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THERMAL STRESSES DUE TO A NUCLEUS OF THERMOELASTIC STRAIN IN AN INFINITE TRANSVERSELY ISOTROPIC CYLINDER

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The thermal stress distribution in a transversely isotropic, infinite circular cylinder resulting from a nucleus of thermo-elastic strain of unit intensity on the circumference of a concentric circle is obtained. The results generalise those found in the existing literature.

Key Words : Thermal Stress; Nucleus of Thermoelastic Strain; Transversely Isotropic; Displacement Potential

INTRODUCTION

THE problem considered here, has already been investigated by Sokolowski[1] for isotropic case. Also a large number of steady state thermal stress problems under the action of various types of continuous and discontinuous temperature fields have been treated in Nowacki.[2] Sharma[3] determined the state of stress due to the action of a prescribed temperature distribution on the bounding plane of an elastic body showing transversal isotropy using the method of thermoelastic displacement potential. Later, Mossakowska and Nowacki[4] introduced displacement function method to determine the stresses due to the action of continuous and discontinuous temperature field within or on the surface of a transversely isotropic body.

This paper is concerned with the determination of stresses in an infinite circular cylinder possessing transverse isotropy due to a nucleus of thermoelastic strain along the circumference of a circle. Assuming axial symmetry and stress free lateral boundary, the stresses within the cylinder are obtained using displacement function method. The approach is similar to that used by Mossakowska and Nowacki[4] and also by the present authors.[5]

BASIC EQUATIONS

The solution of the problem of steady state thermal stresses in an infinite or a semi infinite elastic medium with transverse isotropy about the Z-axis (the axis of symmetry being also the same axis) can be expressed in terms of two functions χ_3 and χ_4 in the cylindrical polar co-ordinates (r, φ, Z) satisfying the following differential equations as in Mossakowska and Nowacki[4]

$$\left(\mu_1^2\nabla_r^2+\frac{\partial^2}{\partial Z^2}\right)\left(\mu_3^2\nabla_r^2+\frac{\partial^2}{\partial Z^2}\right)\left(\mu_5^2\nabla_r^2+\frac{\partial^2}{\partial Z^2}\right)\left(\mu_7^2\nabla_7^2+\frac{\partial^2}{\partial Z^2}\right)\chi_3=0 \qquad ...(1a)$$

and

$$K^1A_{33}A_{44}^2\left(\mu_1^2\nabla_r^2+\frac{\partial^2}{\partial Z^2}\right)\left(\mu_3^2\nabla_r^2+\frac{\partial^2}{\partial Z^2}\right)\left(\mu_5^2\nabla_r^2+\frac{\partial^2}{\partial Z^2}\right)\left(\mu_7^2\nabla_r^2+\frac{\partial^2}{\partial Z^2}\right)\chi_4 = -W, \qquad ...(1b)$$

where $\nabla_r^2=\dfrac{\partial^2}{\partial r^2}+\dfrac{1}{r}\dfrac{\partial}{\partial r}$;

μ_1, μ_3 and μ_7 are expressible in terms of elastic constants;

$$\mu_5^2=\frac{K}{K'};$$

K is thermal conductivity in the plane perpendicular to Z-axis;

K' = thermal conductivity along Z-axis;

A_{33}, A_{44} = elastic constant; and

W = the heat intensity term.

The stresses and displacements can be expressed in terms of χ_3 and χ_4. Now introducing two displacement functions ϕ and ψ defined by[4]

$$\phi=A_{44}\left(\mu_7^2\nabla_r^2+\frac{\partial^2}{\partial Z^2}\right)\left(\mu_5^2\nabla_r^2+\frac{\partial^2}{\partial Z^2}\right)\chi_3 \qquad ...(2a)$$

and

$$\psi=A_{44}\left(\mu_7^2\nabla_r^2+\frac{\partial^2}{\partial Z^2}\right)\chi_4 \qquad ...(2b)$$

