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ENERGY PARTITIONS AT A SOLID-SANDY BILATERAL INTERFACE DUE TO AN INCIDENT ANTIPLANE SHEAR WAVE

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This paper is concerned with the reflection and refraction of elastic waves due to an SH wave incident at an solid-sandy bilateral interface. The effects of sandiness on the relative amplitudes of both the reflected and refracted SH waves and also of the relative energy partitions are distinctly marked and are shown in the graphs.

Key Words : Energy Partitions; Solid-Sandy Bilateral Interface; Antiplane Shear Wave

Introduction

The study of the reflection and refraction of elastic waves from discontinuities plays an important role in the field of Geophysics. As the earth surface might be supposed to consist of layers of different types of material properties, it is imperative to deal with the reflection and refraction of elastic waves in an earthy soil viz., sandy medium.

Problems on the propagation of surface waves in sandy media have been discussed by several authors including Paul,[1] Chattopadhyay and Sharma,[2] Dey and Mukherjee,[3] Dey and De,[4] and Chakraborty *et al.*[5] Weiskopf[6] in one of his papers gave the idea of sandiness by comparing the rigidity parameter of a sandy medium with that of classical isotropic elastic medium. Although Paul[1] was the first to introduue the concept of a sandiness parameter but because of the incorrect formulation of the modified stress-strain relation, the effect of sandiness on the propagation of waves was not found to be in accordance with the physical conditions. Accordingly, the problems of wave propagation in sandy media solved by those authors who followed Paul[1] all appear to give wrong results and conclusions. Pal[7] was the first to point out and rectify the fault.

The problems of the reflection and refraction of elastic waves due to an incident SH wave have been discussed by several authors including Ravindra,[8] Thapliyal,[9] Henneke[10] and others. A general approach developed by Achenbach[11] has been adopted here to solve the problem.

Formulation of the Problem

We have assumed the upper elastic half-space (M_2) to be sandy and the lower half-space (M_1) homogeneous and free of sandiness both being isotropic and supposed to be in welded contact at the horizontal plane interface $x_2 = 0$ (Fig. 1).

Plane harmonic incident, reflected and refracted waves in displacement are represented as

$$\mathbf{u}^{(n)} = A_n \mathbf{d}^{(n)} e^{ik_n(\mathbf{x}.\mathbf{p}^{(n)} - c_n t)}, \qquad ...(1)$$

where the values 0, 1 and 2 of n are associated with the incident, reflected and refracted SH waves. Also $\mathbf{d}^{(n)}$ and $\mathbf{p}^{(n)}$ are the unit displacement and the unit propagation vectors respectively with k_n denoting the corresponding wave number.

In the absence of body forces the existing dynamical equations of motion for the propagation of SH waves in the lower (M_1) and upper (M_2) half-spaces are

$$u_{3,11}^{(n)} + u_{3,22}^{(n)} = \frac{1}{\beta_i^2}\ddot{u}_3^{(n)}, \qquad i = 1, 2, \qquad ...(2)$$

where

$$\beta_1^2 = \frac{\mu_1}{\rho_1} \text{ and } \beta_2^2 = \frac{\mu_2}{\eta\rho_2} \qquad ...(3)$$

