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# ROTON-LIKE EXCITATION IN SUPERFLUID HELIUM-4

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A second quantized technique is used to study the thermodynamic behaviour of roton-like excitations, in liquid helium-4. Temperature dependence and density dependence of the energy-gap needed for roton-like excitations are obtained from this model.

**Key Words : Roton-like Excitation; Superfluid Helium-4; Energy-gap; Bogolubov Quasi-particles; Landau Model; Hartree-Fock Equation; Bose-Einstein Condensation**

## INTRODUCTION

THE theory of roton excitation is still far less developed than the phonon excitations in superfluid helium-4. Much trouble crops up whenever one wants to describe the overall spectrum of the system. Landau[1] tried to obtain some justification for the spectrum from the study of quantum hydrodynamics. However, this is not a complete detailed atomic approach, for the reason that here one attempts to describe the liquid by a few quantities, such as density, current or velocity and then one makes these quantities as operators with reasonable commutation relations to study the spectrum of the system. According to Feynman[2] such an approach cannot give us an ultimate detailed understanding for two reasons : (*a*) The numerical values of the constants recurring in the Landau model of roton spectrum show that these quantities are characteristic of the atomic structure of the liquid; (*b*) A theory based on average variables and which cannot represent the idea that the liquid does in fact have atomic structure, cannot lead to definite values for excitation energy. He further asserted that the description of the liquid He-II with average variables gives no assurance that there are no excitations of a level below the coarseness of these averages.

According to Anderson,[3] vortex nucleation in superfluid He-4 is the bigger puzzle in its theory and hence the dynamical formulation of the energy gap needed for the roton excitation is still to be understood by a refined microscopic theory.

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Much work has been done from 1900 onwards on the microscopic description of roton-type excitations in superfluid helium.[4,7]

Recent developments high energy physics reveal that the model for elementary particles involve quarks, which are assumed to be dynamically generated. In the same manner, it has been hypothesized that the energy-gap, needed for roton-like excitation can be generated dynamically[8] and this energy-gap is essential to describe the low temperature behaviour of helium II.[6] This idea becomes pertinent due to the fact that there exists a complete analogy between the production of hadrons and the creation of excitations in condensed matter physics.[9] Encouraged with such a notation, one of us[7E] has formulated a theory of roton-like excitations in He II, by exploiting the 3-dimensional rotation group and the introduction of roton operations, coupling roton-like excitations, including many states in the roton region of the Landau spectrum of He II. This model is justified on two accounts :

(*i*) Liquid helium has a close resemblance to the system of solids;[10]

(*ii*) Roton is the smallest of the vortices, which are produced when helium II moves with greater velocity[2] and the mixture of quantized vortex lines and the superfluid components can mimic the rigid body rotation.[11]

Here with the above ideas in mind, we give different structures to our roton operators. Our formalism of operators is similar to the quasiboson or random phase approximations for phonon like spectrum. Accordingly, we establish in the next section the relation between the roton excitation and the Bogolubov quasi-particles. The structure of the roton operators enables us to study the low temperature behaviour of superfluid helium-4. For this study we introduce the idea of pairing for roton-like spectrum for the on set of energy gap. This pairing is possible due to a very large density of states of roton-like excitation in the neighbourhood of roton-minimum of Landau's model. Further, the physical origin of pairing can be associated with the small group velocity of the excitation near the roton minimum.[6] In this way, we in the present analysis in third section are able to establish the already known concept that the energy-gap is essential to explain the thermodynamic behaviour of superfluid helium-4. In the last two Sections we then discuss the density and temperature dependence of the energy-gap.

## Second Quantized Operators

In order to take account of the residual interaction and to establish a relation between the approach utilized by one of us[7] and the time-dependent Hartree-Fock-Bogolubov theory we write the Hamiltonian in the form :

$$H = \sum_{ij} h_{ij} A_2^+ A_j + \tfrac{1}{2} \sum_{i,j,k,l} < i, k \mid V \mid j, l > A_2^+ A_k^+ A_j A_l, \qquad \ldots(1)$$

where $A_1^+$, $A_j$; are the creation and annihilation operators for single particle state and we assume that the single particle functions corresponding to $A_i$ are the solutions of the Hartree-Fock equation for the system.