we see that ϕ and ψ satisfy the following equations

$$K'A_{33}A_{44}\left(\mu_1\nabla_r^2+\frac{\partial^2}{\partial Z^2}\right)\left(\mu_3^2\nabla_r^2+\frac{\partial^2}{\partial Z^2}\right)\left(\mu_5^2\nabla_r^2+\frac{\partial^2}{\partial Z^2}\right)\psi=-W \qquad ...(3a)$$

and

$$\left(\mu_1^2\nabla_r^2+\frac{\partial^2}{\partial Z^2}\right)\left(\mu_3^2\nabla_r^2+\frac{\partial^2}{\partial Z^2}\right)\phi=0 \qquad ...(3b)$$

The displacements, temperature and stresses in the axisymmetric case can be expressed interms of ϕ and ψ as in Mossakowska and Nowacki[4]

$$u_r=\beta A_{44}\frac{\partial}{\partial r}\left[\nabla_r^2+a_0\frac{\partial^2}{\partial Z^2}\right]\psi-A_{44}S\frac{\partial^2\phi}{\partial r\partial Z},$$

$$u_\varphi=0,$$

$$w=\beta A_{44}\frac{\partial}{\partial Z}\left(b\nabla_r^2+c\frac{\partial^2}{\partial Z^2}\right)\psi+A_{44}\left(t\nabla_r^2+\frac{\partial^2}{\partial Z^2}\right)\phi$$

and

$$T = A_{33}A_{44}\left(\mu_1^2\nabla_r^2 + \frac{\partial^2}{\partial Z^2}\right)\left(\mu_3^2\nabla_r^2 + \frac{\partial^2}{\partial Z^2}\right)\psi. \qquad \ldots(4)$$

$$\sigma_{rr} = \beta A_{33}A_{44}\left[\frac{\partial^2}{\partial Z^2}\left(d\nabla_r^2 + e\frac{\partial^2}{\partial Z^2}\right) - q\frac{1}{r}\frac{\partial}{\partial r}\left(\nabla_r^2 + a_0\frac{\partial^2}{\partial Z^2}\right)\right]\psi$$
$$- A_{33}A_{44}\frac{\partial}{\partial Z}\left[a_1\nabla_r^2 - (a_1 - b_1)\frac{1}{r}\frac{\partial}{\partial r} - \gamma_0\frac{\partial^2}{\partial Z^2}\right]\phi,$$

$$\sigma_{\varphi\varphi} = \beta A_{33}A_{44}\left[\frac{\partial^2}{\partial Z^2}\left(d\nabla_r^2 + e\frac{\partial^2}{\partial Z^2}\right) - q\frac{\partial^2}{\partial r^2}\left(\nabla_r^2 + a_0\frac{\partial^2}{\partial Z^2}\right)\right]\psi$$
$$- A_{33}A_{44}\frac{\partial}{\partial Z}\left[a_1\nabla_r^2 - (a_1 - b_1)\frac{\partial^2}{\partial r^2} - \gamma_0\frac{\partial^2}{\partial Z^2}\right]\phi,$$

$$\sigma_{zz} = \beta A_{33}A_{44}\nabla_r^2\left(d\nabla_r^2 + e\frac{\partial^2}{\partial Z^2}\right)\psi + A_{33}A_{44}\frac{\partial}{\partial Z}\left(p\nabla_r^2 + \frac{\partial^2}{\partial Z^2}\right)\phi$$

and

$$\sigma_{rz} = -\beta A_{33}A_{44}\frac{\partial^2}{\partial r\partial Z}\left(d\nabla_r^2 + e\frac{\partial^2}{\partial Z^2}\right)\psi$$
$$+ A_{33}A_{44}\frac{\partial}{\partial r}\left(a_1\nabla_r^2 - \gamma_0\frac{\partial^2}{\partial Z^2}\right)\phi \qquad \ldots(5)$$

