SATYENDRA NATH BOSE MEDAL AWARD
LECTURE–1986
TWO DECADES OF QUARK PHYSICS

A N Mitra FNA

Department of Physics and Astrophysics, University of Delhi, Delhi-110007, India

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INTRODUCTION AND SUMMARY

It is a privilege to be asked to give the Satyendra Nath Bose Lecture of the Indian National Science Academy. I have taken the liberty to choose as the theme of my talk a subject which has grown out of a discovery ranking in importance next only to two others in the entire century, namely Quantum Theory and Relativity, whose successful marriage gave birth to the profound concept of Antimatter, subsequently to be confirmed by experiment. It should perhaps be stressed at the outset that the “discovery” of quarks (if at all they have been discovered;) was never as dramatic as some other major discoveries such as antimatter, parity violation and W-bosons, but the concept has presumably shaken the very foundations of physical reality in a manner no less profound than e.g., what quantum theory itself has done in this century. And this has to do with their apparent “invisibility” by the conventional yardsticks as have helped identify the existence of most other elementary particles in physics. Indeed the very theory of quark interactions has had to be designed in such a way as to make it impossible to observe” them as free particles, in the sense all other elementary particles have been observed. Such a strategy is not as opportunistic as might appear a priori for there exists at least one more example in the history of physics where a firm experimental fact was taken as cornerstone of the theoretical foundations. And this was the celebrated Theory of Relativity whose observed invariance of the velocity of light in the (apparently Galilean) frames of reference had led Einstein to incorporate this fundamental fact in his basic postulates, without further scrutiny. Perhaps the same was also true of the gradual evolution of the quantum theory (again motivated by some crucial observations) though not in an equally sharp time scale. The only obvious lesson to be learnt from such examples is that theoretical foundations are more often than not attuned to observational evidence, so that one must be prepared at any stage to give them up unceremoniously, if need be, in response to the demands of fresh experimental evidence of a contrary nature. In the meantime any sustained “faith” in the existing theory must be continuously tested through as wideranging and discriminating experimental data as may be physically possible and, in today’s experimental world of high energy physics characterized by strong choice of priorities which are economically viable. This is

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as much true of “all time theories” (e.g., GUT, Supergravity and Super Strings) which rely heavily on theoretical self-consistency principles and only marginally on concrete experimental support, as of more mundane “effective” theories which must necessarily stay closer to experiment as a means of testing their predictive power, if only to make up for their lack of comparable mathematical elegance.

The quark picture, by and large has come to be accepted by the physics community, if only because of the external manifestations of two of its major attributes, color and flavour, even without going into the technical details of its underlying theory (QCD) which is designed to incorporate a vital property of confinement as a means of satisfying the “invisibility” requirement. In the first part of this talk I shall try to provide a bird’s eye view of the major developments in the theoretical understanding of quark interactions, and in the process offer arguments for effective theories with a wide applicational base to make contact with diverse data (in preference to “all-time-theories” which are getting increasingly harder to confront with experiment). The second part of the talk will be devoted to a particular type of effective theory based on the so-called Bethe-Salpeter Equation which incorporates confinement through an effective $g-g$ or $g-ar{q}$ interaction kernel. The theory developed by the Delhi Group and continuously refined over the years has a wide applicational base, with a corresponding degree of predictive power from low energy hadron spectroscopy and coupling constants to high energy structure and fragmentation functions. To bring out the predictive capacity of the theory, the result of several illustrative applications will be presented, while relegating the technical details of the needed formalism to recent publications. (Because of the time limitations, the written version for both the papers will contain some more details than can be actually presented). Now to put the role of such effective theories in a “physics” perspective, their vast correlative powers, if successfully contacted with matching data, lend them a uniqi status as effective bridges between the data and more fundamental approaches aimed at a deeper understanding of confinement and to that extent leave them free to pursue the latter objective without the need for an equal degree of concern for the former. Unfortunately, though the literature abounds in data-based models, their limited frameworks often do not permit meaningful physical applications beyond relatively narrow sectors. The Bethe-Salpeter framework to be presented here, hopefully provides a wide enough theoretical sweep to warrant a much more integrated view of many sectors covering a very wide range of energies, and thus qualifies for the bridge concept. The third and last part of this talk will be briefly addressed to the general question of “beyond quarks what?” which has assumed considerable importance in recent years in view of the rapid proliferation of quark and lepton varieties. This is reminiscent of the situation up to the early sixties which saw a corresponding proliferation of meson and baryon “resonances” (excited states) as a pointer to their (eventual) composit structure in terms of quarks. If substructures underlying quarks and leptons do indeed exist, these will presumably open up a fourth stage of elementarity, whose observational signatures may not be easy to disentangle in to-day’s state of the experimental art. As a result, efforts in the direction of composite models for quarks and leptons are showing a tendency to
get subsumed within the framework of more-fashionable "all-time" theories (e.g. superstrings) which prefer to avoid the question of such substructures, much like the (then powerful) "Bootstrap" philosophy of the sixties. In this talk, however only the substructure point of view will be addressed.

**The Quark Picture**

It was about 25 years ago that a revolutionary proposal was made almost simultaneously by George Zweig and Murray Gell-Mann concerning the inner constitution of hadrons, till then considered to be the basic building blocks of matter. But the chances of "acceptance" of the proposal were greatly handicapped by (i) the bizarre nature of the assumptions involved (fractional attributes) which ran counter to all the "conventional" ideas of the time, and (ii) the failure to detect such objects in various conceivable experiments. On the other hand, the amazingly simple manner in which numerous hadronic properties ranging from their simplest SU(6) classifications and various low energy properties to an equally simple "understanding" of their high every scattering amplitude could fit into the "quark picture" with no extra assumptions, proved good enough reason for a large fraction of the HEP community to "hold on" to this picture against strong resistance from powerful opponents. The nature of the dichotomy between the two opposing groups may be gauged by the emergence of two distinct schools of thought, viz; 'mathematical' (or fictitious) quarks versus 'constituent' (or physical) quarks. The former (subsequently termed "current" quarks) proved amenable to current-algebraic techniques and gave rise to certain (broken) symmetries of the Chiral SU(3) \times SU(3) types. The constituent picture on the other hand was extensively employed in dynamical calculations patterned after the standard N. R. potential techniques of conventional nuclear physics. These two approaches proved largely complementary both as regards techniques as well as predictive power. the former being successful for several high energy applications and the latter for correlating various low energy data. They showed little evidence of mutual compatibility, let alone the possibility of a synthesis of their conceptual frameworks into a single integrated mathematical formulation, at least until the early seventies which once again saw a renaissance of Field Theory since its eclipse in the late fifties by the (then powerful) Bootstrap Philosophy.

**The Gauge Principle : Yang-Mills Fields**

The language of Field Theory paved the way to an eventual unification of the two approaches through the emergence of two new principles; Asymptotic Freedom and Permanent Confinement. These two principles, both of which were firmly rooted in experimental observations at high energy (deep inelastic scattering data) and low energy (lack of observation of free quarks) respectively, found a natural resting place in a new form of field theory, viz., Quantum Chromodynamics or QCD. This theory, together with its counterpart in Quantum Electrodynamics (QED) are two outstanding examples of one of the most fruitful principles to be enunciated in Field Theory viz., the Principle of Gauge Invariance.
whose central idea may be described (in the context of electric charge) as follows. In order to conserve the total "charge" of the Universe, all particles (electrons, protons, quarks,...) possessing the attribute of charge must couple in a certain mathematically self-consistent manner to its universal carrier field—the electromagnetic field—such that the total Lagrangian for the coupled e.m. and matter fields remains invariant under simultaneous variations (gauge transformations) of the respective fields. By Noether's Theorem, this "gauge invariance" implies the conservation of the "charge" attribute. This circumstance represents a kind of symmetry, analogous to (but more specialized than) rotational or translational symmetry. It should perhaps be stressed that this remarkable symmetry had all along been present \( \textit{albeit} \) in an unmanifest form in the original structure of Maxwell-Logentz equations, but its full flavour could be appreciated only after such a "gauge formulation" which thus gave a new meaning to the attribute called "charge" not only as a measure of the force between charged particles but as something that \( \textit{is strictly conserved} \) as a result of its universal coupling to the (carrier) "gauge field." Since the electric charge is a \textit{single} attribute, the carrier field has a simple \textit{abelian} structure and its identification with the e. m. field is justified by the massless character of the photon (a necessary condition for any field to qualify as a gauge field). The generalization from an abelian gauge field (carrier of a single charge attribute) to a non-abelian, multiple component carrier field (for a multiplicity of charges) was first suggested in a seminal paper by Yang and Mills\(^3\) to incorporate the isospin or "flavour"-degree of freedom within the broad "gauge" concept. Though the carrier field for this "flavour" attribute was not then in sight, it was subsequently to be identified\(^4\) as the massive \( W^\pm \) and \( Z^0 \) mesons of the celebrated Glashow-Weinberg-Salam Electroweak Theory\(^5\) by virtue of which their heavy masses could be reconciled with the basic gauge theoretic requirement (of zero mass carrier fields) through the ingenious device of Spontaneous Symmetry Breaking (proposed earlier by Higgs–Kibble\(^6\) to circumvent the Nambu–Goldstone\(^7\) problem). It thus represented a major vindication (at the level of weak interactions) of the non-abelian extension of the gauge principle by Yang and Mills\(^3\) at a time when its experimental confirmation was unthinkable. In the meantime temporary candidates for the carriers of the flavour attribute had been suggested by Sakurai\(^8\) in the form of certain vector mesons \((\rho, \omega, \phi)\) which were however more appropriate for the mediation of strong interactions. These candidates turned out to be "wrong" on the intrinsic ground of \( q\bar{q} \) compositeness which would disqualify them for the role of proper gauge field carriers. (Additionally, their massive character was a further impediment to the gauge principle but this problem was already subsumed in the more serious problem of reconciling a composite field to the role of carrier of a fundamental non-abelian attribute). Nevertheless, the Sakurai Principle has served as a practical means for utilizing these massive vector mesons \((\rho, \omega, \phi)\), together with their pseudoscalar counterparts \((\pi, \eta)\), to serve as certain background fields for the mediation of strong interactions at the nuclear level, a subject being rapidly developed under the head of Quantum Hadrodynamics (QHD).\(^9\)
QCD and Confinement