In the weak coupling limit, Bogolubov[12] assumed that the term describing the non-interaction among the quasi-particles is longer than the term describing the interaction between them. The operators for the Bogolubov quasi-particles are

$$\alpha_i = u_i A_i - v_i A_2^+$$

and $$\alpha_i^+ = u_i A_i^+ = v_i A_i, \qquad \ldots(2)$$

where

$$u_i^2 - v_i^2 = 1,\ v_i^+ = v_i;\ u_i^+ = u_i,$$

$$[\alpha_i, \alpha_i^+] = \delta_{ii'} \qquad \ldots(3)$$

where the coherent factors $u_i$ and $v_i$ are taken so as to satisfy the Bose commutation relations between $\alpha_i$ and $\alpha_i^+$'s and to satisfy the Bogolubov variation principle. This latter condition implies that the vacuum state $|0>$ corresponding to Eq. (2) is defined by

$$\pi_i|0> 0 \qquad \ldots(4)$$

and $$<0 \mid [A_i^+, A_i], H] \mid 0> 0. \qquad \ldots(5)$$

The results obtained by one of us,[7] reveal that it is possible to assume the following structure of the roton operators :

$$T_{i,m}^+ \begin{cases} \dfrac{i(l_-)_{k,i}}{E_k - E_{i'}} & \text{for } m = 1 \\ 1 & \text{for } m = 0 \\ \dfrac{-i(l_+)_{ki}}{E_k - E_{i'}} & \text{for } m = +1 \end{cases} \qquad \ldots(6)$$

where the energy eigenvalue $E_k$ is visualized from the eigenvalue equation

$$H_0 \mid k> = E_k \mid k>. \qquad \ldots(7)$$

$H_0$ is a certain one-body operator chosen in such a way so that $v = H - H_0$ can be taken into account by some iterative procedure.

The roton operators $T_{im}^+$ are related to the quasi-particle operators by

$$T_i = T_i^+ = \sum_{i,j} (T_{i,j}\, \alpha_i^+ \alpha_i + T_{ij}^* \alpha_j \alpha_i), \qquad \ldots(8)$$

where the matrix elements $T_{ij}$ of $T_i$ are found from the equations describing the dynamical equation of the roton operators. These equations are :

$$<0 \mid \alpha_{10}^+ \alpha_{i0}' \{ i[H, T] - \sqrt{2}\, al\} \mid 0> = 0, \qquad \ldots(9)$$

$$i<0 \mid [I, T] \mid 0> = -\sqrt{2} \qquad \ldots(10)$$

and $$< 0 \mid \sum_i A_i^+ A_i, T \mid 0 > 0. \qquad ...(11)$$

The matrix $T_{ij}$ is proportional to the first order correction to the density matrix counting for the adiabatic slow rotation of the field of roton-like excitation. The interactions of the operator $T_{ij}$ is seen from equation

$$T = i \sqrt{2}\, a(\tilde{I})_{kl'} \frac{n_k - n_{l'}}{E_k - E_{l'}}, \qquad ...(12a)$$

where the operator $\tilde{I}$ is found from the following equations :

$$(\tilde{I}_{ij}) = i \sqrt{2}\; a(\tilde{I})_{kl'} < ik \mid V \mid jl' > \frac{n_{l'} - n_k}{E_{l'} - E_k} (I)_{l'k} \qquad ...(12b)$$

$E_i$ stands for the Hartree-Fock energy of the single particle state $i$ and $n_i$ is the occupation number in the state.

The moment of inertia is now represented by

$$\frac{1}{a} = \sum \frac{(\tilde{I})_{ij}(I_+)_{ij} - (I_+)_{ij}(\tilde{I})_{ij}}{E_i - E_j}. \qquad ...(13)$$

In this way we are able to establish the connection between the roton operators and the density matrix. Similarly, the roton operators producing roton-like spectrum at higher temperatures can also be written in terms of the density matrix. To this end $T_{2m}$ (including the first order terms) have values

$$T_{2m} = \begin{cases} 0 & \text{for } m = 2 \\ \dfrac{-i\sqrt{3}\,(I_-)_{kl'}}{E_k - E_{l'}} & \text{for } m = -1 \\ 1 & \text{for } m = 0 \\ \dfrac{+i\sqrt{3}\,(I_+)_{kl}}{E_k - E_{l'}} & \text{for } m = 1 \\ 0 & \text{for } m = 2 \end{cases} \qquad ..(14)$$