where

$$a_0 = \eta - \kappa(1 + \gamma_0\eta),\ b = \mu_1^2\mu_3^2\kappa\eta - \gamma_0\eta - 1,\ c = \kappa - \frac{\beta'}{\beta},$$

$$d = \gamma_0 - \mu_1^2\mu_3^2\kappa,\ e = \kappa\gamma_0 - 1,\ p = \mu_1^2 + \mu_3^2 + \gamma_0,\ q = \frac{2\mu_7^2}{\eta},$$

$$S = (1 + \gamma_0\eta),\ t = \mu_1^2\mu_3^2\eta,\ a_1 = \mu_1^2\mu_3^2,$$

$$b_1 = \mu_1^2\mu_3^2 - 2\mu_7^2(\gamma_0 + \gamma_0^{-1}),$$

$$\eta = \frac{(\mu_1^2 + \mu_3^2 + 2\gamma_0)}{(\mu_1^2\mu_3^2 - \gamma_0^2)} = \frac{A_{33}}{A_{44}}$$

and

$$\gamma_0 = \frac{A_{13}}{A_{44}},\ A_{13} = \text{elastic constant.} \qquad \ldots(6)$$

The solutions of the problems relating to the infinite cylinder can be expressed as the sum of the functions representing (*i*) the particular solution of (1*b*) and (*ii*) the general solution of (1*a*). Equivalently the solution may be expressed as the sum of the particular solution and the complementary function of (1*b*) while ignoring the function χ_3. It also follows from (2*a*), (2*b*) and (3*a*), (3*b*) that the solution may be expressed in terms of the particular solution $\bar{\psi}$ and the complementary function $\bar{\bar{\psi}}$ of (3*a*) while omitting the equation involving ϕ. Hence, we may express ψ as

$$\psi = \bar{\psi} + \bar{\bar{\psi}} \qquad \ldots(7)$$

In problems where the temperature distribution is given it will be more convenient to determine ψ from the last of equation (4) :

$$A_{33}A_{44}\left(\mu_1^2\nabla_r^2+\frac{\partial^2}{\partial Z^2}\right)\left(\mu_3^2\nabla_r^2+\frac{\partial^2}{\partial Z^2}\right)\psi=T. \qquad ...(8)$$

Problem

We shall determine the stresses within the infinite circular cylinder of radius a due to the action of a nucleus of thermoelastic strain of unit intensity distributed over the circumference of a circle of radius r_0 ($< a$) situated within the cylinder, located on the plane $Z = 0$.

Following Nowacki[2] we express the temperature field in the form

$$T=\delta(Z)\,\delta(r-r_0), \qquad ...(9)$$

where $\delta(x)$ is the Dirac delta function.

To solve this problem we determine ψ with the help of last equation of (4) where T is given by (9) and hence we have

$$A_{33}A_{44}\left(\mu_1^2\nabla_r^2+\frac{\partial^2}{\partial Z^2}\right)\left(\mu_3^2\nabla_r^2+\frac{\partial^2}{\partial Z^2}\right)\psi=\delta(Z)\,\delta(r-r_0). \qquad ...(10)$$

The solution ψ of (10) is composed of two parts $\psi=\bar{\psi}+\bar{\bar{\psi}}$ where $\bar{\psi}$ is the particular solution of (10) which is due to thermoelastic strain and $\bar{\bar{\psi}}$ is the complementary function which corresponds to the isothermal state that corrects the boundary conditions.