The eventual progress towards a “correct” gauge theory of strong interactions was made after the discovery of the 3-fold color charge as a basic non-abelian attribute of quark fields governed by the group SU(3). The carrier gauge field of this attribute was identified as the massless gluon field. In this respect it is necessary to recognize a fundamental difference between abelian U(1) and non-abelian SU(N) gauge fields: The abelian U(1) e.m. field, “carrying” as it does a single attribute (electric charge), does not “possess” this attribute (it is electrically neutral). On the other hand since the non-abelian SU(3) gluon field has to carry the color triplet(3) attribute, as well the color antitriplet (3*), it must also “possess” an octet of color-quark and color-anti-quark fields. Thus was born Quantum chromodynamics QCD,\(^1\) ending a long search for an appropriate strong interaction mechanism within the precincts of gauge field theory, on lines closely patterned after the highly successful Quantum Electrodynamics (QED) of the fifties. Table I summarises the points of similarity and contrast between QED, and QCD together with an interpolating mechanism in QHD, thus completing a 3-stage hierarchy at successive levels of compositeness to date, with corresponding “gauge” fields operative at each level:

<table>
<thead>
<tr>
<th>Gauge Th</th>
<th>QED</th>
<th>QCD</th>
<th>QHD</th>
</tr>
</thead>
<tbody>
<tr>
<td>System</td>
<td>Atomic (Bohr)</td>
<td>Hadronic (H) (MGM-Zweig)</td>
<td>Nuclear (WIGNER)</td>
</tr>
<tr>
<td>constituents</td>
<td>e(^+), e(^-)</td>
<td>q, (\bar{q})</td>
<td>H, (\bar{H})</td>
</tr>
<tr>
<td>carrier field</td>
<td>e.m.</td>
<td>gluonic</td>
<td>mesonic ((\sigma, \omega))</td>
</tr>
<tr>
<td>Attribute</td>
<td>Charge</td>
<td>Color</td>
<td>Isospin, etc.</td>
</tr>
<tr>
<td>Gauge Group</td>
<td>U(1) SU(3)</td>
<td>(\Rightarrow) non-abelian</td>
<td>SU(2), etc.</td>
</tr>
<tr>
<td>Conservation status</td>
<td>exact</td>
<td>exact</td>
<td>approximate</td>
</tr>
<tr>
<td>Coupling Const.</td>
<td>(\alpha = \frac{1}{137})</td>
<td>(\alpha_s \approx 1)</td>
<td>(G^2/4\pi = 2.2)</td>
</tr>
<tr>
<td>Q(^2)-variation</td>
<td>(\frac{\text{d}\alpha}{\text{d}Q^2} &gt; 0)</td>
<td>(\frac{\text{d}\alpha_s}{\text{d}Q^2} &lt; 0)</td>
<td>(Effective theory)</td>
</tr>
<tr>
<td>Short distance behaviour</td>
<td>(\alpha) increases at small (r) (\Rightarrow) Asymptotic barrier</td>
<td>(\alpha_s) decreases at small (r) (\Rightarrow) Asymptotic Freedom</td>
<td></td>
</tr>
<tr>
<td>Long distance behaviour</td>
<td>(\alpha) decreases at large (r) (\Rightarrow) Infrared freedom</td>
<td>(\alpha_s) increases at large (r) (\Rightarrow) Infrared Slavery</td>
<td></td>
</tr>
<tr>
<td>Pert. Theory status</td>
<td>OK at large (r); not good at small (r)</td>
<td>OK at small (r); invalid at larger</td>
<td></td>
</tr>
</tbody>
</table>

\(G^2/4\pi = 2.2\)

(Effective theory)
Inter-atomic (e.m.) \(\rightarrow\) Internuclear (mesonic) \\
\(\rightarrow\) Inter-quark (gluonic) \(\rightarrow\) (?)

An important distinction between abelian QED and non abelian QCD consists in the variation of their respective "Charges" as a function of momentum:

\[
\frac{d}{dQ^2} \alpha_s (Q^2) < 0; \quad \frac{d}{dQ^2} \alpha (Q^2) > 0 \quad \text{for } Q^2 < \text{QED} \]

... (1)

The decrease of \(\alpha_s\) with \(Q^2\) has the important consequence that the "color" interaction becomes weaker at shorter distances, thus providing a natural mechanism for Asymptotic Freedom (AF). In the same vein, the increase of \(\alpha\) with small \(Q^2\) provides a formal basis for confinement of the color attribute (and hence of the quarks and gluons which color resides, thus absolving them of the obligation to exist as free particles). Now Asymptotic Freedom has been convincingly demonstrated through extensive calculation in perturbative QCD (a procedure justified by AF) and their comparison with the data in deep inelastic electron scattering on protons and other nuclei. On the other hand, an equally ‘convincing’ demonstration of confinement is vitiated by the non-perturbative character of \(\alpha_s (Q^2)\) operative at non-so-large \(Q^2\) and the consequent unreliability of (perturbative) calculations in the long distance regime. Though there is little doubt that QCD incorporates the confinement feature within its conceptual premises, one of the major theoretical challenges of the day is the problem of how to bring out this remarkable feature explicitly. This has literally given rise to a whole new industry (less than a decade old) called Lattice Gauge Theory designed to solve the strong interaction problem of QCD through discretization techniques (path integrals over Wilson loops, etc.) requiring more and more elaborate techniques of Computer Science. Since these techniques are generally not amenable to analytical methods, the results derived therein must rely solely on numerical accuracy. Inevitably the progress on these lines, while truly impressive on the absolute scale, has been not only "slow" by the yardsticks of progress in other areas of physics, but the actual results obtained are yet to evolve a consensus as the lattice parameter \((a)\) is made smaller and smaller. Considering the vast amount of effort needed to subject QCD predictions to a rich variety of observational checks before its claim as the Candidate Theory of Strong Interactions acquires universal acceptance, there seems to be little concrete prospect of an early resolution of the QCD problem through the complex channels of Lattice Gauge Theory.

Before ending this section it is perhaps pertinent to note that AF and "Confinement", implied by eq.(1) for QCD, are by no means characteristic of QCD alone, but of all types of non-abelian gauge field couplings. Thus in principle it should also be true of the 'non-abelian flavour gauge group, which characterises the mean couplings of the W-boson fields. However, its lack of manifestation in the latter is partly due to the effect of spontaneous symmetry Breaking and partly due to the "mixing" with the (abelian) QED field which has just the opposite characteristic [see Eq.(1)]. The same "mixing" is also responsible for suppressing the asymptotic increase of the QED coupling constant \(\alpha\) at short distances, thus
preserving the effective validity of perturbation theory in QED, even at the shortest distances.

Effective Confinement in QCD: An Integrated View

A still more ambitious approach to the entire gamut of interactions—strong, electroweak and gravitational—is illustrated by the modern theory of strings and superstrings\(^{12}\) which seems to bear little resemblance to its original modest beginning at the hands of Nambu\(^{13}\) as an offshoot of the semi-intuitive-ideas of quark or duality diagrams on the one hand,\(^{14}\) and the mathematical structure of the dual models\(^{15}\) on the other. While the theory has no doubt retained the basic quark language and the gauge principle, in common with QCD, its galloping developments have nevertheless taken a totally unexpected turn characterised by an increasing degree of theoretical sophistication. This last has reached such gigantic heights of complexity as to render any possibility of a meaningful contact with experiment more and more remote.