In this approximation we have the quantum number $m$ occurring in our operators, not changing by more than two units for the reason that the motion of rotons in liquid helium is strongly damped[13] and produces a diffusing effect on the excitation energy.[14] This approximation is also stipulated regarding the information on the collective variables contained in the solution of the equation with the Bogolubov quasi particles. Again the terms found in the equation[7]

$$< k \mid I.\, T \mid l > \sum_{n \neq k,l} \left\{ V_{k,n} \frac{1}{E_n - E_l} < n \mid I.\, T_1^+ \mid l > \right.$$

$$\left. - < k \mid I.\, T^+ \mid n > \frac{1}{E_n - E_l} V_{nl} \right\}$$

$$= 2I \begin{cases} I_- & \text{for } m = -1 \\ 0 & \text{for } m = 0 \\ I_+ & \text{for } m = 1 \end{cases} \qquad \cdots(15)$$

result as the first approximation in the parameter containing the radius of the rigid rotator. The higher order approximation can be achieved by the treatment of perturbation theory. However, to improve the terms including the first order in $T^+$, the equations are solved further. This results in incorporating in $T^+$ terms with 4-quasi-particle operator $\alpha_i(\alpha_i^+)$. Such a procedure has been adopted in Bogolubov's approximation in the weak coupling limit of the quasi-particles.[12] In this way, this procedure gives an intimate relationship between the quasi particles and our roton-like excitation.

## Energy-Gap and Superfluidity

According to Landau's[14] modified theory of excitation spectrum, both roton and phonon type excitations are treated as two parts of one and the same excitation in the energy momentum curve $E(p)$, which because of rather high energies required in the intermediate region can for most practical purpose be treated as two separate parts (*a*) The phonon region from $p = 0$ upwards and (*b*) the region for roton-like excitation around $p \simeq p_0$, the number of roton-like excitations being appreciable only in the region $p \simeq p_0$ close around the minimum in the dispersion curve of Landau. The excitations would require a finite excitation energy $\Delta$ called the energy gap. This energy gap may be expected to be due to some kind of binding energy between the rotons. This binding energy requires an effective attractive interaction among the rotons. If it is essentially a pairing interaction it would lead to the formation of some kind of bound pairs. Moreover, the binding energy of pair will be modified by the presence of all other rotons and this will give rise to an energy-gap in the excitation of the total system. The experimental measurement of the binding energy of two rotons confirms the existence of two roton bound states with zero total momentum.[14] The possibility of forming two roton resonance with total momentum zero has been demonstrated independently by Iwamoto[16] using a phase shift analysis of the scattering amplitude for two rotons coupled by a separable potential.

Accordingly, in the roton region of the superfluid He-4, a new kind of ground state can be assumed to have formed by having a fraction of the roton-like quasi-particles in pairing state.[5,6] In this way the total energy would be reduced from that of the normal state of the superfluid He-4. Thus we are enabled to give to the roton region a theory based on microscopic formalism.

Now in order to minimize the energy of the new ground state, the roton states (pairing states) from which the paired states are to be formed must be chosen so as to maximize the binding energy of each roton pair and to obtain the greatest possible number of paired states of roton-like quasi-particles. In the particular ground state for the roton-like excitation, the best possible pairing is between the

states of equal and opposite $m$, both being occupied simultaneously.[7] As the pairs of rotons remain bound, these have certain aspects of single particles (quasi-particles) and these remain condensed in the sense of Bose-Einstein condensation in the ground states.

In setting up an explicit form for the ground state wave function in the roton region we assume that the probability with which a specific configuration of pair is visualized, can be expressed as a product of probabilities that each individual pair state is occupied .

Hence

$$| lm > = \prod_i [u_i - v_i T_i] \mid 0 >, \qquad ...(16)$$

where $| 0>$ is the vacuum state and $u_i^2$ and $v_i^2$ obey the relation (3). Evidently, this describes a state in which the total number of pairs is not well defined because it mixes states with any number of rotons. But if $u_i$ and $v_i$ are properly chosen, the best solution of the probability is highly peaked in the neighbourhood of the maximum number of rotons *i.e.*, around $p_0$ in the model of Landau and the fluctuations are small which thus cause no serious difficulties in the formalism.