By making use of the following well-known representation :

$$\left.\begin{aligned}&\delta(Z)=\frac{1}{\pi}\int_0^\infty \cos\gamma Z\,d\gamma\\ \text{and}\\ &\delta(r-r_0)=\frac{2r_0}{a^2}\sum_{n=1}^{\infty}\frac{J_0(\alpha_n r_0)\,J_0(\alpha_n r)}{J_1^2(\alpha_n a)},\ J_0(\alpha_n a)=0,\end{aligned}\right\} \qquad ...(11)$$

where $J_\nu(\nu = 0, 1)$ are Bessel function of order ν, the particular solution $\bar{\psi}$ of the equation (10) may be assumed in the form

$$\bar{\psi}=\frac{2r_0}{\pi a^2A_{33}A_{44}}\int_0^\infty\sum_{n=1}^{\infty}A(\alpha_n,\gamma)\frac{J_0(\alpha_n r_0)\,J_0(\alpha_n r)}{J_1^2(\alpha_n a)}\cos\gamma Z\,dr. \qquad ...(12)$$

Now from (10) and (12), we have

$$A(\alpha_n,\gamma)=\frac{1}{(\mu_1^2\alpha_n^2+\gamma^2)(\mu_3^2\alpha_n^2+\gamma^2)}. \qquad ...(13)$$

Hence, the particular solution finally takes the form

$$\bar{\psi} = \frac{2r_0}{\pi a^2 A_{33} A_{44}} \int_0^\infty \sum_{n=1}^\infty A_n J_0(\alpha_n r) \frac{\cos \gamma Z}{(\mu_1^2 \alpha_n^2 + \gamma^2)(\mu_3^2 \alpha_n^2 + \gamma^2)} d\gamma, \quad \ldots(14)$$

where

$$A_n = \frac{J_0(\alpha_n r_0)}{J_1^2(\alpha_n a)}.$$

Since $\bar{\bar{\psi}}$ satisfies the equation

$$\left(\mu_1^2 \nabla_r^2 + \frac{\partial^2}{\partial Z^2}\right)\left(\mu_3^2 \nabla_r^2 + \frac{\partial^2}{\partial Z^2}\right)\bar{\bar{\psi}} = 0, \quad \ldots(15)$$

we assume $\bar{\bar{\psi}}$ in the form

$$\bar{\bar{\psi}} = \frac{2r_0}{\pi a^2 A_{33} A_{44}} \int_0^\infty \sum_{n=1}^\infty \left[B_n(\gamma)\, I_0\left(\frac{\gamma r}{\mu_1}\right) + C_n(\gamma)\, I_0\left(\frac{\gamma r}{\mu_3}\right)\right] \frac{\cos \gamma Z}{\gamma^3} d\gamma, \quad \ldots(16)$$

where I_ν, $\nu = 0$ or 1 denotes the modified Bessel function of the first kind of order ν and $B_n(\gamma)$ and $C_n(\gamma)$ are arbitrary constants.

Hence the general solution of (10) is

$$\psi(r, Z) = \frac{2r_0}{\pi a^2 A_{33} A_{44}} \int_0^\infty \left[\sum_{n=1}^\infty A_n J_0(\alpha_n r) \frac{\cos \gamma Z}{(\mu_1^2 \alpha_n^2 + \gamma^2)(\mu_3^2 \alpha_n^2 + \gamma^2)} + \sum_{n=1}^\infty \left\{B_n(\gamma)\, I_0\left(\frac{\gamma r}{\mu_1}\right) + C_n(\gamma)\, I_0\left(\frac{\gamma r}{\mu_3}\right)\right\} \frac{\cos \gamma Z}{\gamma^3}\right] d\gamma. \quad \ldots(17)$$

It follows from (5) with $\phi = 0$ and (17) that

$$\sigma_{rr} = \frac{2\beta r_0}{\pi a^2} \int_0^\infty \sum_{n=1}^\infty A_n \left\{h(\gamma)\, \gamma^2 J_0(\alpha_n r) - q \frac{1}{r} k(\gamma)\, \alpha_n J_1(\alpha_n r)\right\} \times \frac{\cos \gamma Z}{(\mu_1^2 \alpha_n^2 + \gamma^2)(\mu_3^2 \alpha_n^2 + \gamma^2)} - \sum_{n=1}^\infty \left\{B_n(\gamma)\left(\frac{1}{\mu_1^2} \gamma f(\mu_1)\, I_0\left(\frac{\gamma r}{\mu_1}\right) + q \frac{1}{r} \frac{1}{\mu_1^3} g(\mu_1)\, I_1\left(\frac{\gamma r}{\mu_1}\right)\right) - C_n(\gamma)\left(\frac{1}{\mu_3^2} \gamma f(\mu_3)\, I_0\left(\frac{\gamma r}{\mu_3}\right) + q \frac{1}{r} \frac{1}{\mu_3^3} g(\mu_3)\, I_1\left(\frac{\gamma r}{\mu_3}\right)\right)\right\} \cos \gamma Z\Big] d\gamma$$

