PROTON OPTICAL POTENTIAL FOR 
\(^{118^{120^{122}}}{Sn}\) ISOTOPES AT 6.5 MeV

N Z DARWISH and F M EL-ASHRY

Department of Physics, Faculty of Science, Tanta University, Tanta, Egypt

(Received 28 October 1986; Accepted 21 May 1987)

Within the framework of optical model, an analysis has been carried out for the angular dependences of the elastic scattering cross-sections of protons from nuclei \(^{118^{120^{122}}}{Sn}\) for \(E_p = 6.5\text{MeV}\). The potentials obtained on the basis of \((p, n)\) reaction cross-sections explain the elastic scattering data as well.

Key Words: Proton Optical Potential; Isotopes; \(^{118^{120^{122}}}{Sn}\); Elastic Scattering

INTRODUCTION

The proton optical model potential has been very successful for interpreting the average properties of proton interaction with nuclei. It has been used especially for protons incident on nuclei at energies above the Coulomb barrier.

Johnson et al.\(^1\) analyzed the data on \((p n)\) reaction cross-sections for nuclei in the region \(89 \leq A \leq 130\) and \(2.5 \leq E_p \leq 7.0\text{MeV}\). This region of proton energies is a sub-barrier region. The same group determined the optical potential parameters for the region \(117 \leq A \leq 124\).\(^2\)

In the present work, the suitability to the optical potential parameters obtained\(^1\)\(^2\) for describing cross-sections for protons by nuclei \(^{118^{120^{122}}}{Sn}\) at \(E_p = 6.5\) MeV is checked.

CALCULATIONS

The optical model used here is the usual form for low energies and is a sum of Wood-Saxon real, surface absorptive, spin-orbit, and Coulomb potentials.\(^3\)

\[
V(r) = -V_R f(r, R_R, a_R) + i4a_D W_D f(r, R_D, a_D) + V_{SO} \frac{\sigma l}{r} \left(\frac{\hbar}{m_e c}\right)^2 
\times \frac{d}{dr} f(r, R_{SO}, a_{SO}) + V_C(R_c),
\]

where

\[
f(r, R, a) = (1 + \exp [-r - R/a])^{-1}
\]

\[
R_x = r_x A^{1/3}
\]

The strength of the real local potential is energy-dependent;

\[
V_R(E) = V_R(0) - b_0 E.
\]
The Coulomb potential is known. A uniformly charged sphere of charge $Z e$ and radius $R_C$ is valid\textsuperscript{3} providing $R_C$ gives the same rms radius as the actual charge distribution. The charge radii for the Sn isotopes are known.\textsuperscript{41}

In comparing the experimental data on scattering of protons by $^{118-120-122}$Sn nuclei with calculations based on the optical model (OM), we used the parameters obtained,\textsuperscript{2} which are displayed in Table I.

**Table I**

*Values of optical potential parameters*\textsuperscript{3}

<table>
<thead>
<tr>
<th>$A$</th>
<th>$V_R(o)$, MeV</th>
<th>$W_D$, MeV</th>
<th>$a_D$, fm</th>
</tr>
</thead>
<tbody>
<tr>
<td>118</td>
<td>63.49</td>
<td>11.71</td>
<td>0.391</td>
</tr>
<tr>
<td>120</td>
<td>63.51</td>
<td>11.49</td>
<td>0.402</td>
</tr>
<tr>
<td>122</td>
<td>63.53</td>
<td>11.27</td>
<td>0.413</td>
</tr>
</tbody>
</table>

In all our calculations,

\[
\begin{align*}
V_{S,o} &= 6.4 & \quad r_{S,o} &= 1.03 & \quad a_{S,o} &= 0.63 \\
A_r &= 0.73 & \quad r_D &= 1.3 & \quad b_o &= 0.32 & \quad r_D &= 0.4
\end{align*}
\]

where $V_{S0}$ is in MeV and $r_x$ and $a_x$ are in (fm).

For all nuclei the parameter $W_D$ was varied to obtain the minimum value of

\[
\chi^2 = \frac{1}{N} \sum_{1}^{N} \left( \frac{\sigma_{\text{exp}} - \sigma_{\text{th}}}{\sigma_{\text{error}}} \right)^2
\]

where $N$ the number of data, $\sigma_{\text{exp}}$ and $\sigma_{\text{th}}$ are the experimental and theoretical cross-sections, respectively, and $\sigma_{\text{error}}$ is the experimental uncertainty.

In these calculations the values of $W_D$ from Table I were also employed. The expression for $V_R(O)$ obtained\textsuperscript{1} differs some what from the analogous one in the Becchetti-Greenlees Scheme\textsuperscript{3} and is given by

\[
\begin{align*}
V_R(0) &= V_0 + 24 \frac{(N - Z)}{A} + 0.40 \frac{Z}{A^{1/3}} \\
V_0 &= 55.4 \text{MeV}
\end{align*}
\]

All the geometrical parameters\textsuperscript{1} (except for $a_D$ are) close to the parameters.\textsuperscript{3}

In order to check the possibility of employing the standard set of Becchetti-Greenlees parameters, we also compared the experimental data on $\sigma_{(o)}$ and $\sigma_R$ with calculations using these parameters. Within the framework of this parameterization, all the parameters except for $W_D$ were fixed in the calculations.