These practical problems associated with such “all-time” or ultimate theories are enough reason to justify the use of less ambitious approaches to quark confinement, while respecting the basic principles of Lorentz and Colour gauge invariance at more modest levels of sophistication. Thus while the full content of the QCD Lagrangian in the usual notation, viz:

\[
\mathcal{L}_\text{QCD} = - \bar{\psi} (\gamma \cdot D + m_\psi) \psi - \frac{i}{2} [D_\mu, D_\nu] \psi_\nu D_\mu = \bar{c}_\mu - \frac{i}{2} g A^a_\mu \ldots (2)
\]

may not yet be amenable to an exact solution, the literature is full of QCD orientated efforts to go ahead with extensive applicational programmes to different sectors of hadron physics, assuming some effective form of confinement. Since, on the other hand such empirical approaches can not be a substitute for a more formal theory, their credibility must be judged by the depth and range of their predictive power, and the extent of success or otherwise of their contact with observations. Unfortunately, most efforts of this nature are limited to relatively narrow sectors of hadronic phenomena, individually ranging from heavy to light quarkonia on the one hand, and the lowest to the highest energies on the other. For example, the MIT Bag Model\(^{16}\) (in which hadrons as confined quark composites, are individual “bubbles” imbedded in an infinite medium known as the true vacuum), though relativistic in content, is too macroscopic for a realistic description of a sufficiently wide class of relativistic transition amplitudes. On the other hand, non-relativistic (N.R.) models with effective \(q-q\) and/or \(q \bar{q}\) (confining) interactions\(^{17}\) are more microscopic to be sure, but their N.R. character again stands in the way of calculating anything meaningful beyond the spectra and a few of the simplest N.R. transition amplitudes only. In short there appears to be little evidence of a more integrated formulation, albeit along less fundamental lines, consciously designed to give a unified description of the physics of these vastly different sectors. Needless to say, any serious effort towards a more integrated view of hadron physics in terms of quark compositeness would be a
small price to pay for taking for granted so fundamental a hypothesis as confinement (in advance of a QCD-level derivation).

Such an integrated view of quark physics, while falling far short of an "ultimate" theory should nevertheless constitute a major advance over various "piecemeal" approaches through its capacity to correlate a wide range of phenomena with a vastly reduced number of free parameters resulting from a more judicious use of the field theoretic principles bearing on the composite structure of hadrons. Indeed, a reasonably consistent formulation of the mathematical machinery for a quantitative computation of the dynamical effects of compositeness at the quark level would, by any reckoning, constitute a vital theoretical ingredient for even a zero-order understanding of the vast complex of hadronic data that are rapidly accumulating at the major high energy accelerator centres around the world. Such an approach which may be regarded as an effective bridge between a full-fledged theory (yet to emerge) and various data-based phenomenological models can be defended from still another angle: It can serve as a highly economical language of data analysis, with vast correlative powers firmly rooted in theoretical principles, when it is remembered that modern methods of data analysis in themselves involve an increasing number of model dependent assumptions and parameters.

Role of the Wave Function: Need for a Causal Form of Dynamics

What kind of a dynamical framework should suffice for a broad-spectrum applicational programme with a QCD-oriented form of quark confinement? Since both Relativity and the Gauge Principle must be vital ingredients for any such framework, it is possible to argue that the Bethe-Salpeter Equation (BSE)\(^\text{18}\) at the quark level\(^\text{19}\) which can naturally incorporate these features because of its field-theoretic basis, constitutes a minimal framework for bringing out the dynamical effects of compositeness on a vast complex of hadronic phenomena without further (ad hoc) assumptions along the way. Since the key ingredient of the BSE is the hadron's wave function, this approach facilitates the determination, in a causal manner, of this vital quantity which serves as the vehicle for spanning an entire physics all the way from low energy spectroscopy to high energy structure and fragmentation functions with diverse types of transition amplitudes at intermediate energies. To appreciate the stakes involved in a causal determination of the wave function, it is good to bear in mind that such a "determination" from the formal premises of QCD is a formidable task at the present state of its art. The QCD Sum rules,\(^\text{20}\) with the higher twist effects parameterized by the vacuum expectations values of certain key operators,\(^\text{21}\) are generally thought to represent a powerful method of analysis designed to bridge the gap between the high and low energy manifestations of QCD; yet a good deal of information gets lost in this approach, especially the mechanism that bears on the hadron mass predictions. In the process, the causal structure of the wave function gets somewhat relegated to the background, giving way to certain "variational forms"\(^\text{22}\) constrained only by the QCD Sum rules. (This is certainly a heavy price for a quantity
which is supposed to provide the basic vehicle for maintaining the causal links between high and low energy physics). The Bethe-Salpeter method of "determination" of the hadron wave function, in contrast, strictly maintains the causal structure of the wave function within its dynamical framework: In this approach the emphasis is on the extrapolation of the wave function from low to high energies. In this last respect its philosophy is opposite to that of QCD Sum rules (from high to low energies) but additionally it preserves the causal character of the wave function.

The Bethe-Salpeter Equation: Early History

The Bethe-Salpeter Equation,18 (which may be regarded as a truncated version of the Schwinger-Dyson Equation to any desired order in the successive "vertex" insertions), has had a long history. It represented the first serious effort to write down a consistent relativistic two-body equation in 4-dimensional form, firmly rooted in field theory, without recourse to perturbation theory. However, the fuller potential of BSE remained essentially unexplored at the time since the small coupling constant of QED had by and large proved a disincentive to its non-perturbative treatment. The only serious application at the early stages was to the \((e^+ e^-)\) positronium system23 where even its relativistic features could not find adequate exposure due to the basically non-relativistic character of \((e^+ e^-)\). Thus most of the early BSE investigations had been of a purely "mathematical" type with "soluble" kernels, designed to explore the singularity structures, normalization aspects and the problems of probability interpretation of the 4-dimensional BS wave function \(\Psi\) arising out of its dependence on the relative time of the two constituents.24 Since the quantity \(\Psi(r,t)\) reduces to the 3-dimensional wave function \(\psi(r)\) with a more natural probability interpretation in the limit \(t = 0\). [the so-called 'Instantaneous' Approximation], considerable effort was spent in making various 3-dimensional formulations of BSE. It should perhaps be stressed that the physical content of the Instantaneous Approximation (or IA) that is manifested in the 3-dimensional formulations of BSE extends well beyond the N. R. limit, so that such formulations in principle, make a good deal of sense for relativistic systems without giving up the simple-minded probability interpretation. The literature has been full of such attempts through the sixties.25

Following a considerable period of lull the BSE again came into prominence in the quark context,19-25 after the initial successes with the N.R. quark model26 had suggested more serious investigations of quark composites with relativistic dynamics. This also coincided with the efforts27-28 to consider \((V/c)^2\) corrections to heavy quarkonia within the framework of the Bethe-Salpeter technology.29-27 The emergence of QCD in the seventies as the pre-eminent theory of strong interactions10 now provided a natural dynamical basis for the strong interaction kernel of the hadronic BSE mediated by gluon exchanges. However, in the absence of a formal resolution of the QCD confinement problem the dominant (confining) part of the kernel is not yet derivable from the QCD Lagrangian, so that
any serious BSE calculation of $q\bar{q}$. or $qqq$ composite dynamics must use the long range $q\bar{q}$ or $q-q$-kernel as an effective input. Nevertheless the very wide (4-dimensionally covariant) framework of the BSE opens up a vast field for hadronic applications (even before a fuller understanding is achieved for the input kernel!) so that such a framework is ideally suited to the need for an integrated view of hadronic phenomena (from the lowest to the highest energies) without further (ad-hoc) assumptions.

The 4-dimensional form of the BSE has not been without its practical problems, however (apart from the conceptual problems of probabilistic interpretation). These show up quite sharply in its mass spectral predictions which are analogous to those of an O(4) theory in the sense of a Wick rotation. An O(4) like theory gives "one quantum number too many" compared with the conventional O(3) theory whose predictions are not only in line with the N.R. quark model but have continued to be in agreement with the data for more than two decades. Such a situation has all along pointed to the need for a 3-dimensional formulation of BSE by integrating out over the time-like degree of freedom for the internal quark motion, before comparing its predictions with observation. In this respect the traditional reduction has been generally in terms of the Instantaneous Approximation (IA) which is a reasonable mechanism for the suppression of the associated (virtual) $q\bar{q}$ effects without sacrificing the relativistic effects of the internal quark motion. This last is an important requirement for light quarkonia without giving up the O(3)-like features in the $q\bar{q}$ dynamics. The disadvantage of this approximation lies in its lack of Lorentz covariance so that the corresponding wave function of such a $q\bar{q}$ composites can not be trusted for a hadron in arbitrary motion. This problem is not serious if only the calculation of mass spectra is desired (since the $q\bar{q}$ systems may be taken in the rest frame) but becomes increasingly acute for the calculation of transition amplitudes involving fast moving hadrons as $q\bar{q}$ composites.