$$\sigma_{\varphi\varphi} = \frac{2\beta r_0}{\pi a^2} \int_0^\infty \left[\sum_{n=1}^{\infty} A_n \left\{ h(\gamma)\, \gamma^2 J_0(\alpha_n r) - qk(\gamma)\, \alpha_n^2 \left(J_0(\alpha_n r) - \frac{J_1(\alpha_n r)}{\alpha_n r} \right) \right\} \right.$$

$$\times \frac{\cos \gamma Z}{(\mu_1^2 \alpha_n^2 + \gamma^2)\,(\mu_3^2 \alpha_n^2 + \gamma^2)} - \sum_{n=1}^{\infty} \left\{ B_n(\gamma) \left(\frac{1}{\mu_1^2} f(\mu_1)\, I_0 \left(\frac{\gamma r}{\mu_1} \right) \right. \right.$$

$$\left. + q \frac{1}{\mu_1^4} g(\mu_1) \right) \left(I_0 \left(\frac{\gamma r}{\mu_1} \right) - \frac{I_1(\gamma r/\mu_1)}{\gamma r/\mu_1} \right) + C_n(\gamma) \left(\frac{1}{\mu_3^2} f(\mu_3)\, I_0 \left(\frac{\gamma r}{\mu_3} \right) \right.$$

$$\left. \left. \left. + q \frac{1}{\mu_3^4} g(\mu_3) \left(I_0 \left(\frac{\gamma r}{\mu_3} \right) - \frac{I_1(\gamma r/\mu_3)}{\gamma r/\mu_3} \right) \right\} \gamma \cos \gamma Z \right] d\gamma,$$

$$\sigma_{zz} = \frac{2\beta r_0}{\pi a^2} \int_0^\infty \left[\sum_{n=1}^{\infty} A_n \alpha_n^2\, h(\gamma)\, J_0(\alpha_n r) \frac{\cos \gamma Z}{(\mu_1^2 \alpha_n^2 + \gamma^2)\,(\mu_3^2 \alpha_n^2 + \gamma^2)} \right.$$

$$\left. + \sum_{n=1}^{\infty} \left\{ B_n(\gamma) \frac{1}{\mu_1^4} f(\mu_1)\, I_0 \left(\frac{\gamma r}{\mu_1} \right) + C_n(\gamma) \frac{1}{\mu_3^4} I_0 \left(\frac{\gamma r}{\mu_3} \right) \right\} \gamma \cos \gamma Z \right] d\gamma$$

and

$$\sigma_{rz} = \frac{2\beta r_0}{\pi a^2} \int_0^\infty \left[\sum_{n=1}^{\infty} A_n \alpha_n h(\gamma)\, J_1(\alpha_n r) \frac{1}{(\mu_1^2 \alpha_n^2 + \gamma^2)\,(\mu_3^2 \alpha_n^2 + \gamma^2)} \right.$$

$$\left. + \sum_{n=1}^{\infty} \left\{ B_n(\gamma) \frac{1}{\mu_1^3} f(\mu_1)\, I_1 \left(\frac{\gamma r}{\mu_1} \right) + C_n(\gamma) \frac{1}{\mu_3^3} f(\mu_3)\, I_1 \left(\frac{\gamma r}{\mu_3} \right) \right\} \right]$$

$$\gamma \sin \gamma z d\gamma, \quad \ldots(18)$$

where

$$\left. \begin{aligned} & h(\gamma) = d\alpha_n^2 + e\gamma^2,\ k(\gamma) = \alpha_n + a_0 \gamma^2 \\ & f(u_i) = d - e\mu_i^2,\ g(\mu_i) = 1 - a_0 \mu_i^2,\ i = 1, 3. \end{aligned} \right\} \quad \ldots(19)$$

