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UNIVERSAL CROSS SECTIONS FOR *K*-SHELL IONIZATION BY LOW ENERGY PROTONS AND DEUTERONS

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Recent data for *K*-shell ionization of $_{60}$Nd, $_{62}$Sm, $_{64}$Gd, $_{65}$Tb, $_{69}$Tm and $_{73}$Ta by 0.8-3.8MeV protons and deuterons are analyzed in terms of Semiclassical Approximation (SCA) and Plane Wave Born Approximation (PWBA) models included the three main corrections: binding, nuclear coulomb repulsion, and relativistic effects. The agreement is rather good with both models. The data are presented as reduced cross sections and compared with a "universal" cross section functions.

Key Words : **Universal Cross Sections; *K*-Shell Ionization; Low Energy Protons Deuterons; Plane Wave Born Approximation; Semiclassical Approximation; Effects Binding; Nuclear Coulomb and Repulsion**

Introduction

The *K*-shell ionization process induced by heavy charged particles has been the subject of intensive studies, both experimental and theoretical, during the last decade. The basic description of coulomb ionization is a first order Bron treatment with the incident projectiles represented either by a plane wave (PWBA) or by particles moving along classical trajectories with well-defined impact parameter (SCA). When the interaction with the target nucleus is ignored, the two formulations are equivalent.[1] Still theoretical results based on the two pictures often deviate considerably due to the differences in the further approximations made in calculation.

To get reasonable agreement with the experimental *K*-shell ionization cross sections, the theories have to consider three types of corrections to a first-order Born treatment.

Firstly, at low projectile velocities, the perturbation of the state of the bound electron by the coulomb interaction with the projectile cannot always be considered small. In the adiabatic limit the transition probabilities become small, not because the perturbation is small, but because it is slowly varying, and an appropriate description should therefore allow the electron wave function to adjust nearly adiabatically to the presence of the projectile. This leads to a change in binding energy. Secondly, deceleration and deflection of projectile by a coulomb repulsion from the target nucleus may cause a significant reduction of ionization probability at low velocities. This effect is the most important at small impact parameters.

Thirdly, relativistic effects on the electron wavefunction may be important, in particular of course for high target atomic numbers, mainly the high momentum tail of the wave function is affected by a relativistic description and since the minimum decreasing projectile velocity, the correction is the most important at low velocities.

As these three effects have their largest influence at low projectile velocities the present work was aimed at obtaining total *K*-shell ionization cross sections for heavy ions impinging on high *Z* targets $_{60}$Nd, $_{62}$Sm, $_{64}$Gd, $_{65}$Tb, $_{69}$Tm, and $_{73}$Ta by 0.8–3.8MeV protons and deuterons.

CALCULATIONS

1. Plane Wave Born Approximation (PWBA)

The recent experimental data for *K*-shell ionization cross sections of $_{60}$Nd, $_{62}$Sm, $_{64}$Gd, $_{65}$Tb, $_{69}$Tm and $_{73}$Ta was taken from references.[2-4]

To display jointly the universal ionization curve for all the *K*-shell ionization cross sections, the (PWBA) model is chosen as a basis on which, following the idea of Basbas *et al.*[5] the reduced (dimensionless) cross section σ_{red} is constructed, as:

$$\sigma_{\text{red}} = \frac{\sigma \exp \epsilon\theta_k}{C\, R\, \sigma_0}.$$

In the formula σ^{exp}-experimental *K*-shell ionization cross section, ϵ, θ_k and σ_0 are standard (PWBA) ionization parameters defined,[6] while *C* and *R* are Coulomb repulsion[7] and electron relativistic correction, respectively. The parameter ϵ includes only the binding correction, because for low *Ep* the polarization effect is negligible. Fig. 1 displays all σ_{red} values versus the dimensionless velocity parameter $\eta_k(\epsilon\theta_k)^2$.[6] The solid line is the universal (PWBA) *K*-shell ionization function, $F_k\,[\eta_k/(\epsilon\theta_k^2)]$ tabulated.[6] This curve reproduces will the general pattern of experimental σ_{red} values.

2. Semi-Classical Approximation (SCA)

In our calculation we used the experimental *K*-shell binding energies from Lederer and Shirly compilation.[8] The model predictions are sensitive to the electron binding energy because it enters in high (—8) power dependence.