A happy solution of the problem of fast moving hadrons within a 3-dimensional BSE framework is offered by the use of the null plane an satz or NPA in which the light front variable $x_\pm = x_0 \pm x_3$ effectively play the roles of $x_3$ and $x_0$ respectively. The use of light front coordinates for quantum mechanical applications was first suggested by Dirac as a concrete step for enlarging the stability group of the Poincare generators (from 6 to 7), but their field theoretic applications were to come two decades later in the aftermath of Bjorken (or Feynman) Scaling. The main advantage of NPA lies in its natural covariance for longitudinal boosts, a feature which readily lends itself of the application of the BSE framework to transition amplitudes involving hadrons in arbitrary motion. Further, the 3-dimensional character of NPA lends the resulting BSE an effectively O(3) like character which as noted above, is important for comparing its mass-spectral predictions with data.

*The effects of associated $q\bar{q}$ pairs (virtual) can equivalently be described in terms of higher Fock states in a null-plane formulation of BSE for a given kernel.*
TWO DECADES OF QUARK PHYSICS

TWO-TIER APPROACH TO BSE: A PRACTICAL PROGRAMME

The next question that arises in this context is whether the time-like degree of freedom to be integrated out for obtaining such a 3-dimensional reduction of the BSE has any observational manifestation at all, for it did not have one there would be little justification in formulating a 4-dimensional BS framework in the first place. Now the fact that the spectral signatures of the 4-dimensional BSE are mostly 3-dimensional, strongly suggests that the effects of the extra (relative-time) dimension must not be pronounced at "low" enough energies, so that it would be reasonable to assume that these open up in a gradual fashion as the composite system is probed much deeper through an increase in the energy scale of the probing mechanism. To that end, it should make sense to develop a parturbative mechanism to investigate the effect of this fourth degree of freedom (through virtual $q\bar{q}$ couplings) through the lowest order Feynman diagrams for the processes under study. To implement such a programme in a natural way will in turn require a reconstruction of the 4-dimensional BS wave function (by retracing the steps that had led to the 3-dimensional BSE) and a consequent identification of the hadron quark (composite) vertex function as the key ingredient in the whole enterprise. Such a strategy in which the input parameters of the BSE kernel are thus firmly rooted in low energy spectroscopy having only 3-dimensional signatures, should offer a practical as well economical alternative to a completely 3-dimensional formulation of BSE through an infinite chain of coupled integral equation involving successively higher Fock states\textsuperscript{34} (much like the original Tamm-Dancoff approach of the Fifties).\textsuperscript{35}

To summarise, it seems possible to envisage a two-tier approach within the BS framework:

A. A 3-dimensional reduction of BSE (termed NPA for short) by integrating out over the internal momentum component $q_\perp$ as a first step for making contact of its predictions with mass spectral data.

B. Reconstruction of the 4-dimensional hadron-quark ($Hq\bar{q}$) vertex function as the basic ingredient for a perturbative inclusion of virtual $q\bar{q}$ effects, (higher Fock States,)\textsuperscript{34} through lowest order Feynman diagrams for a systematic evaluation of successive transition amplitudes of different types arranged in ascending order in the energy scale. The NPA structure of the $Hq\bar{q}$ vertex functions ensures the Lorentz covariance of these transition amplitudes for rapidly moving composite hadrons.

The rest of this section is a summary of some technical details necessary for implementing such a programme and the results of contact of a few crucial findings with data for typical energies. The success of such a programme should

\textsuperscript{34}Indeed the latter approach\textsuperscript{34} would leave much scope for additional assumptions on the input kernels connecting the chain of higher Fock states, which would be a premature exercise in mathematical elegance in the input assumptions without a continuous logistic support at the observational level.
in turn be judged by its simultaneous performance on both \( q \bar{q} \) and \( \text{qqq} \) systems, whose dynamics are intimately linked together, as befits two dual partners in keeping color "confined" through the common "force"—mechanism of QCD. However, as mentioned in the last Section at the present state of the QCD art, confinement must be put in by hand through effective \( q \bar{q} \) and \( q-q \) interactions carrying visible signatures of a (hopefully) common origin.

**B. S. Formalism in NPA**

The following is a summary of the BS formalism as developed by the Delhi Group since 1981, incorporating the various refinements in the formulation and input parameters.

The effective BS kernel, which must incorporate confinement in a Lorentz-invariant manner, has a spin part of the form \( \gamma_\mu^{(1)} \gamma_\mu^{(2)} \), appropriate to a vector confinement\(^{36}\) and the usual color structure \( \frac{1}{2} \lambda_1 . \frac{1}{2} \lambda_2 \). The vector form ensures the same sign for the colour interaction in the \( q \bar{q} \) singlet and \( q \bar{q} \) anti-triplet,\(^{37}\) thus providing a strong motivation for a common dynamical approach to both the confined systems \( q \bar{q} \) and \( \text{qqq} \). The vector form also provides a natural place for the Gauge Principle, to the extent the same makes sense within an effective form of dynamics. The spatial part of the \( q \bar{q} \) and \( q \bar{q} \) interaction kernel is given by a Lorentz-invariant generalization\(^{38}\) of the 3-dimensional harmonic oscillator in the following form

\[
\langle q | V | q' \rangle = \frac{3 \pi \omega_{qq}^2}{m \to 0^+} \lim_{m \to 0^+} - \frac{\partial^3}{\partial m^3} \left( m + q_n^2 \right)^{-1}
\]

where \( \omega_{qq}^2 \left( \approx \omega_{qq}^2 \right) \) is the effective spring constant for the \( q \bar{q} \) (or \( q \bar{q} \)) interaction\(^*\).

This corresponding BSE's (with equal mass quarks for simplicity) for \( q \bar{q} \) and \( \text{qqq} \) systems are of the respective forms\(^{36,38}\)

\[
(m_q + i \gamma^{(1)} . p_1) (m_q - i \gamma^{(2)} . p_2) \Psi(q) = \frac{(2 \pi)^{-4}}{i} \left( \frac{1}{2} \lambda_1 . \frac{1}{2} \lambda_2 \right)
\]

\[
\times \int \gamma_\mu^{(1)} \gamma_\mu^{(2)} V(q - q') \Psi(q') \, d^4q'
\]

where \( (q, q') \) are the relative 4-momenta defined by \( p_{1,2} = \frac{1}{2} P_{12} \pm q_{12} \) etc., and\(^{36,38}\)

\[
\prod_{i=1}^{3} (m_q + i \gamma^{(1)} . p_i) \Psi(p_1, p_2, p_3) = \prod_{i=1}^{3} (m_q + i \gamma^{(3)} . p_3) \left( \frac{2 \pi)^{-4}}{i} \frac{1}{2} \lambda_1 . \frac{1}{2} \lambda_2 \right)
\]

\[
\times \int d^4q'_{12} \gamma_\mu^{(1)} \gamma_\mu^{(2)} V(q_{12} - q'_{12}) \Psi(p'_{1}, p'_{2}, p_3),
\]

where \( q_{12}, q'_{12} \) are the relative 4-momenta of the (12) subsystem with quark \( \neq 3 \) as

\(^*\)Note that this from differs from the more usual 4-dimensional h.o. from \( \Box^2 \delta^4(q) \).
the spectator. The subsequent handling of these equations is greatly simplified after defining a "reduced" BS wave function $\Psi(q)$ as

$$\Psi(q) = (m_q - i\gamma^{(1)} p_1) (m_q + i\gamma^{(2)} p_2) \Phi(q)$$

and a similar definition for $\Phi(p_1, p_2, p_3)$ related to $\Psi(p_1, p_2, p_3)$.

To implement the two-tier formalism it is first necessary to effect a 3-dimensional reduction for $\Phi(q)$ in the null-plane ansatz (NPA) which consists in integrating out over the $q_-$ variable in the meson amplitude $\Phi(q)$ and the pair of variables $q_{12}$ and $p_3$ in the baryon amplitude $\Phi(p_1, p_2, p_3)$ to give the 3-dimensional wave function:

$$\phi(q) = \int \frac{1}{2} dq_- \Phi(q)$$

and

$$\phi(p_1 p_2 p_3) = \int \frac{1}{2} dq_{12} \frac{1}{2} dp_3 \Phi(p_1 p_2 p_3)$$

respectively. To get the maximum simplification out of such a reduction, it is far more convenient to resort to a prior Gordon-reduction than to adopt the traditional $(\pm \pm)$ and $(\pm \pm \pm)$ component reduction in the two cases, as is usually done in connection with the instantaneous approximation. [The objection to the Gordon reduction on the ground that on-shellness is not satisfied, gets greatly diluted in the context of the NPA where the fourth component momentum $p_\perp$ of any quark 4-momentum $p_\perp$ is itself defined as $p_\perp = \omega q_1 / p_\perp$ corresponding to the on-shell condition $p_\perp^2 = m_q^2 = 0$. The fuller procedures are described in ref (36) and (38) for the $q\bar{q}$ case and in ref (38) and (41) for $qqq$, to which we refer the interested reader for details.]

