{c_{12}}\right) \times \left(\frac{b}{r}\right)^{(c_{12}-c_{22})/c_{12}}\right\},$$

$$\tau_{zz} = -\frac{c_{22}}{c_{12} - c_{22}} \cdot y^* \left(\frac{a}{b}\right)^{(c_{12}-c_{22})/c_{12}} \cdot \frac{c_{12}}{c_{12} - c_{22}} \cdot \left\{2 - 2\left(\frac{b}{r}\right)^{(c_{12}-c_{22})/c_{12}} + \frac{c_{12} - c_{22}}{c_{12}}\left(\frac{b}{r}\right)^{(c_{12}-c_{22})/c_{12}}\right\} + \frac{a^2 p}{b^2 - a^2}\left[1 + \frac{2c_{22}}{c_{12} + c_{22}}\right] \quad \ldots(24)$$

$$p = \frac{c_{12}}{c_{12} - c_{22}} \cdot y^* \cdot \left(\frac{a}{b}\right)^{(c_{12}-c_{22})/c_{12}} \left\{\left(\frac{b}{a}\right)^{(c_{12}-c_{22})/c_{12}} - 1\right\}.$$

Particular Case

For isotropic materials, the constants reduced to two only[14] i.e. $c_{11} = c_{22} = c_{33}$, $c_{21} = c_{12} = c_{31} = c_{13} = c_{23} = c_{32} = c_{11} - 2c_{66}$, $\lambda = c_{12}$, $\mu = (c_{11} - c_{12})/2$, $c = (c_{11} - c_{12})/c_{11}$.

The eqn. (23) becomes

$$\tau_{rr} = \frac{1}{c} \cdot \left(\frac{a}{b}\right)^{c} \cdot y^* \left\{1 - \left(\frac{b}{r}\right)^{c}\right\},$$

$$\tau_{\theta\theta} = \frac{1}{c} \cdot \left(\frac{a}{b}\right)^c \cdot y^* \left\{1 - (1-c)\left(\frac{b}{r}\right)^c\right\}, \qquad \ldots(25)$$

$$\tau_{zz} = \frac{1}{c} \cdot \frac{1-c}{2-c} \cdot y^* \left(\frac{a}{b}\right)^c \left\{2 - 2\left(\frac{b}{r}\right)^c + c\left(\frac{b}{r}\right)^c\right\} + \frac{a^2 p}{b^2 - a^2}\left[\frac{c}{2-c}\right]$$

and $$p = \frac{1}{c} \cdot \left(\frac{a}{b}\right)^c y^* \left\{\left(\frac{b}{a}\right)^c - 1\right\},$$

where $$c = \frac{c_{11} - c_{21}}{c_{11}}.$$

For fully plastic case i.e. $(C \to 0)$[9,10,11] eqn. (25) reduces to

$$\tau_{rr} = -\, y^* \log \frac{b}{r},$$

$$\tau_{\theta\theta} = y^* \left[1 - \log \frac{b}{r}\right],$$

$$\tau_{zz} = y^* \left[\tfrac{1}{2} - \log \frac{b}{r}\right]. \qquad \ldots(26)$$

These expressions are the same a obtained by Nadai.[15]

Also, $\tau_{\theta\theta} - \tau_{rr} = y^*$,

which is Tresca's yield conditions.

Solution through the Stress-Difference

For finding the creep stresses, we define the transition function R_1 through the principal stress differences as :[7,11-13]

$$R_1 = \tau_{rr} - \tau_{\theta\theta} = \frac{1}{n^m}\left[(c_{11} - c_{21})\{1 - \beta^n(p+1)^n\}^m + (c_{12} - c_{22})\{1 - \beta^n\}^m + (c_{13} - c_{23})\{1 - (1-\mathrm{d})^n\}^m\right] \ldots(27)$$