The experimental data of elastic scaterring cross-sections of protons by nuclei of $^{118-120-122}$Sn at $E_p = 6.5 \text{MeV}$ were taken.\textsuperscript{5} In determining $\chi^2/N$ it is assumed that the absolute error is $\pm 2$ per cent. The optimal $W_D$ values in the present study, obtained from the analysis of data on elastic proton scattering are tabulated in Table II at $E_p = 6.5 \text{MeV}$. 
Table II

<table>
<thead>
<tr>
<th>Optical potential parameters</th>
<th>$\chi^2/N$</th>
<th>$\sigma_{r}(mb)$</th>
<th>$\sigma_{ps}(mb)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{118}$Sn Table I,</td>
<td>0.4</td>
<td>119</td>
<td>$110 \pm 1$</td>
</tr>
<tr>
<td>$^{118}$Sn formula</td>
<td>1.0</td>
<td>121</td>
<td>$124 \pm 1$</td>
</tr>
<tr>
<td>$^{118}$Sn (5)</td>
<td>0.6</td>
<td>123</td>
<td>$123 \pm 1$</td>
</tr>
</tbody>
</table>

Fig. 1 shows a comparison between the data regarding scattering of $F = 6.5$ MeV protons by $^{118}$Sn nucleus and calculations based on the (OM) with parameters using formula 5 and Table I (curve 2), here $\chi^2/N = 0.4$.

---

Fig. 1 Angular distribution of elastic scattering of protons by $^{118}$Sn nucleus, $E = 6.5$MeV. The curves represent calculations on the basis of the optical model: 1-with parameters and $W_D = 2$MeV, 2-parameters and Table I, 3-parameters and $W_D = 9$MeV.
Calculation using the Bechetti-Greenlees parameters\(^3\) where \(W_D\) is specified by the formula

\[ W_D = 11.8 - 0.25 E + 12 \frac{(N-Z)}{A} \]

Yield \(\sigma_R = 150\) mb, a value that is much too high. If we leave all parameters of [3] except for \(W_D\) unchanged, then for \(W_D = 2\) MeV we have \(\sigma_R = \sigma_{pn}\) but the calculated value of \(\sigma(\theta)\) does not agree with the experimental one (curve 1). Satisfactory agreement between theory and experiment is obtained for \(W_D = 9.0\) MeV (curve 3). However, for the value \(W_D = 9.0\) MeV \(\sigma_R\) does not agree with the experimental value (\(\sigma_{pn} = 110\) mb).

Fig. 2 shows a comparison of \(\sigma(\theta)\) data for \(^{120}\text{Sn}\) for \(E = 6.5\) MeV and calculations based on the (OM) using the parameters of Table I and formula (5). The curve represents calculation with \(W_D = 11.49\) MeV,\(^2\) here \(\sigma_R = 121\) mb (\(\sigma_{pn} = 124\) mb) (curve 1). The calculation curve\(^3\) based on the OM is in good agreement with the experimental (data \(\chi^2/N = 0.2\)) for \(W_D = 9.0\) MeV Fig. 2 (curve 2). However, \(\sigma_R = 145\) mb higher than the experimental value \(\sigma_{pn} = 124\) mb. Fig. 3. shows a comparison of the \(\sigma(\theta)\) data for \(^{122}\text{Sn}\) at \(E = 6.5\) MeV.

---

**Fig 2** Angular distribution of elastic scattering of protons by \(^{120}\)Sn nucleus; \(E = 6.5\) MeV. The curve 1 represents calculation based on the optical model with parameters\(^4\) and Table I, curve 2–parameters\(^5\) and \(W_D = 9\) MeV.
Fig. 3 Angular distribution of elastic scattering of protons by $^{120}$Sn nucleus; $E = 6.5$ MeV. The curve represents calculation based on the optical model with parameters and Table I, curve 2-parameters and $W_D = 9$ MeV.

CONCLUSION

From the present study for the analysis of $\sigma(\theta)$ data the following conclusions can be drawn.

1. Within the limits of the experimental error, the optical potential parameters obtained on the basis of data regarding $(p, n)$ reaction cross-sections are in good agreement with the available experimental data of the elastic scattering cross-sections of protons by these nuclei.

2. By employing the optical parameters obtained by Becchetti and Greenlees we can obtain good agreement with experiment by varying only the parameter $W_D$. At the same time, the calculated cross-section $\sigma_R$ is larger by roughly 20 per cent than the experimental $\sigma_{p,n}$ values of the reactions.

REFERENCES