The main effect of the NPA on the Kernel (3) is to reduce it to the 3-dimensional form

$$V(q - q') = 3\pi \omega_{qq}^2 \nabla_q^2 \delta^3(q - q')$$

where

$$\nabla_q^2 = \nabla_1^2 + \frac{p_\perp^2}{M^2} \delta_+^2,$$

and $p_\perp$ is the 4-momentum of the composite hadron of mass $M$. Note that in this formalism, the third component of any vector $A$ is effectively given by $A_3 = A_+ M / P_+$, which preserves "null-plane covariance" insofar as the numerator and denominator transform in the same manner. In this case, the confinement interaction has the harmonic form $r^2$ with $z$ as the canonical conjugate of $q_3 = q_+ M / P_+$. For the spring constant, the following ansatz

$$\omega_{qq}^2 = 4\mu_{12} \omega_{q_3}^3 \alpha_q; \mu_{12} = \frac{m_1 m_2}{m_1 + m_2}$$

provides an explicit QCD motivation (through the proportionality to $\alpha_q$) and a flavour dependence for different $q\bar{q}$ ($qq$) sectors through the reduced mass $\mu_{12}$. 
The reduced quantity $\omega_0$ is, hopefully, constant over different flavour sectors, thus giving it a nearly universal status. (Actual comparison with the data\textsuperscript{31} does seem to bear out this expectation). For the quantity $\alpha_s$ we also use\textsuperscript{32} the standard from

$$
a_s(M^2) = \frac{12\pi}{(33 - 2f)} (\ln M^2/\Lambda^2)^{-1}, \text{ with } M \to (m_1 + m_2) \quad \ldots (13)
$$

For a more realistic representation, the harmonic form (9) needs to be modified as follows:\textsuperscript{32}

$$
r^2 \to W(r) = r^2 \left[ 1 + A_0 m_1 m_2 r^2 \right]^{-1/2} - \frac{C_0}{\omega_0^3} \quad \ldots (14)
$$

where $-r^2$ must be read as (10). The constant term is designed to take account of the correct zero-point energies in the different flavour sectors, while the $A_0$-term ($A_0 \ll 1$) simulates an effect of an almost linear confinement for the heavy quark sectors (large $m_1 m_2$), while retaining the harmonic form for the light sectors (small $m_1 m_2$). This representation is thus asymptotically consistent with linear confinement (as is believed to be true for QCD), through the intervening length scales in the light quark sectors give it an effectively harmonic appearance. A more formal justification for such a representation has been found in teams of a Fock-Schwinger expansion of the gluon fields,\textsuperscript{43} wherein the first term in the expansion corresponds to the harmonic form.\textsuperscript{43}

With these physical inputs the Gordon-reduced NPA treatment becomes a 3-dimensional equation for $\phi(q)$, eq. (7), of the form\textsuperscript{38}

$$
D_+(q) \phi(q) = \omega_q^2 \frac{P_+}{M} \tilde{D}(q) \phi(q) \quad \ldots (15)
$$

where

$$
D_+(q) = 2P_+ \left( m_\nu^2 + q^2 - \frac{1}{4} M^2 \right) \quad \ldots (16)
$$

is the NPA denominator function defined by\textsuperscript{38}

$$
\frac{2\pi i}{D_+(q)} = \int \frac{d\xi}{\Delta_1 \Delta_2} \Delta_{1,2} = m_\nu^2 + p_{1,2}^0 \quad \ldots (17)
$$

and $\tilde{D}(q)$ is a differential operator\textsuperscript{38} appropriate to the vector confinement with the shape (14). Its fuller structure for $m_1 \neq m_2$ which is described in ref (42), is reducible to that of a 3-dim h.o. with coefficients dependent on the hadron mass $M$ and the total quantum number $N$. The ground state form of $\phi(q)$ deducible from the equation is of the form\textsuperscript{38}

$$
\phi_0(q) = \text{Exp} \left[ - \frac{1}{2\beta^2} q^2 - \frac{1}{2\beta^2} M^2 \frac{q^2}{P_+} \right], \quad \ldots (18)
$$

where $\beta$ is an inverse range parameter defined by eq. (12) of ref (42). This is apart from an overall multiplicative constant Dirac matrix\textsuperscript{38,39} whose precise form ($\gamma_5, i\gamma, \hat{e}$) depends on the actual $q\bar{q}$ state (pseudoscalar, vector) under study. For
heavy quarkonia, one can still use the representation (16) as basis, but must take a linear combination of several radial excitations.42

For completeness the corresponding 3-dimensional BS equation after NPA has the analogous structure38-41

\[ \phi(p_1p_2p_3) = \sum_{123} \frac{P_{12}}{D_{12}^+} \frac{M}{2} \omega_{qq}^2 D_{12} \phi(p_1p_2p_3) \]  

where

\[ D_{12}^+ = 2P_{12}^+ \left( m_1^2 + q_{12}^2 - \frac{1}{3} M^2 + \frac{1}{4} p_{13}^2 + R_{12} \right) \]

is a characteristic energy denominator for the (12) subsystem and all 3-vectors have to be read in the covariant NPA sense \((A_3 = A_+ M/P_+)\). \(\tilde{D}_{12}\) is a differential operator38-41 expressing the effect of the confining interaction within the (12) pair. The details of the various terms as well as the techniques of simplification for the equal mass case are fully described.41 The resulting equation can again be expressed in a form similar to (15) but with two independent harmonic oscillators, corresponding now to the existence of two interval momenta \((\xi, \eta)\) which may be chosen so as to satisfy the requirements of \(S_3\)-symmetry. (The interested reader is referred to ref (41) for details). The corresponding ground state \(qqq\) wave function is a 6-dim gaussian : 36-41

\[ \phi(\xi, \eta) = \text{Exp} \left[ - \frac{1}{2b_N^2} (\xi^2 + \eta^2) \right], \]

where \(\beta_N\) is a known function of the input parameters \((\omega_0, m_0, C_0)\). Finally, the eigenvalue solutions of both the \(qq\) and \(qqq\) equations are expressible as

\[ F_M(M) = N + \frac{3}{2}; \quad F_\theta(M) = N_\xi + N_\eta + 3 \]

respectively, where \(F(M)\) is a known function \(M\) in each case, and \(N\) and \((N_\xi, N_\eta)\) are the corresponding principal quantum numbers. Eq. (22) contains the predictions on the respective mass spectra in terms of a common set of parameters \((\omega_0, \epsilon_0\) and \(m_0\)). This completes the first stage of the two-tier approach advocated in sec. 3 wherein the 3-dimensional form of the BSE was deemed appropriate for making contact with the spectral data31 since these have continued to respect an \(O(3)\)-type symmetry. Physically this has been achieved by integrating out over the \(p_-\)-type variables (NPA) which amounts to suppression of the (virtual) \(qq\) effects or higher Fock states34 on the mass-spectra. And the agreement with the data without this effect does indeed suggest that this effect is small.

Reconstruction of BS Vertex Function in NPA

The second stage of this two-tier formalism consists in reconstructing the 4-dimensional BS amplitude as a means of restoring the virtual \(qq\), etc. effects (higher Fock states) on different types of hadronic transition amplitudes perturbatively through standard Feynman diagrams. Such transition amplitudes—and
there are many-carry a vast complex of complementary information about the fuller 4-dimensional BS dynamics, which is not reflected in the mass-spectral predictions based on its 3-dimensional NPA form. The crucial ingredient for this exercise is the hadron-quark vertex function which can be extracted from the 4-dim. BS amplitude in the following manner.\(^{39}\) The connection (7) between the quantities \(\phi(q)\) and \(\Phi(q)\) can be “reversed” by the observation that the BSE for \(\Phi(q)\) after NPA reduction (9) is expressible as

\[
\Delta_1 \Delta_2 \Phi(q) = \frac{1}{2\pi i} \omega_{q}^{2} \frac{P_{\pm}}{M} \tilde{D}(q) \phi(q)
\]  

...(23)

Comparison with (15) gives the identification

\[
\Phi(q) = D_{\pm} \phi(q) / 2\pi i \Delta_1 \Delta_2
\]  

...(24)

whence the BS amplitude \(\Psi(q)\) is inferred from (6) is

\[
\Psi(q) = N\tilde{H}^\pm S_F(p_\pm) \Gamma_1 S_F(-p_\pm) D_{\pm} \phi(q) / 2\pi i
\]

...(25)

where the constant Dirac matrix \(\Gamma_1\) corresponds to the \(q\bar{q}\) state in question and \(N\tilde{H}^\pm\) is the BS normalization factor. It was calculated in ref (38) for \(\Gamma_1 = \gamma_5\) and found to be proportional to \(p_\pm^{-1}\) (in accordance with the demands of NPA covariance). The calculations are similar for other forms of \(\Gamma_1\). Eq. (25) permits the identification of the normalized \(Hq\bar{q}\) vertex function \(\Gamma(q)\) as

\[
\Gamma(q) = N\tilde{H}^\pm \Gamma_1 D_{\pm} \phi(q) / 2\pi i \equiv \frac{\Gamma_1}{(2\pi)^4} W_1
\]

...(26)

which is the basic ingredient for the evaluation of various transition amplitudes. A very similar construction holds for a baryon \(qqq\) vertex function which was given earlier for the instantaneous approximation and is easily generalized to the NPA framework\(^{38}\) but it is omitted for brevity.