For the determination of the energy gap, we split up the Hamiltonian of the superfield helium-4 into two parts[7]

$$H = H_0 + h, \qquad ...(17)$$

where $H_0$, the interinsic Hamiltonian is angular momentum free and $h$, the residual part govern the roton spectrum of the superfiuid He-4 system. Accordingly, the expectations value of $H$ for the state (16) is given by

$$\bar{E} = < H_0 + h > = \sum_i E_i v_i^2 - \sum_{ii'} V_{ii'} u_i v_i u_i' v_i. \qquad ...(18)$$

If now $v_i$ is taken as a variational parameter and the relation (3) is treated as a constraint, thus the value of $v_i$ which minimizes $E$ is given by

$$v_i^2 = \tfrac{1}{2}\left[ \frac{E_i}{(E_i^2 - \Delta_i^2)^{1/2}} - 1 \right], \qquad ...(19)$$

where $\Delta_i$, the so called energy-gap parameter, is given by

$$\Delta_i = \Sigma \bar{V}_{i,i'} u_i v_i' \qquad ...(20a)$$

with

$$\bar{V}_{ii'} = \tfrac{1}{2}(V_{ii'} + V_{i'i}). \qquad ...(20b)$$

By inserting Eqs. (8) and (19) into Eqs. (20) we get

$$\Delta_i = + \sum \frac{\bar{V}_{i,i'} \Delta_{i'}}{(E_{i'}^2 - \Delta_i^2)^{1/2}} = \sum \frac{\bar{V}_{i,i'} \Delta_{i'}}{E_i'}, \qquad ...(21)$$

where $E_i' = (E_i^2 - \Delta_i^2)^{1/2}$ which corresponds to the energy of the roton-like quasi particles (roton paired particles).

This equation (21) is a non linear integral equation for the energy gap parameter.

One possible trivial solution $\Delta_i = 0$ corresponds to the normal helium for which we have

$$E_N = \sum_{E_i < 0} E_i, \qquad \ldots(22)$$

where $E_N$ is the total energy of the normal ground state.

The non-trivial solution of Eq. (21) corresponds to the superfluid state for which we have

$$E_s = \sum \left\{ E_i \left( \frac{E_i}{E_i'} - 1 \right) - \frac{\Delta^2 i}{E_2'} \right\} \qquad \ldots(23a)$$

It can be proved that

$$E_S < E_N \quad . \qquad \ldots(23b)$$

Now in particular, if the following assumptions are made

$$\Delta_2 = \Delta \text{ (constant) for } | E_2 | < E_D$$

$$\Delta = 0 \qquad \text{for } | E_2 | > E_D. \qquad \ldots(24)$$

(where $E_D$ is the cut off energy) and the density of states $N(E_i)$ can be replaced by a constant $N(0)$ in the interval $| E_i | < E_D$, then Eq. (23) turns out to be

$$E_S = E_N - N(0)E_1 [(E_0 - \Delta^2)^{1/2} - E_D]. \qquad \ldots(25)$$

But as $\Delta < < E_D$ therefore

$$| E_N - E_S | \simeq \tfrac{1}{2} N(0)\Delta^2. \qquad \ldots(26)$$

Thus we find that whenever there is a finite energy-gap to give a nontrivial solution for Eq. (21), the energy of the state is lower than the normal component of the system. This shows that the on-set of energygap generated describe the thermodynamic behaviour of the system is a low temperature phenomenon.

## Density Dependence of Energy Gap

Presently, we have assumed that the roton-like quasi particles have '$l$' and '$m$' as definite quantum numbers and the system of Eqs. (2) and (3) define a transformation from the one roton creation and annihilation operators $A_i^+$, $A_i$ to new quasi-particle (paired roton) creation and annihilation operators $\alpha_i^+$, $\alpha_i$ under Bogolubov's canonical transformation. Since $\alpha_i^+$, $\alpha_i$ obey the usual Bose commutation relations,

therefore the quasi particles partaining to these operators may be regarded as independent bosons.

To determine the excitation energy of one quasi particle, we complete the excitation value of $H_0 + h$ for the wave functions :

$$\psi^{\alpha_i} = \alpha_i^+ \psi = A_i^+ \sum_{i \neq i'} (u_{i'} - v_{i'} T_{i'}^+) \mid 0 >$$

and $$\psi \alpha_i^+ = \alpha_i \psi = A_i \sum_{i \neq i'} (u_{i'} - v_{i'} T_{i'}) \mid 0 > \quad ...(27)$$

which after some algebraic manipulations gives

$$E_i^{\alpha_i} = E_i^{\alpha_i^+} = E_s + (E_i^2 - \Delta_i^2)^{1/2}.$$

$$= E_s + E_i' \quad ...(28)$$

For $\Delta_i \neq 0$ we have the minimum energy of the excitations as

$$E_{io} = \Delta_{io} \equiv \Delta_0. \quad ...(29)$$

Further, since the quasi-particles must always be created as pairs of rotons in order to conserve at least in the average, the total number of roton excitations, the minimum energy of the excited state is :

$$E_{exc} \geqslant (E_s + 2\Delta_0) = E_s + E_0. \quad ...(30)$$

It is also evident from the norm of the subsidiary conditions of the final result of one of us.[7]

