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# MHD BOUNDARY LAYER GROWTH IN A ROTATING LIQUID WITH SUSPENDED PARTICLES

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An analysis of the effects of suspended particles on the flow of electrically conducting fluid past an impulsively started infinite plate in a rotating fluid is presented, on taking into account the transverse magnetic field. Exact solutions are derived to the transverse and axial velocity field by Laplace transform technique. It is observed that the axial component of the skin friction increases due to increasing $f$ (mass concentration of the dust particles), $\omega$ (rotation parameter) or $M$ (the magnetic field parameter). But the transverse component of skin-friction decreases with increasing $f$ but increases with increasing $\omega$ or $M$.

**Key Words : MHD Boundary Layer ; Rotating Fluid ; Particles (Suspended); Laplace Transform ; Skinfriction ; Dusty Gas Motion**

## INTRODUCTION

MOTION of a dusty gas in terms of a large number density $N(x, t)$ of very small particles was first studied by Saffman.[1] It was assumed that the bulk concentration of the dust particles is small, but the density of the dust material is taken large compared with the fluid density, so that mass concentration of the dust is an appreciable fraction of unity. Michael and Miller[2] studied the flow of dusty gas past an impulsively started plate and an oscillating plate. The flow of dusty gas past an infinite rotating disc was studied by Zung.[3] Liu[4,5] also studied Michael and Miller's problem independently. The dynamics of dusty gas has been receiving the attention of many workers as it has good applications in pollution problems. The study of the effects of transversely applied magnetic field magnetic field on the flow of electrically conducting gases containing suspended particles is also very important from practical point of view as MHD generators are now being considered as alternative sources of energy. MHD boundary layer phenomenon in dusty gases needs extensive study. Hence, it is now proposed to present the analysis of the effects of transversely applied magnetic field on the boundary layer growth in a rotating liquid with suspended particles. In the absence of magnetic field, such a study has been presented by Gupta and Pop.[6]

## Mathematical Analysis

Let $z = 0$ be the position of an infinite plate lying in an electrically conducting viscous fluid containing dust particles having a small bulk concentration. The plate and the fluid are assumed to be rotating as a solid body rotation with angular velocity $\Omega$ about $z$-axis. At $t = 0$, the plate is given an impulsive motion and thus the plate starts moving with a uniform velocity $U$ in its own plane in the rotating frame of reference. The equation of conservation of the dust particles is given by

$$\frac{\partial N}{\partial t} + \nabla \cdot (N \cdot \bar{Q}) = 0 \qquad \ldots(1)$$

where $\bar{Q}$ $(u', v', w')$ is the velocity of the dust particles. Neglecting induced magnetic field and following Saffman.[1], we can show that the flow of an electrically conducting viscous incompressible fluid is governed by the following equations :

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2} + 2\Omega v + \frac{KN_0}{\rho}(u' - u) - \frac{\sigma B_0^2}{\rho} u \qquad \ldots(2)$$

$$\frac{\partial v}{\partial t} = \nu \frac{\partial^2 v}{\partial y^2} - 2\Omega u + \frac{KN_0}{\rho}(v' - v) - \frac{\sigma B_0^2}{\rho} v \qquad \ldots(3)$$

Here $\sigma$ is the scalar electrical conductivity of the fluid, $B_0$ is the transverse magnetic field component parallel to the $z$-axis, $\rho$ the density, $\nu$ the kinematic viscosity, $x$, $y$, rectangular coordinate system perpendicular to $z$-axis, and $K$ is the Stokes resistance coefficient. The momentum equations of dust particles are

$$\frac{\partial u'}{\partial t} = \frac{K}{m}(u - u') + 2\Omega v' \qquad \ldots(4)$$

and

$$\frac{\partial v'}{\partial t} = \frac{K}{m}(v - v') - 2\Omega u'. \qquad \ldots(5)$$

Here $m$ is the mass of a dust particle.

On introducting the following non-dimensional quantities

$$q' = \frac{u' + iv'}{U}, \quad q = \frac{u + iv}{U}, \quad T = \frac{t}{\tau}, \quad y = \frac{z}{\sqrt{\nu\tau}},$$

$$\tau = \frac{m}{K}, \quad \Omega\tau = \omega,$$

equations (2) to (5) reduce to the following

$$\frac{\partial q}{\partial T} = \frac{\partial^2 q}{\partial y^2} - (2i\omega + f)\, q + fq' - Mq \qquad \ldots(6)$$

and

$$\frac{\partial q'}{\partial T} = q - (2i\omega + 1)\, q', \qquad \ldots(7)$$

where $f = N_0 m/\rho$ is the mass concentration of the dust and is known as the relaxation time of the dust.

