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ELASTIC-PLASTIC AND CREEP TRANSITION IN AN ORTHOTROPIC ROTATING CYLINDER

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Seth's transition theory, which neither requires any ad hoc semi-empirical laws nor the yield criteria has been used to derive the Elastic-Plastic and creep stresses in an orthotropic rotating hollow and solid circular cylinders. For isotropic materials, as a particular case, the stresses are the same as obtained by Rimrott and Luke, Gupta and Dev, which they derived by assuming Von-Mises yield condition and Norton's Law.

Key Words : Elastic-Plastic and Creep Transition; Orthotropic, Rotating Cylinder; Von-Mises Yield Condition; Norton's Law

Introduction

For an ideal plastic material without strain-hardening, the stress distribution in solid rotating cylinders has been described by Nadai.[1] The addition of a central hole and the consideration of a rigid-plastic material with linear strain hardening have been discussed by Davis and Connelly[2] and by considering finite strain and a power relationship between the stress and strain by Rimmrott.[3] For isotropic creep theory, the problem has been analysed by Rimrott and Luke[4] for large strain's and by Dev[5] for plane strain. In analysing the problem, these authors assumed incompressibility of the material, an yield condition and a power relationship of stress and strain. Incompressibility of material in plasticity and creep problems is one of the important assumptions which simplifies the problem. In fact, in most of the cases, it is not possible to find a solution in closed form without these assumptions. Seth's transition theory does not require any of these assumptions. It utilizes the concept of generalized strain measure and asymptotic transition through the critical points of differential system defining the deformed field. Seth[6,7] has defined the generalized principal strain measure as :

$$\epsilon_{ii} = \int_0^{e^A_{ii}} [1 - 2e^A_{ii}]^{(n/2)-1}\, de^A_{ii} = \frac{1}{n}\,[1 - (1 - 2e^A_{ii})^{n/2}].$$

where n is the measure and e^A_{ii} is the principal Almansi strain component.

In this paper, problem of elastic-plastic and creep transition of an orthotropic rotating hollow cylinders has been solved by using the concept of generalized strain measure. For isotropic incompressible materials our results are the same as those of Gupta,[17] Rimrott and Luke[4] and Davis.[2]

GOVERNING EQUATIONS

Consider a hollow cylinder rotating about its axis with an angular velocity ω. Let the inner and outer radii of the cylinder in the deformed state be a and b respectively. The components of displacement in cylinderical coordinates are given by Seth,[18]

$$u = r(1 - \beta), \quad v = 0, \quad w = \mathrm{d}z, \qquad \ldots(1)$$

where β is a function of $r = (x^2 + y^2)^{1/2}$ only and is d a constant.

The components of generalised strain are given by Seth,[6]

$$\left.\begin{aligned} e_{rr} &= \frac{1}{n^m}\,[1 - (r\beta' + \beta)^n]^m, \\ e_{\theta\theta} &= \frac{1}{n^m}\,[1 - \beta^n]^m, \\ e_{zz} &= \frac{1}{n^m}\,[1 - (1 - \mathrm{d})^n]^m \\ \text{and} \qquad e_{r\theta} &= e_{\theta z} = e_{zr} = 0. \end{aligned}\right\} \qquad \ldots(2)$$

The stress-strain relation for an orthotropic material are given by Sokolinikoff,[9]

$$\begin{aligned} \tau_{rr} &= c_{11}e_{rr} + c_{12}e_{\theta\theta} + c_{13}e_{zz}, \\ \tau_{\theta\theta} &= c_{21}e_{rr} + c_{22}e_{\theta\theta} + c_{23}e_{zz}, \\ \tau_{zz} &= c_{31}e_{rr} + c_{32}e_{\theta\theta} + c_{33}e_{zz} \\ \text{and} \qquad \tau_{\theta z} &= c_{44}e_{\theta z}, \quad \tau_{zr} = c_{55}e_{zr}, \quad \tau_{r\theta} = c_{66}e_{r\theta}, \end{aligned} \qquad \ldots(3)$$

where $c_{11}, c_{12}, c_{13}, c_{21}, c_{22}, c_{23}, c_{31}, c_{32}, c_{33}, c_{44}, c_{55}$ and c_{66} are constants of material. From eqns. (2) and (3) we have,

$$\begin{aligned} \tau_{rr} &= \frac{c_{11}}{n^m}\,[1 - (r\beta^1 + \beta)^n]^m + \frac{c_{12}}{n^m}\,[1 - \beta^n]^m + \frac{c_{13}}{n^m}\,[1 - (1 - \mathrm{d})^n]^m, \\ \tau_{\theta\theta} &= \frac{c_{21}}{n^m}\,[1 - (r\beta^1 + \beta)^n]^m + \frac{c_{22}}{n^m}\,[1 - \beta^n]^m + \frac{c_{23}}{n^m}\,[1 - (1 - \mathrm{d})^n]^m, \\ \tau_{zz} &= \frac{c_{31}}{n^m}\,[1 - (r\beta^1 + \beta)^n]^m + \frac{c_{32}}{n^m}\,[1 - \beta^n]^m + \frac{c_{33}}{n^m}\,[1 - (1 - \mathrm{d})^n]^m \\ \text{and} \qquad \tau_{r\theta} &= \tau_{\theta z} = \tau_{zr} = 0. \end{aligned} \qquad \ldots(4)$$