Now if $\Delta_i$ is assumed to be only a function of $E_i$, then the density of states for the superfluid can be computed, if the corresponding function for the normal component is known. In this case we postulate that the total number of states is conserved. Hence we have,

$$N_N(E)\, dE = N_s(E^i)\, dE^i, \quad ...(31)$$

Then,

$$N_s(E^i) = N_N(E) \frac{dE^i}{dE'}$$

$$= \frac{E^i N_N \{ \pm \sqrt{(E^i)^2 - \Delta^2} \}}{\{(E^i)^2 - \Delta^i\}^{1/2} + \Delta \dfrac{d\Delta}{dE}} \quad ...(32)$$

For $E^i < \Delta_0$; $N_S(E^i) = 0$.

From the second term in the denominator of the above Eq. (32) it is clear that any abrupt change in $\Delta(E)$ will be reflected in the corresponding irregularity of $N_S(E)$. Further, the argument $N_N(E)\, [\pm (E^2 - \Delta^2)^{1/2}]$ is positive for the superfluid component.

Nishiyama,[17] observed a broad peak in the level density near the roton minimum, which can be understood as corresponding to a higher multiphonon peak. In this way we are led to conclude that there are other possible excitations and our consideration of rotons alone is partial. These excitations in the superfluid are created due to special superfluid interactions and a special feature of the superfluid state is the possibility of the collective excitations with energies lying near the energy gap. These excitations are the quantized vortices and the vortex lines etc. The nature of the excitation spectrum having collective features is closely related to the angular dependence of the interaction.[6] At very low temperature and for a constant potential $v$, the collective modes which exist, is phonon that starts the long-range correlations. At higher temperature the potential can be expanded in spherical harmonics and this leads to the creation of rotons.[5] Accordingly, we choose our roton operator in terms of such harmonics and such interactions at finite temperature may also lead to the on set of quantized vortex lines which correspond to the $p$, $d$, $f$ parts of the harmonics. These types of excitations correspond to the angular character with energies lying within the enery gap.[6]

## Temperature Dependence of Energy Gap

The effect of a finite temperature on liquid Helium II is to excite it to a state above the ground state. When the temperature is around the $\lambda$ point, the occupation number of excitations, is given by the Bose function

$$f(E_i) = \left[\exp\left(\frac{E_i}{k_B T}\right) - 1\right]^{-1}. \qquad \ldots(33)$$

However, when the temperature rises, so that the quasi particles (paired rotons) are formed, the occupation probability is given by the modified Bose function

$$g(E_i') = f(E_i') = \left[\exp\left(\frac{E_i'}{k_B T}\right) - 1\right]^{-1}, \qquad \ldots(34)$$

where $E_i'$ is the excitation energy of the quasi particles.

The effects of temperature are complicated on account of the fact that in addition to the existence of the phonon excitation, the roton excitation and others are profusely produced so that it has to read just itself to the distribution of the excitations. This causes a change in the energy gap parameter with the rise of temperature.

Now in order to obtain the occupation probability for the quasiparticles and the temperature-dependence of the energy gap, we define the free energy $F$ and minimize it with recpect to this parameter.

This free energy is given by

$$F = < H_0 > + < h > - TS, \qquad \ldots(35)$$

where $< H_0 >$ and $< h >$ are, respectively, the thermal averages of $H_0$ and $h$, and $s$ is the entropy.

At the roton region, when quasi particles are created in the superfluid, we have

$$< H_0 >_s = 2 \sum_i E_i(1 + g_i)\, v_i^2 + 2 \sum E_i g_i u_i^2$$

$$= 2 \sum_i \{E_i(1 + 2g_i)\, u_i^2 + g_i\}, \qquad ...(36)$$

where $g_i = g_i(E_i, T)$ the probability occupations, of quasi particles and is assumed to be another variational parameter.

This shows that only a fraction $(1 + 2g_i)$ of the quasi particles are effectively occupied at the roton region.