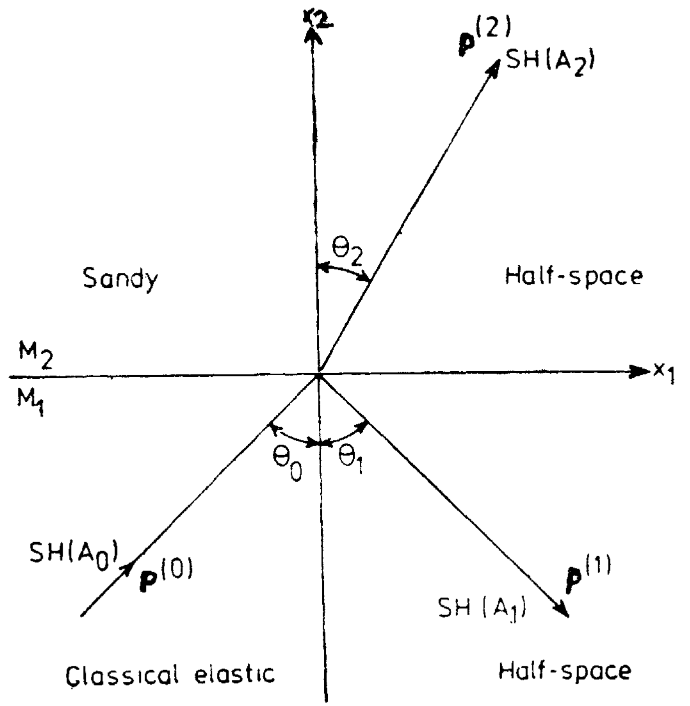

Fig 1

and η is the sandiness parameter. The operators, i and $(\cdot\cdot)$ represent the differentiation once with respect to space coordinates and twice with respect to time coordinate respectively.

The only existing boundary forces at the common interface are

$$\tau_{23}^{(n)} = i\mu_1 A_n k_n d_3^{(n)} p_2^{(n)} e^{i\alpha_n}, \; n = 0, 1 \qquad ...(4)$$

and

$$\tau_{23}^{(n)} = i \frac{\mu_2}{\eta} A_n k_n d_3^{(n)} p_2^{(n)} e^{i\alpha_n}, \qquad n = 2, \qquad ...(5)$$

when the interface is viewed from the lower and the upper half space respectively where

$$\alpha_n = k_n (x_1 p_1^{(n)} + x_2 p_2^{(n)} - c_n t), \qquad n = 0, 1, 2 \qquad ...(6)$$

For the incident wave: $n = 0$,

$$c_T = c_0, \mathbf{d}^{(0)} = (0, 0, 1),$$

$$\mathbf{p}^{(0)} = (\sin\theta_0, \cos\theta_0, 0).$$

For the reflected wave $n = 1$,

$$c_T = c_1, \mathbf{d}^{(1)} = (0, 0, 1),$$

and $\mathbf{p}^{(1)} = (\sin\theta_1, -\cos\theta_1, 0)$.

For the refracted wave: $n = 2$,

$$c_T' = c_2, \mathbf{d}^{(2)} = (0, 0, 1)$$

and $\mathbf{p}^{(2)} = (\sin\theta_2, \cos\theta_2, 0)$,

where

$$c_T = \beta_1, c_T' = \beta_2.$$

Boundary Conditions

As these two media are in welded contact at $x_2 = 0$, the boundary conditions are

$$\left. \begin{aligned} & u_3^{(0)} + u_3^{(1)} = u_3^{(2)} \\ \text{and} \quad & \\ & \tau_{23}^{(0)} + \tau_{23}^{(1)} = \tau_{23}^{(2)} \end{aligned} \right\} \quad \text{at } x_2 = 0. \qquad ...(7)$$

Since the boundary conditions in (7) are valid for all x_1 and t, we have

$$\alpha_0 = \alpha_1 = \alpha_2 \text{ at } x_2 = 0$$

and hence

$$\left. \begin{aligned} & k_0 \sin\theta_0 = k_1 \sin\theta_1 = k_2 \sin\theta_2 \\ & k_0\, c_T = k_1\, c_T = k_2\, c_T' . \end{aligned} \right\} \qquad ...(8)$$

Therefore, the set of boundary conditions in (7) gives two equations for the amplitudes A_1 and A_2 in terms of A_0 and is written with the aid of (8) as

$$\begin{bmatrix} 1 & -1 \\ 1 & R \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = A^0 \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \quad \ldots(9)$$

where $R = \frac{k_2}{k^0} \frac{\mu_2}{\eta\mu_1} \frac{\cos\theta_2}{\cos\theta_0}$.