Equations of equilibrium are all satisfied except,

$$\frac{\mathrm{d}}{\mathrm{d}r}\,(r\tau_{rr}) - \tau_{\theta\theta} + \rho r^2 w^2 = 0, \qquad \ldots(5)$$

where ρ is the density of the material.

From (4) and (5), we obtain

$$p(p+1)^{n-1}\beta[1-\beta^n(p+1)^n]^{m-1}\frac{dp}{d\beta}+p(p+1)^n[1-\beta^n)(p+1)^n]^{m-1}$$
$$+\frac{c_{12}}{c_{11}}p[1-\beta^n]^{m-1}-\frac{1}{mnc_{11}\beta^n}[(c_{11}-c_{21})\{1-\beta^n(p+1)^n\}^m$$
$$+(c_{12}-c_{22})\{1-\beta^n\}^m+(c_{13}-c_{23})\{1-(1-\mathrm{d})^n\}^m]-\frac{n^m\rho w^2r^2}{mnc_{11}\beta^n}$$
$$=0, \qquad \ldots(6)$$

where $r\beta^1 = p\beta$ the transition points of β are $p \to -1$, $p \to \pm\infty$.

For $m = 1$, which holds for secondary state of creep, the eq. (6) reduces to,

$$\frac{d\beta}{dp}\Big[p(p+1)^n+\frac{c_{12}p}{c_{11}}-\frac{\rho w^2r^2}{c_{11}\beta^n}-\frac{1}{nc_{11}\beta^n}\{(c_{11}-c_{21})(1-\beta^n(p+1)^n$$
$$+(c_{12}-c_{22})(1-\beta^n)+(c_{13}-c_{23})(1-c_1-d^n)\}\Big]$$
$$+\beta p(p+1)^{n-1}=0. \qquad \ldots(7)$$

The boundary conditions require that:

(*i*) $\tau_{rr=0}$ at $r = a$ and $r = b$; and ...(8)

(*ii*) the resultant force normal to the plane z = constant must vanish i.e.,

$$\int_a^b r\tau_{zz}dr = 0. \qquad \ldots(9)$$

Transition and Plastic Stress Components

For finding the plastic stresses, the transition function R is taken through the principal stress at the transition point $P \to \pm\infty$ [Seth,[9] Hulsurkar,[10] Hodge and Balaban,[12–14] Gupta and Dharmani,[14,15] Gupta and Rana,[15] Gupta[16]].

We define the transition function R as

$$R = 1-\frac{n\tau_{rr}}{c_{11}+c_{12}+c_{13}}-\frac{n\rho w^2r^2}{2(c_{11}+c_{12}+c_{13})}$$
$$\equiv\frac{1}{c_{11}+c_{12}+c_{13}}\left\{c_{11}\beta^n(p+1)^n+c_{12}\beta^n+c_{13}(1-\mathrm{d})^n-\frac{n\rho w^2r^2}{2}\right\}. \qquad \ldots(10)$$

Taking logarithmic differentiation of eq(10) with respect to r, we get

$$\frac{d}{dr}\log R=\frac{n\beta^n c_{11}}{rR(c_{11}+c_{12}+c_{13})}\Big[P(P+1)^n\beta\frac{dP}{d\beta}+P(P+1)^n$$
$$+\frac{c_{12}P}{c_{11}}-\frac{n\rho w^2r^2}{c_{11}\beta^n}\Big]. \qquad \ldots(11)$$

Substituting the value of $\frac{dP}{d\beta}$ from eq (7) in (11) we get,

$$\frac{d}{dr}\log R = \frac{1}{rc_{11}} \times \frac{[(c_{11}-c_{21})(1-\beta^n(P+1)^n+(c_{12}-c_{22})c_1-\beta^n)+(c_{13}-c_{23})(1-c_1-d)^n)]}{\left[\beta^n(P+1)^n+\frac{c_{12}}{c_{11}}\beta^n+\frac{c_{13}}{c_{11}}(1-d)^n-\frac{n\rho w^2r^2}{2c_{11}}\right]} \quad \ldots(12)$$

Taking asymptotic value of eq. (12) as $P \rightarrow \pm\infty$, we have

$$\frac{d}{dr}\log R = -\frac{1}{r}\frac{(c_{11}-c_{21})}{c_{11}}. \quad \ldots(13)$$

Integration of eq. (13) yields

$$R = A_0 r^{-(c_{11}-c_{21})/c_{11}}, \quad \ldots(14)$$

where A_0 is a constant of integration.