The initial and boundary conditions are

$$q = q' = 0, \quad T<0 \qquad \ldots(8)$$

$$q = 1, \text{ at } y = 0; \quad q = 0 \text{ as } y \to \infty, \; T \geqslant 0$$

Equations (6) and (7) under the initial and boundary conditions (8) are solved by usual Laplace transform technique and the solutions are as follows for large values of time $T$ :

$$q(y, T) = \tfrac{1}{2}\left[\exp\left\{y\left(M + 2i\omega + \frac{2i\omega f}{1 + 2i\omega}\right)^{1/2}\right\}\right.$$

$$\times \operatorname{erfc}\left\{\frac{y}{2\sqrt{t}} + \left(M + 2i\omega + \frac{2i\omega f}{1 + 2i\omega}\right)^{1/2}\sqrt{T}\right\}$$

$$+ \exp\left\{-y\left(M + 2i\omega + \frac{2i\omega f}{1 + 2i\omega}\right)^{1/2}\right\}$$

$$\left.\times \operatorname{erfc}\left\{\frac{y}{2\sqrt{T}} - \left(M + 2i\omega + \frac{2i\omega f}{1 + 2i\omega}\right)^{1/2}\sqrt{T}\right\}\right] \qquad \ldots(9)$$

Substituting for $q$ in (7) and solving by the usual Laplace-transform technique, we get

$$q' = \frac{1}{1 + 2i\omega}\left[q - \tfrac{1}{2}\exp\left(-(1 + 2i\omega)T\right)\right.$$

$$\times\left\{\exp\left(-y\sqrt{1 + M + \frac{2i\omega f}{1 + 2i\omega}}\right)\right.$$

$$\times \operatorname{erfc}\left(\frac{y}{2\sqrt{T}} - \sqrt{1 + M + \frac{2i\omega f}{1 + 2i\omega}}\, T^{1/2}\right)$$

$$+ \exp\left(y\sqrt{1 + M + \frac{2i\omega f}{1 + 2i\omega}}\right)$$

$$\left.\left.\times \operatorname{erfc}\left(\frac{y}{2\sqrt{T}} + \sqrt{1 + M + \frac{2i\omega f}{1 + 2i\omega}}\, T^{1/2}\right)\right\}\right] \qquad \ldots(10)$$

Equation (10) can be written in the asymptotic form as follows :

$$q = \exp\left(-y(a_1 + ib_1)\right) - \frac{\left[y\left[\left(a_1^2 - b_1^2\right)T - \dfrac{\xi^2}{4T} - 2ia_1b_1T\right]\right.}{2\sqrt{\pi T}\left[\left\{\left(a_1^2 - b_1^2\right)T - \dfrac{\xi^2}{4T}\right\}^2 + 4a_1^2b_1^2T^2\right]}$$

$$\left.\exp\left[-\left\{\left(a_1^2 - b_1^2\right)T + \frac{y^2}{4T} + 2ia_1b_1T\right\}\right]\right], \qquad \ldots(11)$$

where

$$a_1 = \left[M + \frac{4\omega^2 f}{1 + 4\omega^2} + \left\{\left(M + \frac{4\omega^2 f}{1 + 4\omega^2}\right)^2\right.\right.$$

$$\left.\left. + \left(2\omega + \frac{2\,\omega f}{1 + 4\omega^2}\right)^2\right\}^{1/2}\right]^{1/2}$$

and $$b_1 = \left[M + \frac{4\omega^2 f}{1+4\omega^2} - \left\{\left(M + \frac{4\omega^2 f}{1+4\omega^2}\right)^2 + \left(2\omega + \frac{2\omega f}{1+4\omega^2}\right)^2\right\}^{1/2}\right]^{1/2} \quad ...(11a)$$

In writing (11), it is noted that

$$\text{erfc } (z) \sim \frac{\exp(-z^2)}{z\sqrt{\pi}} \text{ as } |z| \to \infty \quad ...(12)$$

and $$\text{erfc } (-z) = 2 - \text{erfc } (z)$$

We now separate (11) into its real and imaginary parts as follows :

$$\frac{u}{U} = \exp(-a_1 y) \cos b_1 y$$

$$- \frac{\left[\left\{\left(a_1^2 - b_1^2\right) T - \frac{y^2}{4T}\right\} \cos(2a_1 b_1 T) - 2a_1 b_1 T \sin(2a_1 b_1 T)\right]}{2\sqrt{\pi T}\left[\left\{\left(a_1^2 - b_1^2\right) T - \frac{y^2}{4T}\right\}^2 + 4a_1^2 b_1^2 T^2\right]}$$