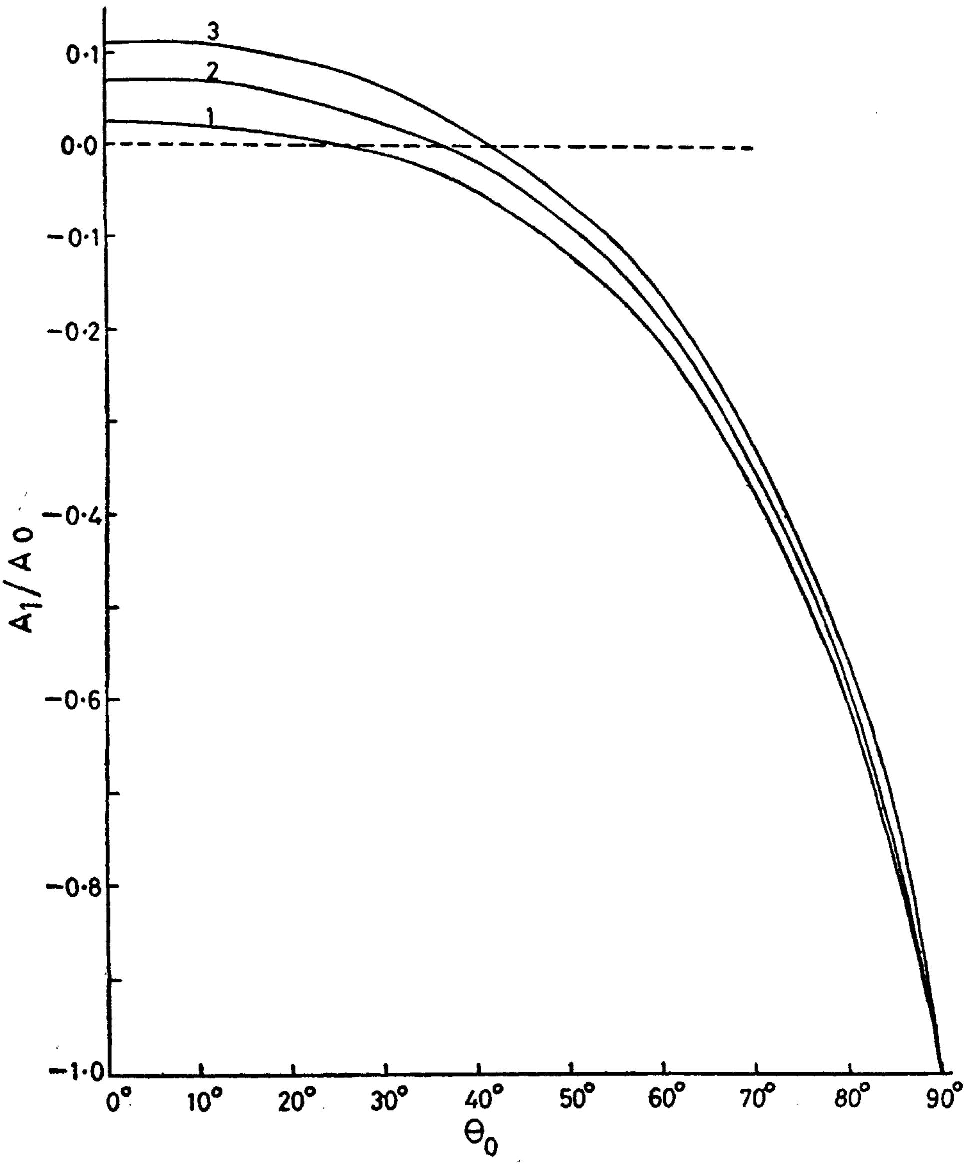

Fig 2

From (8), we find that

$$\sin \theta_2 = \frac{c'_T}{c_T} \sin \theta_0. \quad \ldots(10)$$

Therefore, for a given value of θ_0, we can determine the relative amplitude ratios of both the reflected and refracted waves for different values of the sandiness parameter.