Using eqs. (10) and (14)

$$\tau_{rr} = B_0\{1 - A_0 r^{-(c_{11}-c_{21})/c_{11}}\} - \frac{\rho w^2 r^2}{2}. \quad \ldots(15)$$

where $B_0 = \frac{c_{11}+c_{12}+c_{13}}{n}$.

Substitution of eq. (15) in eq. (5) gives,

$$\tau_{\theta\theta}-\tau_{rr} = B_0\frac{(c_{11}-c_{21})}{c_{11}} A_0 r^{-(c_{11}-c_{21})/c_{11}} \quad \ldots(16)$$

and $$\tau_{\theta\theta} = B_0\left\{1 - A_0 r^{-(c_{11}-c_{21})/c_{11}}\left(1-\frac{c_{11}-c_{21}}{c_{11}}\right)\right\} - \frac{\rho w^2r^2}{2} \quad \ldots(17)$$

From eq. (4) we get,

$$\tau_{zz} = \frac{c_{32}}{c_{12}+c_{22}}(\tau_{rr}+\tau_{\theta\theta}) + \left\{c_{31} - \frac{c_{32}(c_{11}+c_{21})}{c_{12}+c_{22}}\right\} + 2K_1, \quad \ldots(18)$$

where $K_1 = \frac{1}{2}\left\{c_{33} - \frac{c_{32}(c_{11}+c_{21})}{c_{12}+c_{22})}\right\} e_{zz}$

Using boundary conditions eq. (8) and (9) in eqs. (15) and (18), we get

$$A_0 = \left[1 - \frac{\{1-(a/b)^{(c_{11}-c_{21})/c_{11}}\}}{\{(b/a)^2-(a/b)^{(c_{11}-c_{21})/c_{11}}\}}\right] . a^{(c_{11}-c_{21})/c_{11}},$$

$$B_0 = \frac{\rho w^2 a^2}{2}\frac{[(b/a)^2-(a/b)^{(c_{11}-c_{21})/c_{11}}]}{[1-(a/b)^{(c_{11}-c_{21})/c_{11}}]}$$

and $$K_1 = \frac{1}{2}\left\{\frac{c_{32}(c_{11}+c_{21})}{c_{12}+c_{22}} - c_{31}\right\} - \frac{c_{32}}{c_{12}+c_{22}}\rho w^2\frac{(a^2+b^2)}{8}.$$

Substituting the value of A_0, B_0 and K_1 in eq. (15) to eq. (18) we get the orthotropic transitional stresses as

$$\tau_{rr} = \frac{\rho w^2 a^2}{2} \frac{((b/a)^2 - (a/b)^{(c_{11}-c_{21})/c_{11}})}{(1-(a/b)^{(c_{11}-c_{22})/c_{11}})} \left[1 - \left\{\frac{(1-(a/b)^{(c_{11}-c_{21})/c_{11}})}{((b/a)^2-(a/b)^{(c_{11}-c_{21})/c_{11}})}\right\} \times (a/r)^{(c_{11}-c_{21})/c_{11}}\right] - \frac{\rho w^2 r^2}{2},$$

$$\tau_{\theta\theta} - \tau_{rr} = \frac{\rho w^2 a^2}{2} \frac{((b/a)^2 - (a/b)^{(c_{11}-c_{21})/c_{11}})}{(1-(a/b)^{(c_{11}-c_{21})/c_{11}})} \cdot \frac{c_{11}-c_{21}}{c_{11}} \times \left\{1 - \frac{(1-(a/b)^{(c_{11}-c_{21})/c_{11}})}{((b/a)^2-(a/b)^{(c_{11}-c_{21})/c_{11}})}\right\} (a/r)^{(c_{11}-c_{21})/c_{11}},$$

$$\tau_{\theta\theta} = \frac{\rho w^2 a^2}{2} \frac{((b/a)^2 - (a/b)^{(c_{11}-c_{21})/c_{11}}}{(1-(a/b)^{(c_{11}-c_{21})/c_{11}})} \times \left[1 - \left\{1 - \frac{(1-(a/b)^{(c_{11}-c_{21})/c_{11}})}{((b/a)^2-(a/b)^{(c_{11}-c_{21})/c_{11}})}\right\} (a/r)^{(c_{11}-c_{21})/c_{11}} \times \left(1 - \frac{c_{11}-c_{21}}{c_{11}}\right)\right] - \frac{\rho w^2 r^2}{2} \quad \ldots(19)$$

and $$\tau_{zz} = \frac{c_{32}}{(c_{12}+c_{22})}[\tau_{rr} + \tau_{\theta\theta}] - \frac{c_{32}}{c_{12}+c_{22}} \rho w^2 \frac{(a^2+b^2)}{4}.$$