The boundary conditions for the present problem are

$$\sigma_{rz} = 0 \text{ and } \sigma_{rr} = 0 \text{ for } r = a \quad \ldots(20)$$

which represent the requirement that the surface of the cylinder would be free of tractions. Now it follows from the first and fourth of (18) and the boundary conditions (20) that

$$B_n(\gamma)\, F_1 \left(\frac{\gamma a}{\mu_1} \right) + C_n(\gamma)\, F_2 \left(\frac{\gamma a}{\mu_3} \right) = \frac{-A_n q \frac{1}{a} k(\gamma)\, \alpha_n J_1(\alpha_n a)}{(\mu_1^2 \alpha_n^2 + \gamma^2)\,(\mu_3^2 \alpha_n^2 + \gamma^2)} \quad \ldots(21)$$

and

$$B_n(\gamma)\, F_3\left(\frac{\gamma a}{\mu_1}\right) + C_n(\gamma)\, F_4\left(\frac{\gamma a}{\mu_3}\right) = \frac{-\, A_n h(\gamma)\, \alpha_n J_1(\alpha_n a)}{(\mu_1^2\alpha_n^2 + \gamma^2)\,(\mu_3^2\alpha_n^2 + \gamma^2)}, \qquad \ldots(22)$$

where

$$F_1\left(\frac{\gamma r}{\mu_1}\right) = \gamma\, \frac{1}{\mu_1^2} f(\mu_1)\, I_0\left(\frac{\gamma r}{\mu_1}\right) + q\, \frac{1}{r}\, \frac{1}{\mu_1^3}\, g(\mu_1)\, I_1\left(\frac{\gamma r}{\mu_1}\right),$$

$$F_2\left(\frac{\gamma r}{\mu_3}\right) = \gamma\, \frac{1}{\mu_3^2} f(\mu_3)\, I_0\left(\frac{\gamma r}{\mu_3}\right) + q\, \frac{1}{r}\, \frac{1}{\mu_3^3}\, g(\mu_3)\, I_1\left(\frac{\gamma r}{\mu_3}\right),$$

$$F_3\left(\frac{\gamma r}{\mu_1}\right) = \frac{1}{\mu_1^3} f(\mu_1)\, I_1\left(\frac{\gamma r}{\mu_1}\right)$$

and

$$F_4\left(\frac{\gamma r}{\mu_3}\right) = \frac{1}{\mu_3^3} f(\mu_3)\, I_1\left(\frac{\gamma r}{\mu_3}\right). \qquad \ldots(23)$$

From (21) and (22) we have

$$\left.\begin{aligned} B_n(\gamma) &= \frac{A_n \alpha_n J_1(\alpha_n a)}{(\mu_1^2\alpha_n^2 + \gamma^2)\,(\mu_3^2\alpha_n^2 + \gamma^2)} \cdot \frac{G_2\left(\frac{\gamma a}{\mu_3}\right)}{D(\gamma)} \\ \text{and} \qquad & \qquad \ldots(24) \\ C_n(\gamma) &= -\,\frac{A_n \alpha_n J_1(\alpha_n a)}{(\mu_1^2\alpha_n^2 + \gamma^2)\,(\mu_3^2\alpha_n^2 + \gamma^2)} \cdot \frac{G_1\left(\frac{\gamma a}{\mu_1}\right)}{D(\gamma)} \end{aligned}\right\}$$