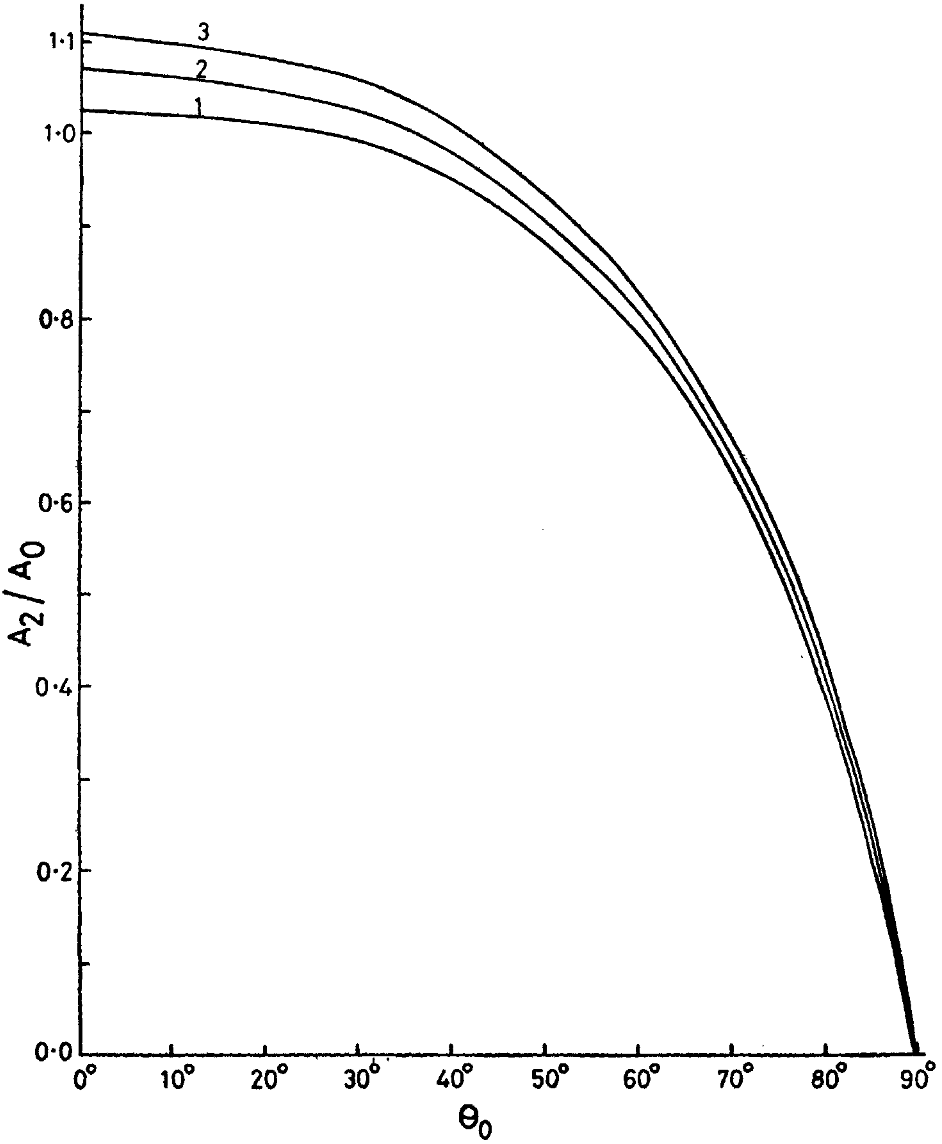

Fig 3

Energy Partitions

Since no energy is dissipated at the interface, the energy of the incident wave is equal to the sum of the energies of the reflected and refracted waves. Thus the energy equation is

$$1 = \left(\frac{A_1}{A_0}\right)^2 + \frac{c_T'}{c_T}\frac{\rho_2}{\rho_1}\left(\frac{A_2}{A_0}\right)^2 \qquad \ldots(11)$$

As the ralative amplitudes of both the reflected and the refracted waves are known, therefore, the relative energy transmitted by both the reflected and the refracted waves can be determined from eq. (11).