It is found that the value of $|\tau_{\theta\theta} - \tau_{rr}|$ is maximum at $r = a$, which means that the yielding of the cylinder will take place at the internal surface. In this case eq. (19) gives,

$$|\tau_{\theta\theta} - \tau_{rr}| = \frac{\rho w^2 a^2}{2} \frac{[(b/a)^2 - (a/b)^{(c_{11}-c_{21})/c_{11}}]}{[1-(a/b)^{(c_{11}-c_{21})/c_{11}}]} \cdot \frac{c_{11}-c_{21}}{c_{11}} \times \left[1 - \frac{[1-(a/b)^{(c_{11}-c_{21})/c_{11}}]}{[(b/a)^2-(a/b)^{(c_{11}-c_{21})/c_{11}}]}\right] \equiv y^*, \quad \ldots(20)$$

where y^* is yield stress in tension.

The stresses for fully plastic case are obtained by using the relations $c_{11} = c_{13} = -c_{12}$ and $c_{23} = c_{21} = -c_{22}$ as given by Seth,[8] then eq. (19) and eq. (20) become,

$$\tau_{rr} = \frac{\rho w^2 a^2}{2} \frac{[(b/a)^2 - (a/b)^{(c_{12}-c_{22})/c_{12}}]}{[1-(a/b)^{(c_{12}-c_{22})/c_{12}}]} \times \left[1 - \left\{1 - \frac{[1-(a/b)^{(c_{12}-c_{22})/c_{12}}]}{[(b/a)^2-(a/b)^{(c_{12}-c_{22})/c_{12}}]}\right\} (a/r)^{(c_{12}-c_{22})/c_{12}}\right] - \frac{\rho w^2 r^2}{2},$$

$$\tau_{\theta\theta} = \frac{\rho w^2 a^2}{2} \frac{[(b/a)^2 - (a/b)^{(c_{12}-c_{22})/c_{12}}]}{[1-(a/b)^{(c_{12}-c_{22})/c_{12}}} \times \left[1 - \left\{1 - \frac{[1-(a/b)^{(c_{12}-c_{22})/c_{12}}]}{[(b/a)^2-(a/b)^{(c_{12}-c_{22})/c_{12}}}\right\} (a/r)^{(c_{22}-c_{12})/c_{12}} \times \left(1 - \frac{c_{12}-c_{22}}{c_{12}}\right)\right] - \frac{\rho w^2 r^2}{2},$$

$$\tau_{zz} = -\frac{c_{22}}{c_{12}+c_{22}}(\tau_{rr}+\tau_{\theta\theta}) + \frac{c_{22}}{c_{12}+c_{22}}\rho w^2 \frac{(a^2+b^2)}{4} \qquad \ldots(21)$$

and

$$|\tau_{\theta\theta}-\tau_{rr}| = \frac{\rho w^2 a^2}{2}\frac{[(b/a)^2-(a/b)^{(c_{12}-c_{22})/c_{12}}]}{[1-(a/b)^{(c_{12}-c_{22})/c_{12}}]}\cdot\frac{c_{12}-c_{22}}{c_{12}} \times\left[1-\frac{[1-(a/b)^{(c_{12}-c_{22})/c_{12}}}{[(b/a)^2-(a/b)^{(c_{12}-c_{22})/c_{12}}]}\right] \equiv y^*.$$

Particular Case

For isotropic materials, the constant reduces to two only,[8]

$$c_{11} = c_{22} = c_{33},$$

$$c_{12} = c_{21} = c_{32} = c_{23} = c_{31} = c_{13} = c_{11} - 2c_{66},$$

$$\lambda = c_{12}, \quad u = \frac{c_{11}-c_{12}}{2}, \quad c = \frac{c_{11}-c_{12}}{c_{11}}.$$

Equations (19) and (20) become

$$\tau_{rr} = \frac{\rho w^2 a^2}{2}\frac{[(b/a)^2-(a/b)^c]}{[1-(a/b)^c]}\left[1-\left\{1-\frac{[1-(a/b)^c]}{[(b/a)^2-(a/b)^c]}\right\}\times\left(\frac{a}{r}\right)^c\right]-\frac{\rho w^2 r^2}{2},$$

$$\tau_{\theta\theta} = \frac{\rho w^2 a^2}{2}\frac{[(b/a)^2-(a/b)^c]}{[1-(a/b)^c]}\left[1-\left\{1-\frac{[1-(a/b)^c]}{[(b/a)^2-(a/b)^c]}\right\}\times\left(\frac{a}{r}\right)^c(1-c)\right]-\frac{\rho w^2 r^2}{2}, \qquad \ldots(22)$$

$$\tau_{zz} = \frac{1-c}{2-c}(\tau_{rr}^{b}+\tau_{\theta\theta}) - \frac{1-c}{2-c}\cdot\frac{\rho w^2}{4}(a^2+b^2)$$

and

$$|\tau_{\theta\theta}-\tau_{rr}| = \frac{\rho w^2 a^2}{2}\frac{[(b/a)^2-(a/b)^c]}{[1-(a/b)^c]}\cdot c\cdot\left[1-\frac{[1-(a/b)^c]}{[(b/a)^2-(a/b)^c]}\right].$$