Before describing some illustrative application to hadronic transition amplitudes for low and high energies in following two sections and a few remarks about the structure (18) of \(\phi(q)\) are in order. As explained in ref (38), this structure is appropriate to the “on mass shell” condition for the hadron in addition to the usual on-shell conditions for the two quarks under NPA \((p_{\pm} = \omega_{q_\pm} / p_{+})\). Such conditions may be realized for heavier (vector) mesons (upwards of \(\rho, \omega\) mesons) but not readily for the higher pseudoscalar mesons. so that some relaxation of the on-shellness for both the quarks and the hadron may be necessary in practice. We shall therefore employ the following device. When the hadron and one of the quarks is “free” (on-shell), such as in the case for structure or fragmentation functions induced by a virtual photon (see section on “Transition Amplitudes at low-energy”), we eliminate the 4-momentum of the propagator quark in favour of the other quark’s and the hadron’s. This amounts to the replacement\(^{45,46}\)

\[
\frac{M^2 q^2}{P^2_+} \rightarrow - q_+ q_- = - (\frac{1}{2} P_+ - p_{2+}) (\frac{1}{2} P_- - p_{2-})
\]
and gives for the wave function of (18) the result
\[ \phi_2 = \text{Exp} \left[ -\frac{1}{4\beta^2} k_1^2 + \frac{1}{4\beta^2} \left( \frac{1}{2} - x_2 \right) \left( M^2 - \frac{2^2}{\lambda_2} \right) \right] \] ...(27)
where \( k_\perp = \sqrt{(1 - x_1)} P_1 - p_{2\perp}/\sqrt{(1 - x_1)} \);
and \( p_{1+}, p_{2+} = (x_1, x_2) P_+ \).

Such situation will be termed “half-off-shell.” On the other hand if the two
quarks are both taken on-shell, the hadron’s on-shellness need not be invoked, and
this amounts to the replacement\(^{38,47}\)
\[ \frac{M^2 q_+^2}{P_+^2} \rightarrow -q_+ q_- = \frac{\omega_+^2 q_+^2}{p_{1+} p_{2+}}. \]
which is appropriate for several pionic processes since the pion’s “on-shellness”
as a \( q\bar{q} \) composite is often not satisfied by virtue of its unusually low mass. The
corresponding hadron wave function is\(^{47}\)
\[ \tilde{\phi}_0 = \text{Exp} \left[ -\frac{1}{2} \beta^{-2} (q_\perp^2 + x^2 m_q^2)/(1 - x^2) \right], \] ...(28)
where \( x = 2q_+/P_+ \).

Such a situation may be termed hadron “off-shell.” We note in passing that the
B. S. normalization constant \( N_{H^+}^- \) of eq. (26), will require appropriate changes in
its definition in accordance with the form of the 3-dimensional wave function
(\( \phi_0, \phi_2, \tilde{\phi}_0 \)) employed.

Why do we need these different types of hadron wave functions for different
kinematical situations? This ambiguity is probably a reflection of the defects of
this effective confinement approach w.r.t. a more complete theory where such
choices would not have to be made. For example, the full content of QCD
not only includes the running constant but also the dynamical quark mass\(^{48,49}\)
which also “runs”. In our effective description these functions have been
“frozen” at their constituent values, so this limitation is sought to be compen-
sated through a choice of the wave function out of the 3 possible forms
(18), (27) and (28) to suit different kinematical situations, largely on physical
grounds.

**Hadron Mass Spectra \((q\bar{q}, qqq, q^2\bar{q}^2)\)**

The first and the most tangible test of any dynamical model of composite
particles must lie in its mass spectral predictions which bear on its relatively low
energy aspects. The latter should prima facie justify the neglect of virtual \( q\bar{q} \)
effects which is achieved through the three-dimensional reduction of the BSE via
the NPA, as described earlier. The resulting predictions are given by equations of
the form (22) for the \( q\bar{q} \) and \( qqq \) systems respectively, where \( F(M) \) in each
case is a known function of \( M, N \) and the input parameters \((\omega_0, C_0, A_0 \text{ and } m_q)\)
and can be inverted to give an explicit determination of \( M \) in term of \( N \). Before
comparison with the actual data, however, it is necessary to take account of the 
one-gluon exchange effects (Coulomb plus Fermi-Breit). These can be included 
perturbatively for light \((u, d, s)\) quark systems, but not for the heavy quarkonia 
\((c\bar{c}, \bar{b}b)\) which require a diagonalization of the strong coulomb term matrix elements 
\(\langle n \mid V_{\text{coul}} \mid n' \rangle\) with respect to the radially excited states \((n, n')\). The latter in turn 
are strongly influenced by the \(A_0\)-term in (14) for the heavy quarkonia, but the h.o. 
basis can still be profitably employed to take its effect into account. The necessary 
details worked out\(^{42}\) and the results are given in Tables II, III & IV for several 
distinct values quarkonia sectors with the following input for the key constants:

\[
\begin{array}{|c|c|c|}
\hline
\text{Meson} & \text{NJLS} & \text{M(calc.)} \\
\hline
\pi (140) & 000 & 163.0 \\
\rho (775) & 0101 & 915 \\
B (1235) & 1110 & 1182 \\
f, A_0 (1320) & 1211 & 1352 \\
\rho' (1600) & 2101 & 1590 \\
A (1680) & 2220 & 1532 \\
g (1690) & 2321 & 1682 \\
h (2030) & 3431 & 2009 \\
\pi (2350 ?) & 4541 & 2288 \\
r (2510 ?) & 5651 & 2571 \\
\phi (1020) & 0101 & 1051 \\
H (1420) & 1111 & 1364 \\
f' (1525) & 1211 & 1460 \\
\phi (1680) & 2101 & 1644 \\
\phi (1850 ?) & 2321 & 1776 \\
\hline
\end{array}
\]

\[\omega_0 = 158\ \text{MeV},\ C_0 = .2958,\ A_0 = .0283 \] \((29a)\)

and the quark masses (in MeV)

\[m_{ud} = 270,\ m_s = 410,\ m_c = 1760,\ m_b = 5210 \] \((29b)\)

The calculation of the baryon mass spectra is much more complex, despite the 
appearance of the simple looking equation \(F_{B}(M) = N + 3\), since the function \(F_{B}\), 
as a non-linear function of \(M\) and \(N\), is itself the result of a sequence of complex 
operations involving a large number anharmonic terms in \(\xi, \eta\) and their canonical 
conjugates. These can be handled by the techniques of \(SO(2,1)\) algebra,\(^{42}\) as are 
also necessitated for the corresponding terms in simpler case of \(q\bar{q}\) states\(^{41}\)\(^{50}\) 
which enter the \(F_M\)-function in the equation \(F_M(M) = N + \frac{3}{2}\). In the \(q\bar{q}\) case, 
the generators of this algebra are essentially given by the operators\(^{50}\)

\[N = a_i^+ a_i,\ A = a_i a_i,\ A^+ = a_i^+ a_i^+ ,\]
### Table III

*Mass spectra of $c\bar{c}$ and $b\bar{b}$ states (in MeV)*

<table>
<thead>
<tr>
<th>Mesons</th>
<th>NJLS</th>
<th>$M$(calc.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$ (2980)</td>
<td>0000</td>
<td>3035</td>
</tr>
<tr>
<td>$\psi$ (3097)</td>
<td>0101</td>
<td>3092</td>
</tr>
<tr>
<td>$\chi$ (3415)</td>
<td>1011</td>
<td>3510</td>
</tr>
<tr>
<td>$\chi$ (3510)</td>
<td>1111</td>
<td>3526</td>
</tr>
<tr>
<td>$\chi$ (3555)</td>
<td>1211</td>
<td>3556</td>
</tr>
<tr>
<td>$\tau$ (3590 ?)</td>
<td>2000</td>
<td>3661</td>
</tr>
<tr>
<td>$\psi$ (3685)</td>
<td>2101</td>
<td>3696</td>
</tr>
<tr>
<td>$\psi$ (3770)</td>
<td>2121</td>
<td>3716</td>
</tr>
<tr>
<td>$\psi$ (?)</td>
<td>3211</td>
<td>4029</td>
</tr>
<tr>
<td>$\psi$ (4030)</td>
<td>4101</td>
<td>4087</td>
</tr>
<tr>
<td>$\psi$ (4160)</td>
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<td>4101</td>
</tr>
<tr>
<td>$\psi$ (4415)</td>
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<td>4419</td>
</tr>
<tr>
<td>$\tau$ (?)</td>
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<td>6486</td>
</tr>
<tr>
<td>$\Upsilon$ (9460)</td>
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<td>9498</td>
</tr>
<tr>
<td>$\zeta$ (9873)</td>
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<td>9859</td>
</tr>
<tr>
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</tr>
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<td>$\chi$ (9915)</td>
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<td>9912</td>
</tr>
<tr>
<td>$\Upsilon$ (10025)</td>
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<td>10001</td>
</tr>
<tr>
<td>$\chi$ (10254)</td>
<td>3111</td>
<td>10269</td>
</tr>
<tr>
<td>$\chi$ (10271)</td>
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<td>10278</td>
</tr>
<tr>
<td>$\Upsilon$ (10355)</td>
<td>4101</td>
<td>10321</td>
</tr>
<tr>
<td>$\Upsilon$ (10575)</td>
<td>6101</td>
<td>10536</td>
</tr>
</tbody>
</table>