The following presentation of the same total volume of data in one figure is inspired also by the idea of a reduced cross section[5] followed by a similar presentation by Bell[9] of a larger range of cross sention for heavy projectile. However, instead of (PWBA) treatment, in the present paper a reduced cross section in the framework of (SCA) model is adopted and based on those above corrections procedure. The general expression for the reduced experimental cross-section is,

$$\sigma_{\text{red}}^{\text{exp}} = \frac{45\, \sigma_{\text{kI}}^{\text{exp}}}{2^{12}\, \text{Ryd}\, z_1^2\, r_k^2\, E_B^{-1}\, F_R},$$

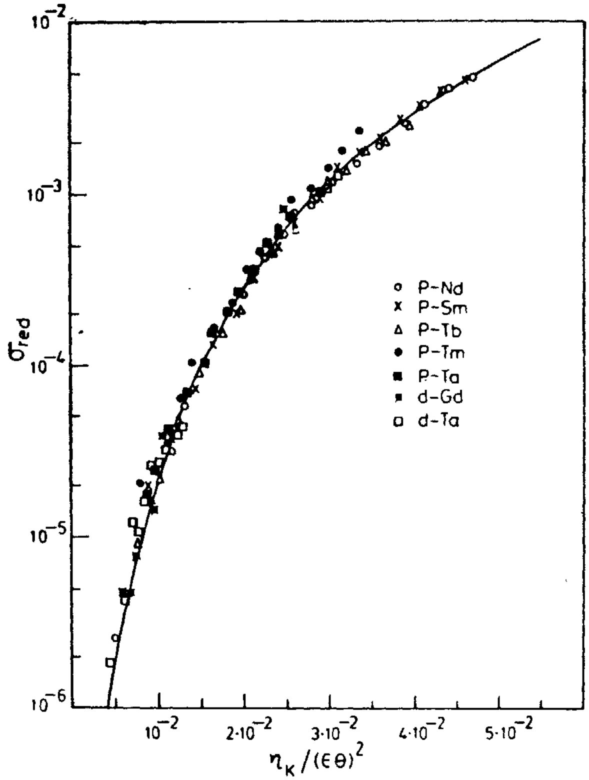

FIG 1.

where Ryd $= 13.6$eV, z_1 is the projectile atomic number, $r_k(= a_0/z_{2k}$; a_0 the Böhr radius and Z_{2k} is the atomic number correct for screening) is a K-shell radius and E_B the K-shell binding energy. The values of z_{2K} and E_B are evaluated in the "United atom" limit to correct for binding effects. The factor F_R is the relativistic correction given by Amundsen *et al.*[10]

The experimental reduced cross sections are shown graphically in Fig. 2, where they are plotted against the dimentionless reduced velocity parameter, which is evaluated with inclusion of Coulomb retardation effects, as given by (SCA) prescription—The full curve is the "Universal" (SCA) theoretical prediction, which derives from the following expression,

$$\sigma_{red}^{theor} = \frac{\xi^8}{(1 + 0.00536\xi + 1.380\xi^2 + 0.219\xi^3)^4}$$

The graphical presentation illustrates the agreement of the present experimental data and the (SCA) model predictions.

In this paper it is very clear that in both approches (PWBA) and (SCA) the agreement is rather good except in the case of Tm especially at relatively high energy. One can see that the authors of these experimental data have used the method