The stresses for fully plastic state are obtained by taking ($c \to 0$) in eqn. (22)

$$\tau_{rr} = \frac{\rho w^2}{2}\frac{(b^2-a^2)}{\log b/a}\cdot\log r/a + \frac{\rho w^2}{2}(a^2-r^2),$$

$$\tau_{\theta\theta} = \frac{\rho w^2}{2}\frac{(b^2-a^2)}{\log b/a}\cdot[1+\log r/a] + \frac{\rho w^2}{2}(a^2-r^2),$$

$$\tau_{zz} = \frac{\rho w^2}{2}\frac{(b^2-a^2)}{\log b/a}[1+2\log r/a] + \frac{\rho w^2}{4}(a^2-b^2-2r^2) \qquad \ldots(23)$$

and

$$|\tau_{\theta\theta}-\tau_{rr}| = \frac{\rho w^2}{2}\frac{(b^2-a^2)}{\log b/a} \equiv y^*.$$

These equations are the same as obtained by Gupta for Hollow rotating cylinder.

SOLUTION THROUGH THE STRESS DIFFERENCE

For finding the creep stresses, the transition function R_1 is taken through the principal stressess at the transition point $P \to -1$ [Seth,[18,19] Hulsurkar,[10] Gupta and Dharmani,[13-15,20] Gupta, Dharmani and Rana[13] and Gupta & Rana[15]].

We defined the transition function R_1 as,

$$R_1 = \tau_{rr} - \tau_{\theta\theta} \equiv \frac{1}{n^m} [(c_{11} - c_{21}) \{1 - \beta^n(P + 1)^n\}^m + (c_{12} - c_{22}) \{1 - \beta^n\}^m + (c_{13} - c_{23}) \{1 - (1 - d)^n\}^m]. \quad \ldots(24)$$

Taking the logrithmic differentiation of eq. (24) with respect to β and using eq. (6), we have,

$$\frac{d}{d\beta} \log R_1 = - \frac{(c_{11} - c_{21})}{c_{11}\beta P} + \frac{\left[\frac{c_{11}c_{22} - c_{12}c_{21}}{c_{11}}\right] mn\beta^{n-1} \{1 - \beta^n\}^{m-1} - n^m \rho w^2 r^2 \frac{(c_{11} - c_{21})}{c_{11}\beta P}}{[(c_{11} - c_{21}) \{1 - \beta^n(P + 1)^n\}^m + (c_{12} - c_{22}) \{1 - \beta^n\}^m + (c_{13} - c_{23}) \{1 -(1 - d)^n\}^m]} \quad \ldots(25)$$

Using the asymptotic value $P \to -1$, in eq. (25) and integrating we have,

$$R_1 = Ar^{k-2} [(c_{11} - c_{21}) + (c_{12} - c_{22}) \{1 - \beta^n\}^m + (c_{13} - c_{23}) \{1 - (1 - d)^n\}^m]^l \exp (f_1), \quad \ldots(26)$$

where $K = 2 - \frac{(c_{11} - c_{21})}{c_{11}}$, $l = \frac{c_{12}c_{21} - c_{11}c_{22}}{c_{11}(c_{12} - c_{22})}$,

$$f_1 = - \rho w^2 n^m \frac{(c_{11} - c_{21})}{c_{11}} \int \frac{r\mathrm{d}r}{[(c_{11} - c_{21}) + (c_{12} - c_{22}) \{1 - \beta^n\}^m + (c_{13} - c_{23} \{1 - (1 - d)^n\}^m]}$$

and A is the constant of integration.

Substituting the value of $\tau_{rr} - \tau_{\theta\theta}$ from eq. (26) in eq. (5) and integrating we have,

$$\tau_{rr} = - \frac{\rho w^2 r^2}{2} - A \int r^{k-3} [(c_{11} - c_{21}) + (c_{12} - c_{22}) \{1 - \beta^n\}^m + (c_{13} - c_{23}) \{1 - (1 - d)^n\}^m]^l \exp (f_1) \, \mathrm{d}r + A_1, \quad \ldots(27)$$

where A_1 is a constant of integration and the asymptotic value of β as $P \to -1$ is D, D being a constant. The constant A and A_1 are evaluated by using boundary condition (8) as,

$$A = - \frac{\rho w^2(b^2 - a^2)}{2 \int_a^b F(r)/r\mathrm{d}r}$$

and
$$A_1 = \tfrac{1}{2}\rho w^2 a^2 - \frac{\rho w^2(b^2 - a^2)\int_0^a F(r)/r\mathrm{d}r}{2\int_a^b F(r)/r\mathrm{d}r},$$

where
$$F(r) = r^{k-2}\,[(c_{11} - c_{21}) + (c_{12} - c_{22})\{1 - D^n r^{-n}\}^m + (c_{13} - c_{23})\{1 - (1 - d)^n\}^m]^1 \exp(f_1).$$