Taking logarithmic differentiation of eqn. (27) with respect to β, and using (5) we get

$$\frac{\mathrm{d}}{\mathrm{d}\beta} \log R_1 = -\frac{mn\beta^{n-1}}{n^m R_1}\left[(c_{11} - c_{21}) \times \frac{[(c_{11} - c_{21})\{1 - \beta^n(p+1)^n\}^m + (c_{12} - c_{22})\{1 - \beta^n\}^m + (c_{13} - c_{23})\{1 - (1-\mathrm{d})^n\}^m]}{mnc_{11}\beta^n} - \frac{[c_{11}c_{22} - c_{12}c_{21}]}{c_{11}}\{1 - \beta^n\}^{m-1}\right] \qquad \ldots(28)$$

The asymptotic value of eqn. (28) at $p \to -1$ is,

$$\frac{d}{d\beta} \log R_1 = \frac{c_{11} - c_{21}}{c_{11}} \cdot \frac{1}{\beta} + \frac{mn\beta^{n-1}\{1 - \beta^n\}^{m-1} [c_{11}c_{22} - c_{12}c_{21}]/c_{11}}{[(c_{11} - c_{21}) + (c_{12} - c_{22}) \{1 - \beta^n\}^m + (c_{13} - c_{23}) \{1 - (1 - d)^n\}^m]} \quad \ldots(29)$$

Integrating eqn. (29), we have

$$R_1 = Ar^{K_1} [K + (c_{12} - c_{22}) \{1 - D^n r^n\}^m]^{K_2}, \quad \ldots(30)$$

where A is a constant of integration, asymptotic value of β as $p \to -1$ is D/r, D being a constant and

$$K = [(c_{11} - c_{21}) + (c_{13} - c_{23}) \{1 - (1 - d)^n\}^m],$$

$$K_1 = -\frac{(c_{11} - c_{21})}{c_{11}}$$

and $$K_2 = -\frac{(c_{11}c_{22} - c_{12}c_{21})}{c_{11}(c_{12} - c_{22})}.$$

Substituting the value of R_1 from eqn. (30) in eqn. (4), we get

$$\tau_{rr} = -A \int r^{K_1 - 1} [K + (c_{12} - c_{22}) \{1 - D^n r^n\}^m]^{K_2} \cdot dr + B, \quad \ldots(31)$$

where B is a constant of integration.

Using boundary conditions eqn. (7) and eqn. (8) in eqn. (31) and (18), we get the transitional creep stresses as :

$$\tau_{rr} = A \int_r^b r^{K_1 - 1} [K + (c_{12} - c_{22}) \{1 - D^n r^n\}^m]^{K_2} \, dr,$$

$$\tau_{\theta\theta} = \tau_{rr} - Ar^{K_1}[K + (c_{12} - c_{22}) \{1 - D^n r^n\}^m]^{K_2} \quad \ldots(32)$$

$$\tau_{zz} = \frac{c_{32}}{c_{12} + c_{22}} \cdot A \left[2 \int_r^b r^{K_1}[K + (c_{12} - c_{22}) \{1 - D^n r^{-n}\}^m]^{K_2} \, dr - r^{K_1}[K + (c_{12} + c_{22}) \{1 - D^n r^{-n}\}^m]^{K_2} \right] + \left\{ c_{31} - c_{32} \frac{(c_{11} + c_{21})}{c_{12} + c_{22}} \right\} + \left\{ c_{33} - c_{32} \frac{(c_{13} + c_{23})}{c_{12} + c_{22}} \right\} \cdot e_{zz},$$

where $$A = -p \Big/ \int_a^b r^{K_1 - 1} [K + (c_{12} - c_{22}) \{1 - D^n r^{-n}\}^m]^{K_2} \, dr.$$

The relation (30) holds good only for one stage of the creep in which the measure indices are m, n. If all the three stages of creep are to be taken into account, we shall add the incremental values[7] of $\tau_{rr} - \tau_{\theta\theta}$. Thus from eqn. (30), we have

$$\tau_{rr} - \tau_{\theta\theta} = Ar^{3K_1} \prod_{m,n} [K + (c_{12} - c_{22})\{1 - D^n r^{-n}\}^m]^{K_2}, \qquad \ldots(33)$$

where m and n having three different sets of values, each corresponding to one stage of creep.