In a similar way we have

$$< h > = - \sum_{ii'} V_{i,i'} u_i v_i (1 + 2g_i)\, u_{i'} v_{i'} (1 + 2g_{i'}) \qquad ...(37)$$

and the entropy of the system is given by

$$- TS = k_B T \sum_i g_i \log g_i - (1 + g_i) \log (1 + g_i) \qquad ...(38)$$

Now when Eqs. (36), (37) and (38) are inserted into Eq. (35) and $F$ is minimized with respect to $u_i$ and $g_i$ keeping Eq. (3) as a constraint, we get the following as equilibrium values :

$$u_i^2 = \tfrac{1}{2}\left[1 + \frac{E_i}{E_i'}\right], \qquad ...(39)$$

$$E_i' = (E_i^2 - \Delta^2)^{1/2}, \qquad ...(40)$$

and

$$\Delta_i = \Sigma \bar{V}_{i,i'} u_{i'} v_i (1 + g_i$$

$$= \sum_{i'} \frac{\bar{V}_{ii'}}{2E_{i'}'} \tanh \left(\frac{E_i'}{2k_B T}\right)$$

$$\simeq \sum \frac{\bar{V}_{ii'}}{2E_{i'}'} \Delta_i' \exp \left\{\frac{-\Delta_{i'0}}{k_B T}\right\} \qquad ...(41)$$

From the above, we find that the expressions (39) and (40) remain unchanged from the $T = 0$ expressions while Eq. (41) gives the new integral equation for the energy gap parameter as a function of temperature. This expression seems to be in some consonance with the experimental results of Yarnel *et al.*[18] and Henshaw and Wods.[19]

## Conclusions

Here we comment on the accuracy of the approximation involved in deriving formulae for roton operators. This limitation of the net result becomes obvious by looking at Eq. (14). In our way we lose the information on the components of roton operators which change '$m$' quantum number of states by more than two

units. The same situation crops up on the collective variables contained in the solution of TDFB equation; but our scheme has some advantage in going beyond the approximation exploited previously. The terms retained in Eq. (15) may be looked upon as the first approximation in the parameter $I(\Delta)^{-1}$ of the final result of Ahmad.[7]

The higher order terms can be taken into account in a perturbative manner. To this end, one has to solve Eq. (15), with the right-hand side, containing the terms linear in I, i.e. the diagonal matrix elements of $T^{+}_{lm}$.

The present usage of second quantized version enables us to include in $T^{+}$ terms with 4 — quasi particle operators $\alpha_i(\alpha^{+}_i)$ alike a scheme analogous to higher order random-phase approximation. Again all the subsidiary conditions are satisfied only if the separation of the Hamiltonian in Eq. (17) is exact. However, it is not so for the real roton spectrum and only a few steps in this procedure make sense. So we adopted the above approximation of roton operators for our roton like excitations (quasi particles). Physically, it is so because, the rotons as visualized by Feynman are smallest of the vortices.[2]

In the presence of normal fluid roton of phonon type excitations collide with vortex lines, take energy from them and then gradually turning this energy gain into more phonon like and roton like quasi particles. Vortices drift through the fluid and the drift of the large vortex has its seat in the force tending to shrink the vortex to decrease the energy of line and ultimately become gryoscope and have nutation of gryoscope.[2] However in roton or roton like excitations no drift motion results because it has no further scope to shrink, hence giving a moment of inertia for the rigid motion in our model to the quasi particles justifies the model. For the description of the thermodynamic behaviour of superfluid helium, we have adopted the idea of pairing of rotons. This pairing of roton-like excitations form a finite condensate which is an important factor in describing the phenomena of superfluidity of this system. The pairing phenomena is visualized due to the existence of small group velocity of the rotons near the roton minimum.[4,6] In this way a finite fraction of rotons remain in pairing state.[5,6] This factor though small cannot be neglected. However, the depletion effect is higher.[17] It is important to realize that in the Bose system of phonon excitations, the ground state is formed kinematically and even the non-interacting Bose gas in this region has a condensate. In the roton region, we visualize condensation due to the dynamical effect of the system and it is because of the dynamical effect we have shown, how the energy gap, playing an important role in describing the low temperature behaviour of the superfluid helium can be generated dynamically. Finally, we have applied the model to describe the density dependence and temperature dependence of the energy gap and our result of the latter one is more or less in consonance with the results obtained experimentally.[18,19]

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