$$\times \exp\left[-\left\{\left(a_1^2 - b_1^2\right) T + \frac{y^2}{4T}\right\}\right] \quad ...(13)$$

$$\frac{v}{U} = -\exp(-a_1 y) \sin b_1 y$$

$$- \frac{y\left[\left\{\frac{y^2}{4T} - (a_1^2 - b_1^2) T\right\} \sin(2a_1\beta_1 T) - 2a_1\beta_1 T \cos(2\alpha_1\beta_1 T)\right]}{2\sqrt{\pi T}\left[\left\{(a_1^2 - b_1^2) T - \frac{y^2}{4T}\right\}^2 + 4a_1^2 b_1^2 T^2\right]}$$

$$\times \exp\left[-\left\{(a_1^2 - b_1^2) T + \frac{y^2}{4T}\right\}\right] \quad ...(14)$$

Equations (13) and (14) are the expressions for the velocity components in the Ekman boundary layer on the plate. The boundary layer thickness is proportional to $1/a_1$ and from (11*a*), we conclude that it decreases with increasing $M$, the Hartmann number and $f$, the mass concentration of the dust particles. But the second terms in (13) and (14) represent the inertial oscillations of the liquid with small amplitude. The frequency of these oscillations is $2a_1 b_1 = 2\omega\left(1 + \frac{f}{1+2\omega}\right)$ and this leads us to conclude that it is not affected by the magnetic field. For constant $\omega$, the frequency of the oscillation is found to increase with increasing $f$.

Knowing the velocity field, it is interesting to study the effects of rotation and the dust particles on the shearing stress. It is given by

$$-\left.\frac{\partial q}{\partial y}\right|_{y=0} = \frac{1}{\sqrt{\pi T}} \exp\left(-M + 2i\omega + \frac{2i\omega f}{1+2i\omega}\right) + \left[M + 2i\omega + \frac{2i\omega f}{1+2i\omega}\right]^{1/2} \times \operatorname{erf}\left[\left(2i\omega + M + \frac{2i\omega f}{1+2i\omega}\right)^{1/2} T\right] \quad \ldots(17)$$

Separating the real and imaginary parts, we have calculated the values of $\tau_u$ and $\tau_v$ and they are entered in Table I.

TABLE I

| | | | Values of | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | $\tau_u$ | | | $\tau_v$ | | |
| $M$ | $T$ | | 0.5 | 1.0 | 1.5 | 0.5 | 1 0 | 1.5 |
| 0 | 2.0 | 0.2 | 0.7710 | 0.8775 | 0.9474 | 0.4480 | 0.4413 | 0.4396 |
| 0 | 4.0 | 0.2 | — | 0.8484 | — | — | 0.3883 | — |
| 0 | 2.0 | 0.4 | — | 1.0289 | — | — | 0.4634 | — |
| 1.0 | 2.0 | 0.2 | 1.1815 | 1.2110 | 1.2395 | 0.1969 | 0.2492 | 0.3003 |
| 1.0 | 4.0 | 0.2 | — | 1.1840 | — | — | 0.2719 | — |
| 1.0 | 2.0 | 0.4 | — | 1.3100 | — | — | 0.4076 | — |
| 4.0 | 2.0 | 0.2 | 2.0279 | 2.0470 | 2.0667 | 0.1378 | 0.1780 | 0.2175 |
| 4.0 | 4.0 | 0.2 | — | 2.0457 | — | — | 0.1792 | — |
| 4.0 | 2.0 | 0.4 | — | 2.1186 | — | — | 0.2993 | — |

We conclude from this table that in the presence or absence of the magnetic field, there is a rise in the value of $\tau_u$ and a fall in the value of $\tau_v$ due to increasing $f$. An increase in $M$ or $\omega$ leads to a decrease in the value of $\tau_u$ and $\tau_v$.

## CONCLUSIONS

1. Due to the application of the magnetic field, the velocity oscillations are damped out.
2. An increase in $f$, $M$ or $\omega$ leads to an increase in the value of $\tau_u$. But $\tau_v$ is found to decrease with increasing $f$ and it increases with increasing $M$ or $\omega$.

## REFERENCES

1. P G Saffman *J Fluid Mech* **13** (1962) 120
2. D H Michael and D A Miller *Mathematika* **13** (1966) 97
3. L B Zung *Phys Fluid* **12** (1969) 18
4. J T C Liu *Phys Fluid* **9** (1966) 1716
5. J T C Liu *Astronautica Acta* **13** (1967) 369
6. A S Gupta and I Pop *Bull Math* **19** (1975) (67) 291