Isotropic Steady State of Creep

For isotropic materials, the constants of the material reduce to two only[14] i.e. $c_{11} = c_{22} = c_{33}$, $c_{21} = c_{12} = c_{31} = c_{13} = c_{32} = c_{23} = c_{11} - 2c_{66}$. In terms of λ and μ, we have the relations

$$\lambda = c_{12}, \mu = \frac{c_{11} - c_{12}}{2}.$$

The stresses for isotropic steady state of creep are obtained by putting $m = 1$ in eqn. (32) and using the above relations as,

$$\begin{aligned}
\tau_{rr} &= -p\frac{[(b/r)^{2n-c(n-1)} - 1]}{[(b/a)^{2n-c(n-1)} - 1]}, \\
\tau_{\theta\theta} &= -p\frac{[(b/r)^{2n-c(n-1)}\{1 - 2n + c(n-1)\} - 1]}{[(b/a)^{2n-c(n-1)} - 1]} \qquad \ldots(34) \\
\tau_{zz} &= -p\cdot\frac{1-c}{2-c}\frac{[(b/r)^{2n-c(n-1)}\{2 - 2n + c(n-1)\} - 2]}{[(b/a)^{2n-c(n-1)} - 1]} \\
&\quad + \frac{a^2p}{b^2 - a^2}\left[1 - \frac{2(1-c)}{(2-c)}\right],
\end{aligned}$$

where $c = \dfrac{c_{11} - c_{12}}{c_{11}}$.

For incompressible material i.e. $(c \to 0)$[9-13] these equations reduce to :

$$\begin{aligned}
\tau_{rr} &= -p\frac{[r^{-2n} - b^{-2n}]}{[a^{-2n} - b^{-2n}]}, \\
\tau_{\theta\theta} &= -p\frac{[r^{-2n}\{1 - 2n\} - b^{-2n}]}{[a^{-2n} - b^{-2n}]} \qquad \ldots(35)
\end{aligned}$$

and $$\tau_{zz} = -p\frac{[(1-n)r^{-2n} - b^{-2n}]}{[a^{-2n} - b^{-2n}]}$$

These expressions are the same as obtained by Bailey[1] and Odqvist[5] by assuming in compressibility of the material, provided we have $n = 1/N$.

REFERENCES

1. R W Bailey *Proc Inst mech Engrs* (1951) 164–425
2. C D Weir *J appl Mech* **24** (1957) 464
3. P J Rimrott *J appl Mech* **24** (1959) 271
4. R H King and W W Mackie *J basic Engng* **87** (1967) 877
5. F K G Odqvist *Mathematical Theory of Creep and Creep Rupture* (1974) 64–66
6. B R Seth *Int J nonlinear Mech* **1** (1966) 35–40
7. ———*J math phys Sci* **6** (1972) 1
8. ———*ZAMM* **44** (1964) 229–233; *Proc XI int Congr appl Mech Munich* (1964) 383–389
9. ———*Int J nonlinear Mech* **5** (1970) 279–285
10. ———*ZAMM* **50** (1970) 617–621
11. S Hulsurkar *ZAMM* **46** (1966) 431–437
12. S K Gupta and R L Dharmani *Indian J pure appl Math* **8** (1977) 1049–1054
13. ——— *ZAMM* **59** (1979) 517–521
14. I S Sokolnikoff *Mathematical Theory of Elasticity* McGraw Hill Book Co 2nd Edn (1956) 66
15. A Nadai *Theory of Flow and Fracture of Solids* Second Edn McGraw Hill Book Co Inc New York (1950)