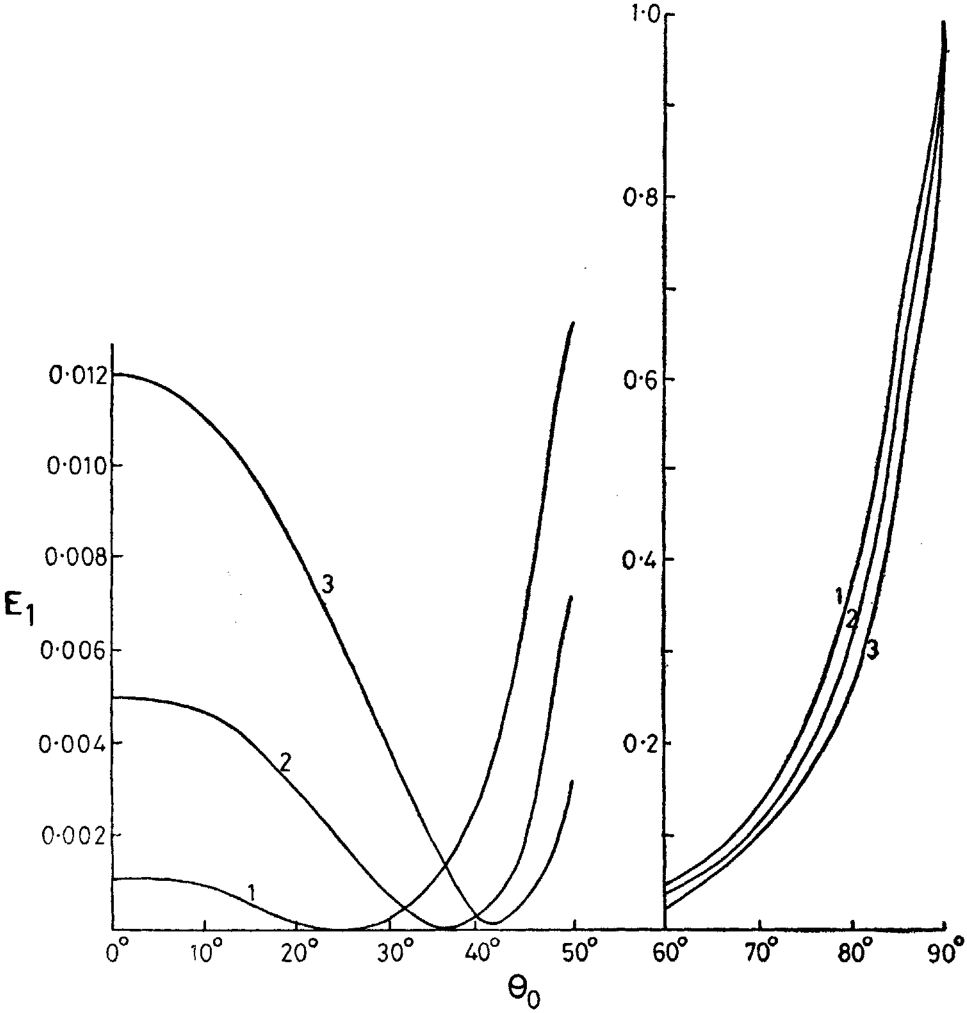

FIG 4

Numerical Calculation and Graphical Representation

For the purpose of the numerical calculation of the amplitudes of the waves and also of the energy partitions the following values are considered

$$\frac{\mu_1}{\mu_2} = .64, \quad \frac{\rho_2}{\rho_1} = 1.4$$

and

$$\eta = 1, \; 1.2 \text{ and } 1.4.$$

The values of the relative amplitudes A_1/A_0 and A_2/A_0 and also of the relative energies E_1 and E_2 are calculated for different values of θ_0 and η. These results are represented graphically in Figs. 2–5.