where

$$\left.\begin{aligned} D(\gamma) &= F_1\left(\frac{\gamma a}{\mu_1}\right) F_4\left(\frac{\gamma a}{\mu_3}\right) - F_2\left(\frac{\gamma a}{\mu_3}\right) F_3\left(\frac{\gamma a}{\mu_1}\right), \\ G_1\left(\frac{\gamma a}{\mu_1}\right) &= h(\gamma)\, F_1\left(\frac{\gamma a}{\mu_1}\right) - q\, \frac{1}{a}\, k(\gamma)\, F_3\left(\frac{\gamma a}{\mu_3}\right) \\ \text{and} \qquad & \qquad \ldots(24) \\ G_2\left(\frac{\gamma a}{\mu_3}\right) &= h(\gamma)\, F_2\left(\frac{\gamma a}{\mu_3}\right) - q\, \frac{1}{a}\, k(\gamma)\, F_4\left(\frac{\gamma a}{\mu_1}\right). \end{aligned}\right\}$$

Thus the arbitrary constants $B_n(\gamma)$ and $C_n(\gamma)$ are known and consequently $\psi(r, Z)$ is known. It follows immediately that the stresses given by (18) are now fully determined and these can be further simplified by using the representations[1]

$$\int_0^\infty \frac{\cos \gamma Z}{\mu_i^2\alpha_n^2 + \gamma^2}\, d\gamma = \frac{\pi}{2}\, \mu_i \exp\left(-\, \mu_i \alpha_n Z\right)$$

$$\frac{2}{a} \sum_{n=1}^{\infty} \frac{\alpha_n}{\alpha_n^2 + \frac{\gamma^2}{\mu_i^2}}\, \frac{J_0(\alpha_n r_0)}{J_1(\alpha_n a)} = \frac{I_0\left(\frac{\gamma r_0}{\mu_i}\right)}{I_0\left(\frac{\gamma a}{\mu_i}\right)}, \; i = 1, 3.$$

We obtain finally the stress components as

$$\sigma_{rr} = -\frac{\beta r_0}{a^2}\sum_{n=1}^{\infty}\frac{J_0(\alpha_n r_0)}{J_1^2(\alpha_n a)}\left[\sum_{j=1,3} C_{jj}\alpha_n\left\{\mu_j^2 f(\mu_j) J_0(\alpha_n r) + qg(\mu_j)\frac{J_1(\alpha_n r)}{\alpha_n r}\right\}\exp(-\mu_j\alpha_n Z)\right]$$

$$-\frac{\beta r_0}{\pi a}\int_0^{\infty}\frac{E(\gamma)}{D(\gamma)}\cos\gamma Z\left[G_2\left(\frac{\gamma a}{\mu_3}\right)F_1\left(\frac{\gamma r}{\mu_1}\right) - G_1\left(\frac{\gamma a}{\mu_1}\right)F_2\left(\frac{\gamma r}{\mu_3}\right)\right]d\gamma,$$

$$\sigma_{\varphi\varphi} = -\frac{\beta r_0}{a^2}\sum_{n=1}^{\infty}\frac{J_0(\alpha_n r_0)}{J_1^2(\alpha_n a)}\left[\sum_{j=1,3} C_{jj}\alpha_n\left\{\mu_j^2 f(\mu_j) J_0(\alpha_n r) + qg(\mu_j)\left(J_0(\alpha_n r) - \frac{J_1(\alpha_n r)}{\alpha_n r}\right)\right\}\exp(-\mu_j\alpha_n Z)\right]$$

$$-\frac{\beta r_0}{\pi a}\int_0^{\infty}\frac{E(\gamma)}{D(\gamma)}\gamma\cos\gamma Z\left[G_2\left(\frac{\gamma a}{\mu_3}\right)F_5\left(\frac{\gamma r}{\mu_1}\right) - G_1\left(\frac{\gamma a}{\mu_1}\right)F_6\left(\frac{\gamma r}{\mu_3}\right)\right]d\gamma,$$