### Table IV

*Mass spectra of strange, charm and beauty states (in MeV)*

<table>
<thead>
<tr>
<th>Meson</th>
<th>NJLS</th>
<th>$M$(calc.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K$ (496)</td>
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<td>564</td>
</tr>
<tr>
<td>$K^*$ (892)</td>
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</tr>
<tr>
<td>$Q_1$ (1270)</td>
<td>1110</td>
<td>1252</td>
</tr>
<tr>
<td>$Q_8$ (1350)</td>
<td>1111</td>
<td>1312</td>
</tr>
<tr>
<td>$K^{**}$ (1430)</td>
<td>1211</td>
<td>1412</td>
</tr>
<tr>
<td>$L$ (1770)</td>
<td>2220</td>
<td>1595</td>
</tr>
<tr>
<td>$K^{***}$ (1780)</td>
<td>2321</td>
<td>1738</td>
</tr>
<tr>
<td>$K^{****}$ (2060)</td>
<td>3431</td>
<td>2064</td>
</tr>
<tr>
<td>$D$ (1869)</td>
<td>0000</td>
<td>2010</td>
</tr>
<tr>
<td>$D^*$ (2010)</td>
<td>0101</td>
<td>2098</td>
</tr>
<tr>
<td>$F$ (1971)</td>
<td>0000</td>
<td>2113</td>
</tr>
<tr>
<td>$F^*$ (2140)</td>
<td>0101</td>
<td>2198</td>
</tr>
<tr>
<td>$B$ (5271)</td>
<td>0000</td>
<td>5253</td>
</tr>
</tbody>
</table>
where $a_i, a_i^+$ are the standard ladder operators of the 3-dim h.o. problem. However, for the baryon $(qqq)$ states, there are as many as five coupled sets of such operators each satisfying SO$(2, 1)$ algebra, thus rendering the exercise much more complex than for the $q\bar{q}$ case. The procedure can be considerably simplified through a generous use of $S_3$-symmetry for the choice of the appropriate SO$(2, 1)$ variables which may be essentially represented in terms of the following sets of operators:

$$(N, N') = N_x \pm N_n; \quad N_x = a_{\xi i}^+ a_{\xi i}, \text{etc.,} \quad \text{...(30a)}$$

$$N'.N^n = a_{\xi i}^+ a_{\eta i}^+ \pm a_{\eta i}^+ a_{\bar{\xi} i}^+ \quad \text{...(30b)}$$

$$A_x = a_{\xi i}, a_{\bar{\eta} i}; \quad A_x^+ = a_{\xi i}^+ a_{\eta i}^+ \quad \text{...(31a)}$$

and

$$(A, A') = A_x \pm A_n; \quad A' = 2a_{\xi i} a_{\eta i} \quad \text{...(31b)}$$

Their algebras tend to get decoupled from one another if the role of the operator $N_a$ (which is primarily responsible for such coupling) can be recognized. Its eventual neglect can be justified on the ground that it corresponds physically to 20-type excitations involving totally antisymmetric states (with at least $N = 2$) which are inhibited from coupling to the more familiar baryon states (of 56 and 70 types) by strong selection rules based on the twin effects of "wrong" symmetry and angular momentum barriers. The actual calculations of their effect on the qqq problem are much too complex to be given here, and interested reader is referred to a recent publication for the details. The results on the non-strange baryon spectra, all the way to $N = 6$, with the same parameters $(29a, b)$ as employed for $q\bar{q}$ spectra, are given in Table V. In this table, comparison with the data has again

**Table V**

_Baryon mass spectra: theory vs. experiment_

<table>
<thead>
<tr>
<th>State</th>
<th>NJLS</th>
<th>$F_B$ (expt)</th>
<th>$F_B$ (th) = $N + 3$</th>
<th>M(Th) (MeV.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$ (938)</td>
<td>56 : 0 1/2 0 1/2</td>
<td>2.972</td>
<td>3</td>
<td>944</td>
</tr>
<tr>
<td>$\Delta$ (1236)</td>
<td>56 : 0 3/2 0 3/2</td>
<td>2.819</td>
<td>3</td>
<td>1264</td>
</tr>
<tr>
<td>$D_{13}$ (1520)</td>
<td>70 : 1 3/2 1 1/2</td>
<td>3.979</td>
<td>4</td>
<td>1524</td>
</tr>
<tr>
<td>$P'_{11}$ (1440)</td>
<td>56 &amp; 70 : 2 1/2 0 1/2*</td>
<td>5.045</td>
<td>5</td>
<td>1432</td>
</tr>
<tr>
<td>$F_{21}$ (1600)</td>
<td>56 &amp; 70 : 2 3/2 0 3/2+</td>
<td>4.148</td>
<td>5</td>
<td>1737</td>
</tr>
<tr>
<td>$F_{18}$ (1680)</td>
<td>56 : 2 5/2 2 1/2</td>
<td>5.012</td>
<td>5</td>
<td>1678</td>
</tr>
<tr>
<td>$P'_{11}$ (1710)</td>
<td>56 &amp; 70 : 2 1/2 0 1/2*</td>
<td>4.955</td>
<td>5</td>
<td>1718</td>
</tr>
<tr>
<td>$F_{27}$ (1950)</td>
<td>56 : 2 7/2 3/2</td>
<td>5.028</td>
<td>5</td>
<td>1945</td>
</tr>
<tr>
<td>$F_{17}$ (1990)</td>
<td>70 : 2 7/2 2 3/2</td>
<td>5.240</td>
<td>5</td>
<td>1950</td>
</tr>
<tr>
<td>$G_{17}$ (2150)</td>
<td>70 : 3 7/2 3 1/2</td>
<td>6.368</td>
<td>6</td>
<td>2128</td>
</tr>
<tr>
<td>$G_{19}$ (2250)</td>
<td>70 : 3 9/2 3 3/2</td>
<td>5.999</td>
<td>6</td>
<td>2250</td>
</tr>
<tr>
<td>$H_{19}$ (2250)</td>
<td>56 : 4 9/2 4 1/2</td>
<td>6.646</td>
<td>7</td>
<td>2304</td>
</tr>
<tr>
<td>$H_{311}$ (2420)</td>
<td>56 : 4 11/2 4 3/2</td>
<td>6.828</td>
<td>7</td>
<td>2447</td>
</tr>
<tr>
<td>$I_{111}$ (2600)</td>
<td>70 : 5 11/2 5 1/2</td>
<td>8.079</td>
<td>8</td>
<td>2587</td>
</tr>
<tr>
<td>$K_{510}$ (2950)</td>
<td>56 : 6 13/2 6 3/2</td>
<td>8.846</td>
<td>9</td>
<td>2973</td>
</tr>
</tbody>
</table>

* Mixing effect included.
* Mixing effect not included.
been given after inclusion of the strong coulomb effects, as in the \( q\bar{q} \) case, but only in a perturbative fashion (since only light quarks are involved here). The comparison in Table 5 is depicted in two different ways: (i) the results for \( F_B \) in terms of the experimental masses versus "expected" values of \( N + 3 \); (ii) direct inversion of \( F_B = N + 3 \) for an explicit prediction of \( M \) within this model. Both comparisons show a good fit to the data with no extra parameters beyond the common inputs (29a & b) for both \( q\bar{q} \) and \( qqq \) systems, except in the region of \( N = 2 \) which is known to be strongly affected by various mixing possibilities. Since the present model "allows" for mixing, the latter is illustrated by the results for \( P_{11}^{'} \) and \( P_{11}^{''} \) before and after the mixing effects, predicted by the model, are taken into account. (The fits after leave little to be desired). The calculations for unequal mass quarks (strange baryons, etc.) are currently in progress.

\( q^2\bar{q}^2 \) States

For completeness we include a few comments on the predictive powers of this model in respect of the so-called "baryonium" states as \( q^2\bar{q}^2 \) systems, again with no free parameters beyond the basic constants 29a, b.). The history of such states which were quite fashionable in the late seventies was unfortunately marred by premature experimental data which subsequently disappeared, despite theoretical attempts to "explain" them in terms of certain "instant" models with inadequate theoretical basis. [In particular, one of the "issues" had been the allegedly observed sharp widths of such states which had provoked those theoretical attempts]. The width question was addressed in terms of the BS model in the initial stages of its formulation which still needed a few loose ends to be tightened (especially the question of zero point energies) through additional assumptions. Even at that stage, the main conclusion, namely, the ground state is sufficiently high (\( > 2 \text{GeV} \)) to make the decay products "fall apart with large widths (without angular momentum barriers) was qualitative enough to rule out any narrow state apriori, without further refinements. The subsequent refinements of the model both in terms of mathematical (NPA) formulation as well as better physical inputs narrated in the foregoing, warranted a fresh, more quantitative, examination of \( q^2\bar{q}^2 \) states with no extra assumption or parametrization beyond those listed in this section. There typical mass predictions (\( u\bar{u}^2, s\bar{s}^2, c\bar{c}^2 \)) for zero orbital excitation, involving equal mass kinematics for simplicity, using the parameters (\( W_0, C_0, A_0 \)), are given in Table VI for two types of admissible color states.