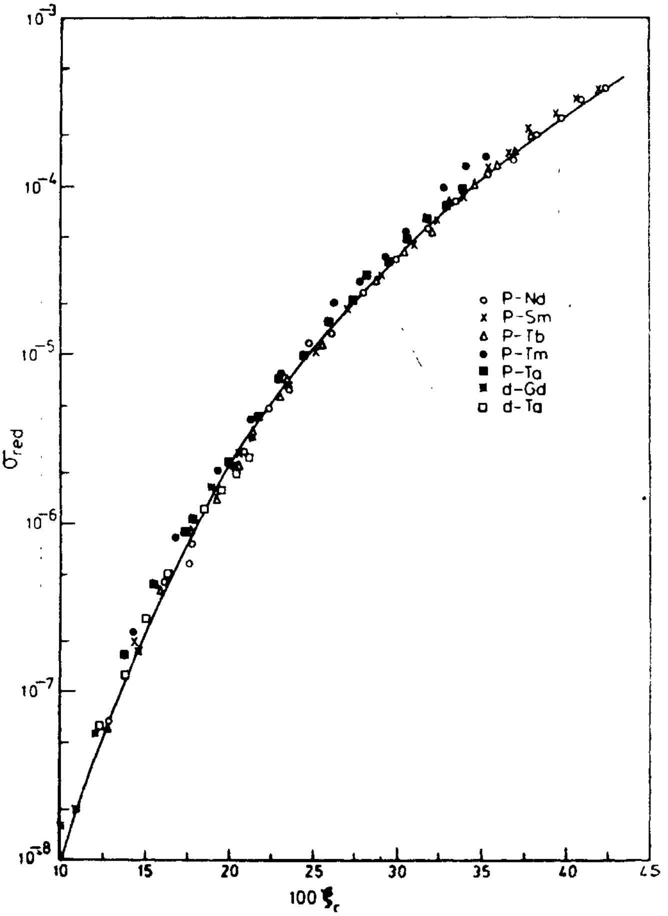

FIG 2.

of Celler *et al.*[11] This method included an intercomparison of two effects, one atomic and the other nuclear which occur in the target under projectile impact. The first is just the desired *K*-shell ionization, and the second is nuclear coulomb excitation (CE) of the target nuclei, the latter being precisely calculable as long as the projectile energy is well under the obtained from the following expression,

$$\sigma_{KI} = \left(\frac{I_{KX}}{I_{\gamma}\,(1+\alpha_T)\,\omega_K}\right)\sigma_{CE},$$

where σ_{CE} is the nuclear CE cross section and I are the measured *KX*-ray and γ-ray intensities. It is evident from expression that a major limitation in the precision, which may be achieved for the ionization cross section, is connected with the accuracy of the reference cross section for the nuclear Coulomb excition. The authors of these experimental data have calculated the CE cross sections from first-order perturbation theory.[12]

First order perturbation procedure is sufficiently accurate because the probability of double and higher order Coulomb excitation by low energy protons is negligibly small. In the case of $_{69}$Tm nuclei it is necessary to use the multiple Coulomb exitation formalism. This follows from the fact that an excited level with a very low excitation energy (8keV) exists in the thulium nucleus and this level becomes excited with high probabilty even by low energy proton and thus double excitions with level as an intermediate state must be consider. Using this approch, one can get better agreement for this nuclei $_{62}$Tm.[13] The results obtained in the present study using (SCA) have been presented in Table I.

TABLE I

Universal ionization cross sections for $_{60}$Nd, $_{62}$Sm; $_{65}$Tb, $_{69}$Tm, *and* $_{73}$Ta, *with* P *projectile*; *for* $_{64}$Gd *and* $_{73}$Ta, *with* d-*projectile*

$E^{eff}_{(MeV)}$	σ^{exp}_{KI} (red)	σ^{SCA}_{KI} (red)
1	2	3
	P on $_{60}$Nd	
0.88	7.624 (−7)	8.475 (−7)
1.10	5.667 (−6)	2.990 (−6)
1.31	6.738 (−6)	7.233 (−6)
1.51	1.353 (−5)	1.401 (−5)
1.72	2.474 (−5)	2.495 (−5)
1.93	3.751 (−5)	4.045 (−5)
2.13	5.786 (−5)	6.016 (−5)
2.33	8.195 (−5)	8.528 (−5)
2.54	1.102 (−4)	1.181 (−4)
2.75	1.447 (−4)	1.579 (−4)
2.95	2.042 (−4)	2.029 (−4)
3.15	2.639 (−4)	2.551 (−4)
3.35	3.340 (−4)	3.148 (−4)
3.55	3.895 (−4)	3.821 (−4)
	P on $_{62}$Sm	
0.74	2.024 (−7)	1.666 (−7)
0.95	9.501 (−7)	8.578 (−7)
1.16	2.580 (−6)	2.659 (−6)
1.37	5.769 (−6)	6.224 (−6)
1.56	1.050 (−5)	1.153 (−5)
1.76	1.859 (−5)	1.981 (−5)
1.97	3.021 (−5)	3.208 (−5)
2.17	4.498 (−5)	4.771 (−5)
2.37	6.462 (−5)	6.768 (−5)
2.57	8.877 (−5)	9.245 (−5)
2.76	1.301 (−4)	1.207 (−4)
2.96	1.957 (−4)	1.559 (−4)
3.16	2.170 (−4)	1.970 (−4)
3.36	2.645 (−4)	2.441 (−4)
3.56	3.340 (−4)	2.976 (−4)
3.77	3.868 (−4)	3.668 (−4)
	P on $_{65}$Tb	
0.73	6.239 (−8)	6.775 (−8)
0.94	4.102 (−7)	4.091 (−7)
1.15	1.448 (−6)	1.391 (−6)
1.36	3.525 (−6)	4.456 (−6)
1.56	7.047 (−6)	6.861 (−6)
1.76	1.134 (−5)	1.209 (−5)
1.96	1.948 (−5)	1.955 (−5)
2.16	2.932 (−5)	2.963 (−5)
2.36	4.190 (−5)	4.271 (−5)
2.56	5.714 (−5)	5.914 (−5)
2.76	8.420 (−5)	7.924 (−5)
2.96	1.043 (−4)	1.033 (−4)
3.16	1.355 (−4)	1.316 (−4)
3.36	1.593 (−4)	1.644 (−4)
3.56	2.015 (−4)	2.019 (−4)
	P on $_{69}$Tm	
0.94	2.289 (−7)	1.614 (−7)
1.14	8.584 (−7)	5.816 (−7)
1.35	2.094 (−6)	1.570 (−6)
1.56	4.218 (−6)	3.410 (−6)
1.76	7.816 (−6)	6.231 (−6)
1.96	1.215 (−5)	1.037 (−5)
2.16	2.122 (−5)	1.608 (−5)
2.36	2.863 (−5)	2.364 (−5)