Putting the value of A_1 in eq. (27) we get

$$\tau_{zz} = -\frac{\rho w^2}{2}(r^2 - a^2) - \frac{\rho w^2(b^2 - a^2)\int_0^a F(r)/r\mathrm{d}r}{2\int_a^b F(r)/r\mathrm{d}r} - A\int F(r)/r\mathrm{d}r. \quad \ldots(28)$$

The asymptotic value of $\tau_{\theta\theta}$ and τ_{zz} are obtained from eq. (26) and eq. (3) as,

$$\tau_{\theta\theta} = \tau_{rr} - AF(r). \quad \ldots(29)$$

and
$$\tau_{zz} = \frac{c_{32}}{c_{12} + c_{22}}(\tau_{rr} + \tau_{\theta\theta}) + A_2, \quad \ldots(30)$$

where
$$A_2 = \left\{c_{31} - \frac{c_{32}(c_{11} + c_{21})}{c_{12} + c_{22}}\right\} + \left\{c_{33} - \frac{c_{32}(c_{13} + c_{23})}{c_{12} + c_{22}}\right\} e_{zz}.$$

Using boundary condition (9) in eq. (30), we get,

$$A_2 = \frac{c_{32}}{c_{12} + c_{22}}\left[\frac{\rho w^2(b^2 - a^2)}{2} + 2\rho w^2\int_a^b r\left(\frac{\int_0^a F(r)/r\mathrm{d}r}{\int_a^b F(r)/r\mathrm{d}r}\right)\mathrm{d}r + \frac{2A}{b^2 - a^2}\int_a^b r\left(\int_0^r 2F(r)/r\mathrm{d}r\right) + \frac{2}{(b^2 - a^2)}\int_a^b rF(r)\,\mathrm{d}r\right].$$

The relation (26) holds good only for one stage of creep, in which measure indices are m, n if all the three stages of creep are to be taken into account, we shall add the incremental values.[18]

For steady state of creep, the stresses are obtained by putting $m = 1$, in eq. (28)–(30) as,

$$\tau_{rr} = -\tfrac{1}{2}\rho w^2(r^2 - a^2) + \frac{\rho w^2(b^2 - a^2)\int_a^r F_1(r)\,\mathrm{d}r}{2\int_a^b F_1(r)\,\mathrm{d}r}$$

$$\tau_{\theta\theta} = \tau_{rr} + \frac{\rho w^2(b^2 - a^2) r F_1(r)}{2 \int_a^b F_1(r)\, dr}$$

and
$$\tau_{zz} = \frac{c_{32}}{c_{12} + c_{22}} (\tau_{rr} + \tau_{\theta\theta}) + A_2, \qquad \ldots(31)$$

where
$$F_1(r) = r^{k-2} [(c_{11} - c_{21}) + (c_{12} - c_{22}) \{1 - D^n r^{-n}\} + (c_{13} - c_{23}) (1 - (1 - d)^n]^{\iota} \exp f_1$$

and
$$f_1 = - \rho w^2 n \frac{(c_{11} - c_{21})}{c_{11}} \times \int \frac{r dr}{[(c_{11} - c_{21}) + (c_{12} - c_{22}) (1 - D^n r^{-n}) + (c_{13} - c_{23}) (1 - (1 - d)^n)]}.$$

For Hencky's measure i.e. ($n \to 0$), which holds for secondary stage of creep eq. (31) reduces to

$$\tau_{rr} - \tau_{\theta\theta} = A_3 r^{k-2}, \qquad \ldots(32)$$

where
$$A_3 = - \frac{\rho w^2(b^2 - a^2)(k - 2)}{2(b^{k-2} - a^{k-2})}$$

Rotating Solid Circular Cylinder

If the cylinder in the previous case is upto origin, (i.e. $a = 0$) then the components of displacement, strains and stresses are still given by eq. (1), (2) and (4) respectively.