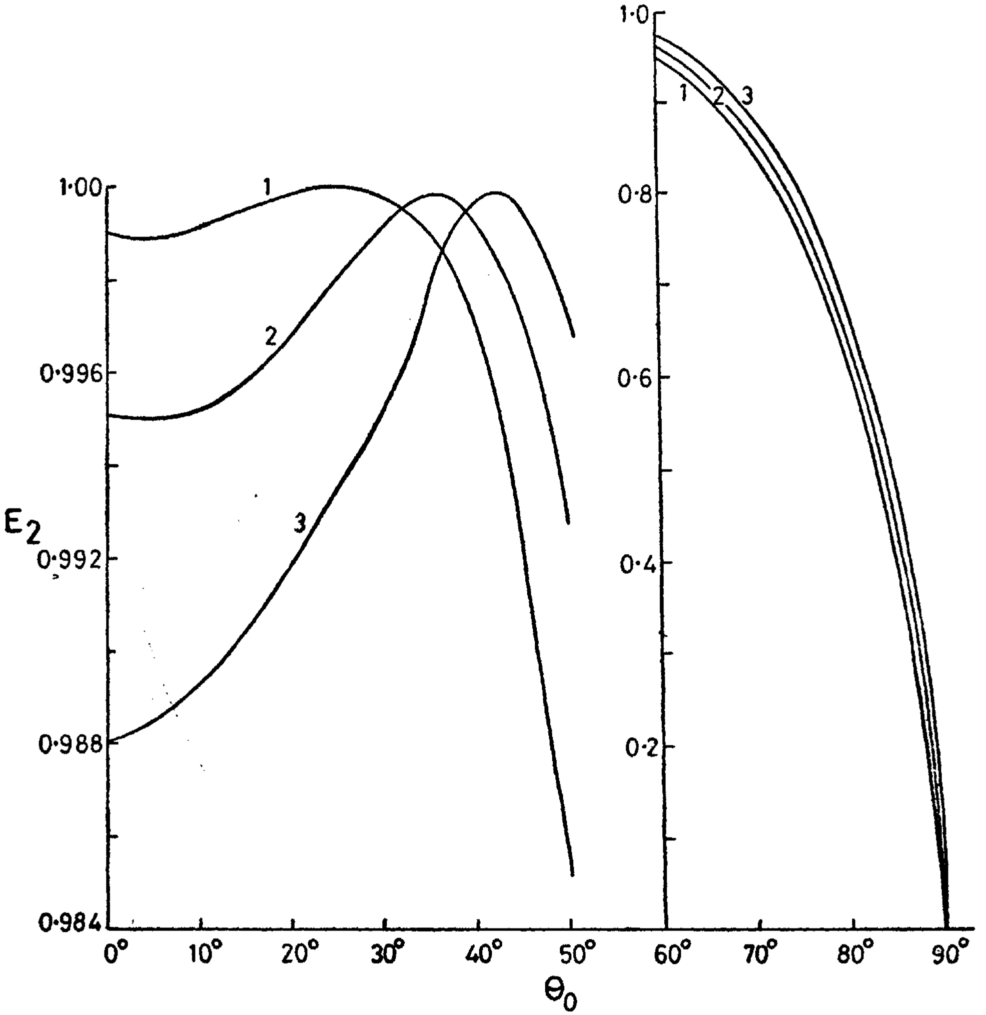

FIG 5

DISCUSSION

On comparing the curve 1 with the curves 2 and 3 in the Fig. 2, we conclude that the sandiness increases the amplitude of the reflected waves upto a certain angle of incidence and then decreases it numerically. The amplitude of the reflected wave vanishes at $\theta_0 = 26.5°$, $35.5°$ and $42°$ for $\eta = 1, 1.2$ and 1.4 respectively. The amplitude of the refracted SH waves increases with the sandiness for all θ_0 which is evident from the curves 1–3 in Fig. 3.

From Fig. 4, it is seen that the relative energy transmission through the reflected wave decreases, vanishes and then increases for all η. Due to sandiness, the energy transmitted through the reflected waves is greater up to a certain angle of incidence then thereafter, there exists a transition zone ($32° < \theta < 42°$) and then becomes less.

Similarly, the energy transmitted through the refracted waves increases, takes the value unity, decreases and finally vanishes for all η. Due to sandiness, the energy transmission through the refracted waves is less up to a certain angle of incidence, after that becomes greater with a transition zone in between. All these results are evident from the curves 1–3 in Fig. 5.

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