Continued

Table 1 (*Continued*)

1	2	3
2.56	3.979 (−5)	3.327 (−5)
2.75	5.299 (−5)	4.457 (−5)
2.95	7.742 (−5)	5.589 (−5)
3.15	1.033 (−4)	7.603 (−4)
3.35	1.353 (−4)	9.600 (−4)
5.56	1.543 (−4)	1.204 (−4)
	T on $_{73}$Ta	
1.03	1.649 (−7)	1.216 (−7)
1.19	4.359 (−7)	3.289 (−7)
1.40	1.117 (−6)	9.015 (−7)
1.62	2.436 (−6)	2.051 (−6)
1.83	4.327 (−6)	3.877 (−6)
2.02	7.445 (−6)	6.312 (−6)
2.18	1.026 (−5)	3.055 (−6)
2.38	1.566 (−5)	1.350 (−5)
2.58	2.151 (−5)	1.928 (−5)
2.75	3.091 (−5)	2.533 (−5)
2.95	3.902 (−5)	3.946 (−5)
3.15	5.097 (−5)	4.432 (−5)
3.35	6.793 (−5)	5.656 (−5)
3.56	8.186 (−5)	7.168 (−5)
3.76	9.667 (−5)	8.827 (−5)
	d on $_{64}$Gd	
0.89	1.586 (−8)	1.107 (−8)
1.07	5.678 (−8)	4.329 (−8)
1.14	6.496 (−8)	6.767 (−8)
1.34	1.701 (−7)	1.873 (−7)
1.55	4.540 (−7)	4.197 (−7)
1.75	5.780 (−7)	8.210 (−7)
1.96	1.615 (−6)	1.431 (−6)
2.16	2.154 (−6)	2.308 (−6)
2.36	3.328 (−6)	3.482 (−6)
2.56	4.947 (−6)	5.004 (−6)
2.76	5.995 (−6)	6.968 (−6)
	d on $_{73}$Ta	
1.09	7.703 (−9)	5.056 (−9)
1.30	2.001 (−8)	2.017 (−8)
1.50	6.041 (−8)	5.488 (−8)
1.71	1.240 (−7)	1.265 (−7)
1.92	2.667 (−7)	2.509 (−7)
2.12	4.987 (−7)	4.356 (−7)
2.32	9.203 (−7)	7.025 (−7)
2.53	1.241 (−6)	1.091 (−6)
2.73	1.582 (−6)	1.584 (−6)
2.93	2.124 (−6)	2.217 (−6)
3.13	2.538 (−6)	3.011 (−6)

CONCLUSION

It may be concluded that the agreement between theory and experiment is satisfactory with both (PWBA) and (SCA) in this range of mass number and energy, multiple Coulomb excitation formalism should be included when the energy of 2^+ state is small as in the case of $_{69}$Tm. In general, it is difficult from comparison with experimental data to draw any conclusion about which theory is better.

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