In this case the boundary conditions are,

$$\tau_{rr} = 0 \quad \text{at} \quad r = b, \qquad \ldots(33)$$

and
$$\int_0^b r \tau_{zz}\, dr = 0. \qquad \ldots(34)$$

Transitional creep stresses are still given by eqs. (24), (29) and (30) but the constants A_1 and A_2 are evaluated by using boundary conditions (33) and (34). The creep stresses for the this case are,

$$\tau_{rr} = \frac{\rho w^2}{2} (b^2 - r^2) + A \int_r^b \frac{F(r)}{r}\, dr,$$

$$\tau_{\theta\theta} = \tau_{rr} - AF(r),$$

$$\tau_{zz} = \frac{c_{32}}{2(c_{12} + c_{22})} [\rho w^2(b^2 - 2r^2) - 2AF(r)] + \frac{Ac_{32}}{c_{12} + c_{22}} \left[\int_r^b \frac{2F(r)}{r}\, dr - \int_0^b \frac{2r}{b^2} \left(\int_r^b \frac{2F(r)}{r}\, dr - F(r) \right) dr \right], \qquad \ldots(35)$$

where $$F(r) = r^{k-2}\left[(c_{11}-c_{21}) + (c_{12}-c_{22})\{1-D^n r^n\}^m + (c_{13}-c_{23})\{1-(1-d)^n\}^m\right]^l \exp f_1,$$

and $$f_1 = -\rho w^2 n^m \frac{(c_{11}-c_{21})}{c_{11}} \times \int \frac{r\,dr}{[(c_{11}-c_{21}) + (c_{12}-c_{22})\{1-D^n r^{-n}\}^m + (c_{13}-c_{23})\{1-(1-d)^n\}^m]}$$

The stresses for stationary state of creep are obtained by putting $m = 1$, in eq. (35) as,

$$\tau_{rr} = \frac{\rho w^2}{2}(b^2 - r^2) + A_4 \int_r^b F_1(r)\,dr,$$

$$\tau_{\theta\theta} = \tau_{rr} - A_4 r F_1(r),$$

$$\tau_{zz} = \frac{c_{32}}{2(c_{12}+c_{22})}\left[\rho w^2(b^2 - 2r^2) - 2A_4 r F_1(r) + \frac{c_{32}}{c_{12}+c_{22}} \times A_4\left[\int_r^b 2F_1(r)\,dr - \int_0^b \frac{2r}{b^2}\left(\int_r^b 2F_1(r)\,dr - rF_1(r)\right)dr\right]\right], \quad \ldots(36)$$

where $$F_1(r) = r^{k-2}\left[(c_{11}-c_{21}) + (c_{12}-c_{22})(1-D^2 r^{-n}) + (c_{13}-c_{23})(1-(1-d)^n)^l\right]\exp f_1$$

and $$f_1 = -n\rho w^2 \frac{(c_{11}-c_{21})}{c_{11}} \times \int \frac{r\,dr}{[(c_{11}-c_{21}) + (c_{12}-c_{22})\{1-D^n r^n\} + (c_{13}-c_{23})\{1-(1-d)^n\}]}.$$

ISOTROPIC STEADY STATE OF CREEP

For isotropic materials, the constants of material reduce to two only.[8]

$$c_{11} = c_{22} = c_{33},$$

$$c_{21} = c_{12} = c_{23} = c_{32} = c_{31} = c_{13} = c_{11} - 2c_{66},$$

$$\lambda = c_{12}, \quad u = \frac{c_{11}-c_{12}}{2}, \quad c = \frac{c_{11}-c_{12}}{c_{11}} = \frac{2u}{\lambda + 2u}.$$

The creep stresses for hollow cylinder become,

$$\tau_{rr} = \frac{\rho w^2}{2}(a^2 - r^2) + \frac{\rho w^2(b^2 - a^2)\int_a^r F_2(r)\,dr}{2\int_a^b F_2(r)\,dr},$$

$$\tau_{\theta\theta} = \tau_{rr} - \frac{\rho w^2(b^2 - a^2)\, r^{(n-1)c-2n}}{2\int_a^b F_2(r)\,\mathrm{d}r} \exp(-c_1 r^{n+2})$$

and
$$\tau_{zz} = \frac{1-c}{2-c}\Bigg[\frac{\rho w^2}{2}(b^2 + a^2 - 2r^2) + \frac{\rho w^2(b^2 - a^2)\int_a^r F_2(r)\,\mathrm{d}r}{\int_a^b F_2(r)\,\mathrm{d}r} + \rho w^2 \frac{(b^2 + a^2)\, rF_2(r)}{2\int_a^b F_2(r)\,\mathrm{d}r} - 2\rho w^2 \int_a^b r \frac{\left(\int_a^r F_2(r)\,\mathrm{d}r\right)}{\int_a^b F_2(r)\,\mathrm{d}r}\,\mathrm{d}r - \frac{\int_a^b \rho w^2 r^2 F_2(r)\,\mathrm{d}r}{\int_a^b F_2(r)\,\mathrm{d}r}\Bigg], \quad \ldots(37)$$

where $F_2(r) = r^{(n-1)c-2n} \exp(-c_1 r^{n+2})$,

and $C_1 = \dfrac{n\rho w^2}{D^n(n+2)(d+2u)}$.