$$\sigma_{zz} = \frac{\beta r_0}{a^2}\sum_{n=1}^{\infty}\frac{J_0(\alpha_n r_0)}{J_1^2(\alpha_n a)}\left[\sum_{j=1,3} C_{jj}\alpha_n f(u_j) J_0(\alpha_n r)\exp(-\mu_j\alpha_n Z)\right]$$

$$+\frac{\beta r_0}{\pi a}\int_0^{\infty}\frac{E(\gamma)}{D(\gamma)}\gamma\cos\gamma Z\left[G_2\left(\frac{\gamma a}{\mu_3}\right)F_7\left(\frac{\gamma r}{\mu_1}\right) - G_1\left(\frac{\gamma a}{\mu_1}\right)F_8\left(\frac{\gamma r}{\mu_3}\right)\right]d\gamma$$

and

$$\sigma_{rz} = -\frac{\beta r_0}{a^2}\sum_{n=1}^{\infty}\frac{J_0(\alpha_n r_0)}{J_1^2(\alpha_n a)}\left[\sum_{j=1,3} C_{jj}\alpha_n\mu_j f(\mu_j) J_1(\alpha_n r)\exp(-\mu_j\alpha_n Z)\right]$$

$$+\frac{\beta r_0}{\pi a}\int_0^{\infty}\frac{E(\gamma)}{D(\gamma)}\gamma\sin\gamma Z\left[G_2\left(\frac{\gamma a}{\mu_3}\right)F_3\left(\frac{\gamma r}{\mu_1}\right) - G_1\left(\frac{\gamma a}{\mu_1}\right)F_4\left(\frac{\gamma r}{\mu_3}\right)\right]d\gamma, \quad \ldots(25)$$

where

$$C_{jj} = \frac{-1}{\mu_j(\mu_j - \mu_k)},\ j, k = 1, 3 \text{ and } j \neq k,$$

$$E(\gamma) = \frac{1}{\gamma^2(\mu_3^2 - \mu_1^2)}\left[\frac{I_0\left(\frac{\gamma r_0}{\mu_1}\right)}{I_0\left(\frac{\gamma a}{\mu_1}\right)} - \frac{I_0\left(\frac{\gamma r_0}{\mu_3}\right)}{I_0\left(\frac{\gamma a}{\mu_3}\right)}\right],$$

$$F_5\left(\frac{\gamma r}{\mu_1}\right) = \frac{1}{\mu_1^2} f(\mu_1)\, I_0\left(\frac{\gamma r}{\mu_1}\right) + q\,\frac{1}{\mu_1^4}\, g(\mu_1)\left(I_0\left(\frac{\gamma r}{\mu_1}\right) - \frac{I_1\left(\frac{\gamma r}{\mu_1}\right)}{\frac{\gamma r}{\mu_1}}\right),$$

$$F_6\left(\frac{\gamma r}{\mu_3}\right) = \frac{1}{\mu_3^2} f(\mu_3)\, I_0\left(\frac{\gamma r}{\mu_3}\right) + q\,\frac{1}{\mu_3^4}\, g(\mu_3)\left(I_0\left(\frac{\gamma r}{\mu_3}\right) - \frac{I_0\left(\frac{\gamma r}{\mu_3}\right)}{\frac{\gamma r}{\mu_3}}\right),$$

$$F_7\left(\frac{\gamma r}{\mu_1}\right) = \frac{1}{\mu_4} f(\mu_1)\, I_0\left(\frac{\gamma r}{\mu_1}\right)$$

and

$$F_8\left(\frac{\gamma r}{\mu_3}\right) = \frac{1}{\mu_3^4} f(\mu_3)\, I_0\left(\frac{\gamma r}{\mu_3}\right). \qquad \ldots(26)$$

It can be shown as in Sokolowski[1] that the above integrals involved in the expressions for the stresses are convergent for all values of $r \neq r_0$, $Z \neq 0$. The formula (25) for the stresses in the infinite cylinder is a generalization of the results of Sokolowski.[1]

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