Eq. (29) reduces to the Tresca yield condition for incompressible material as, ($c \to 0$) i.e.,

$$\tau_{rr} - \tau_{\theta\theta} = A_3 = y. \quad \ldots(38)$$

For incompressible material, creep stresses for a hollow cylinder become

$$\tau_{rr} = -\tfrac{1}{2}\rho w^2\left[(r^2 - a^2) + \frac{(b^2 - a^2)\, a^{-2n}}{(b^{-2n} - a^{-2n})}\right] + \frac{yr^{-2n}}{2n},$$

$$\tau_{\theta\theta} = \tau_{rr} - yr^{-2n}$$

and
$$\tau_{zz} = \tfrac{1}{4}\rho w^2(b^2 + a^2 - 2r^2) - \frac{y}{2n}\left[\frac{b^{2(1-n)} - a^{2(1-n)}}{b^2 - a^1} - (1-n)\, r^{-2n}\right]. \quad \ldots(39)$$

where $y = \dfrac{n\rho w^2(b^2 - a^2)}{b^{-2n} - a^{-2n}}$.

The stresses for an elastic hollow rotating cylinder are obtained by putting $n = 1$, in eq. (39) as,

$$\tau_{rr} = \frac{\rho w^2}{2}\left[(a^2 + b^2) - \left(\frac{a^2 b^2}{2} + r^2\right)\right],$$

$$\tau_{\theta\theta} = \frac{\rho w^2}{2}\left[(b^2 + a^2) + \left(\frac{a^2 b^2}{2} - r^2\right)\right]$$

and $$\tau_{zz} = \frac{\rho w^2}{2}\left[\frac{(b^2 + a^2)}{2} - r^2\right]. \qquad \ldots(40)$$

These equations are the same as obtained by Rimrott and Luke[4] at time, $t = 0$.

For plane strain case, (i.e. $e_{zz} = 0$), as a particular case, eq. (39) becomes,

$$\tau_{rr} = -\tfrac{1}{2}\rho w^2\left[(r^2 - a^2) - \frac{r^{-2n} - a^{-2n}}{a^{-2n} - b^{-2n}}(b^2 - a^2)\right],$$

$$\tau_{\theta\theta} = \tau_{rr} - n\,\frac{\rho w^2(b^2 - a^2)}{(b^{-2n} - a^{-2n})}\,r^{-2n}$$

and $$\tau_{zz} = -\frac{\rho w^2}{2}\left[(r^2 - a^2) - \frac{(b^2 - a^2)}{b^{-2n} - a^{-2n}}\{(1 - n)\,r^{-2n} - a^{-2n}\}\right]. \qquad \ldots(41)$$

These expressions are the same as obtained by Dev[5] provided we put $n = (1/N)$. They were obtained by assuming Norton's Law and Von-Mises yield condition.

Creep stress for an isotropic solid circular cylinder become,

$$\tau_{rr} = \tfrac{1}{2}\rho w^2(b^2 - r^2) + A_4\int_r^b F_2(r)\,dr,$$

$$\tau_{\theta\theta} = \tau_{rr} - A_4 r F_2(r)$$

and $$\tau_{zz} = \tfrac{1}{2}\,\frac{1 - c}{2 - c}\cdot\rho w^2(b^2 - 2r^2) - \frac{1 - c}{2 - c}A_4 r F_2(r)$$

$$+ \frac{1 - c}{2 - c}\cdot A_4\left[\int_r^b 2F_2(r)\,dr - \int_0^b \frac{2r}{b^2}\left(\int_r^b 4F_2(r)\,dr - 2F_2(r)\right)dr\right] \qquad \ldots(42)$$

For incompressible material (i.e. $c \to 0$), eq. (42) becomes,

$$\tau_{rr} = \frac{\rho w^2}{2}(b^2 - r^2) - \frac{A_4}{2n}(b^{-2n} - r^{-2n}),$$

$$\tau_{\theta\theta} = \tau_{rr} - A_4 r^{-2n}$$

$$\tau_{zz} = \tfrac{1}{2}\rho w^2(b^2 - 2r^2) - \frac{A_4}{2n}\{b^{-2n} - (1 - n)\,r^{-2n}\}. \qquad \ldots(43)$$

The stresses for a solid circular elastic cylinder are obtained by putting $n = 1$, in eq. (43) as,

$$\tau_{rr} = \frac{\rho w^2}{2}(b^2 - r^2) - \frac{A_4}{2}(b^{-2} - r^{-2}),$$

$$\tau_{\theta\theta} = \frac{\rho w^2}{2}(b^2 - r^2) - \frac{A_4}{2}(b^{-2} + r^{-2})$$

and $$\tau_{zz} = \frac{\rho w^2}{2}(b^2 - r^2) + \frac{A_4 3u}{2b^2}\,e_{zz}. \qquad \ldots(44)$$

These expressions are same as obtained by Hodge and Balaban.[11]

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