These have been termed in the literature as "true" (T) and "mock" (M), corresponding to the color representations (3* and 6) for the \( q^2 \) pairs and the conjugate representations (3 and 6*) for the \( \bar{q}^2 \) pairs respectively. Note that M-states lie as much as \( 800 \text{ MeV} \) higher than the corresponding T-states, which are themselves as high as \( \sim 2 \text{Gev} \), quantitative accord with earlier findings.

Most of these states will have large widths, except perhaps the \( s\bar{s}^2 \) state at 2.22 GeV whose peculiar selection rules should make it narrow, thus qualifying it as a viable candidate for the observed \( \xi (2.22) \) state.
**Table VI**

*Mass-spectra of $q^3\bar{q}^3$ system: (GeV)*

<table>
<thead>
<tr>
<th>Flavour</th>
<th>$T$-States $O^+$</th>
<th>$(Jp)$ $I^+$</th>
<th>$2^+$</th>
<th>$M$-State $O^+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u^3\bar{u}^3$</td>
<td>1.88</td>
<td>1.97</td>
<td>2.11</td>
<td>289</td>
</tr>
<tr>
<td>$s^3\bar{s}^3$</td>
<td>2.18</td>
<td>2.25</td>
<td>2.36</td>
<td>3.19</td>
</tr>
<tr>
<td>$c^3\bar{c}^3$</td>
<td>6.72</td>
<td>6.73</td>
<td>6.75</td>
<td>7.79</td>
</tr>
</tbody>
</table>

("'True'") $T : q^3 (3^*)$ and $\bar{q}^3 (3)$.  
("'Mock'") $M : q^3 (6)$ and $\bar{q}^3 (6^*)$.

A more interesting possibility emerges from the high mass prediction for the $M$-state, approaching almost the $J/\psi$ mass value (even without a $c$-quark!) solely as a result of the 4-quark dynamics of $uds$ quarks. And this concerns the possibility of identifying it with the narrow $U(3.1)$ state reported by at least two groups. Indeed this question was recently examined by Ono et al\(^{55}\) in the context of the above BS-model predictions and using the corresponding BS-wave functions.\(^{4251}\) It has been found that not only is the mass position of the $U(3.1)$ state consistent with 4-quark BS dynamics\(^{51}\), but that the estimates of its actual width, using the available decay channels (consistent with the necessary selection rules for $M$-state transitions) do suggest that it should be narrow.\(^{55}\) The details may be found in ref \(^{55}\).

**Transition Amplitudes at Low Energy**

The mass spectral predictions described above, (important as these are) constitute but a first test of this two-tier model that has been achieved through the 4-dimensional (NPA) reduction of the BSE which suppresses the virtual $q\bar{q}$ effects. The fact that good agreement with the data has been obtained without their inclusion strongly suggests that the mass spectra are insensitive to these effects. It is therefore reasonable to suppose that these open up gently enough to justify a perturbative approach to test their dynamical consequences through their appearance in the 4-dim. BS amplitude $\overline{\Omega}$, or equivalently the normalized hadron-quark vertex function $\Gamma (q)$, eq. (4.23). (Note that the normalization constant $N_H$ will depend on the from of the 3-dim wave function—$\phi_0, \phi_2, \phi_0$—employed for the purpose). For a systematic analysis of the virtual $q\bar{q}$ effects (higher Fock states), on different types of hadronic transition amplitudes, it is convenient to classify them in ascending order on the energy scale, as well as different flavour sectors. A sample list is the following ($h = q\bar{q}, qqq; H = Q\overline{Q}$):

- mass spectra  
- Form Factors (e.m. and pionic)  
- Transition Amplitudes ($h \rightarrow h^* + h^*, h^* + \gamma,...$)
In this section we shall discuss the results for a few "low energy" transition amplitudes involving the electroweak couplings of the pion to illustrate the possibilities of the second stage of the two-tier model. Apart from this general motivation, the predictions of this composite pion model with no other parameters than \((\omega_0, C_\theta, A_0)\), may be useful to keep in mind in the context of the more popular notion of the pion as an elementary (Goldstone?) particle which has been considerably strengthened by the successes of the chiral-anomaly theories. Such theories which are characterized by the dimensional parameter \(f_\pi\) governing the interactions of the pion field, have moreover been found to be consistent with the low energy limit of QCD with testable predictions such as the low energy theorem:

\[
e f_\pi^2 F_{3\pi} = F_\pi \quad \text{(32)}
\]

connecting the amplitudes for \(\pi \rightarrow \bar{\ell} \ell\), \(\pi^0 \rightarrow \gamma\gamma\), and \(\gamma \rightarrow 3\pi\), among other types of predictions. A recent measurement of the \(\gamma \rightarrow 3\pi\) amplitude shows good agreement with this prediction, thus apparently vindicating the Wess-Zumino-Witten (WZW) theory with SU(3) Color. On the other hand the WZW theory in its present "effective" form does not have a mechanism for probing the pion's internal structure which is revealed through. e.g., (i) its from factor at "low" energies, and (ii) its structure and fragmentation functions at "high" energies. Fuller information, all the way from low to high energies, is more naturally contained in the pion's wave function as a \(q\bar{q}\) composite.

The BS model allows us to calculate all the three quantities \(f_\pi\), \(F_\pi\) and \(F_{3\pi}\) explicitly in terms of the theoretical ingredients of Sec. 4.2, using the simplest Feynman diagrams in Fig. 1. For the process \(\pi \rightarrow \bar{\ell} \ell\), Fig. 1(a), the natural choice of the wave function is the "off-shell" form (28), in which both the quark lines \(p_1\) and \(p_2\) are on-shell (The most symmetrical \(q\bar{q}\) configuration) without having to specify the "state" of the pion. The corresponding normalizer is given by

\[
\tilde{N}(\pi) P_+(8\pi 6\delta 6)^{1/2} = \int d^2 q_1 \int_{-1}^{+1} dx [2 q_1^2 + m_e^2 + \frac{1}{4} M_e^2 (1 + x^2)] \tilde{\phi}^3 \quad \text{(33)}
\]

The amplitude \(f_\pi^+\) is defined through

\[
f_\pi^+ P_\mu = \sqrt{3} i (2\pi)^{-4} \int d^4q \text{ Tr } (\Psi^{\mu} \gamma_5)
\]

Fig 1 Diagrams for \(\pi \rightarrow \bar{\ell} \ell\), \(\pi^0 \rightarrow \gamma\gamma\) and \(\gamma \rightarrow 3\pi\) couplings.
which after integration over $dq_-$ in the usual manner\textsuperscript{39} yields

$$f_\pi^+ = 2\sqrt{3} m_\pi \bar{N}^{(-)} N_+ \int d^2 q_1 \int_{-1}^{+1} d x \bar{\phi}_0$$

leading, after necessary substitutions, to

$$f_\pi^+ = \sqrt{2} f_\pi = 157 \text{ MeV} \ (\text{Vs} \ 135 \text{ MeV})^{31} \quad \ldots(34)$$

Next, the invariant amplitude for $\pi^0 \rightarrow \gamma \gamma$, Fig 1. (b, c), is given by\textsuperscript{47}

$$A(\pi^0 \gamma \gamma) = 6^{-1/2} e^2 Tr \int d^4 q [\bar{\Psi}(p_1, p_2) i \gamma \cdot \epsilon^{(1)} \Sigma(q - Q) i \gamma \cdot \epsilon^{(2)} + (1 \leftrightarrow 2)],$$

where $Q = k_1 - k_2$, and the second term interchanges the two photons. The quantity $F_\pi$ is defined through

$$A(\pi^0 \gamma \gamma) \equiv F_\pi \epsilon_{\mu \nu \rho \sigma} \epsilon_\mu^{(1)} \epsilon_\nu^{(2)} P_\rho Q_\sigma$$

and works out as

$$F_\pi = \frac{e^2 N_\pi^{(-)}}{\sqrt{6}} \int d^4 q \frac{4m_\pi D_\pm \phi_0}{\Delta_1 \Delta_2} \frac{1}{2\pi i} \left[ \frac{1}{\Delta_3^+} + \frac{1}{\Delta_3^-} \right]$$

and

$$\Delta_1, \Delta_2 = m_\pi^2 + p_{1,2}^2; \quad \Delta_3^\pm = m_\pi^2 + (q \mp Q)^2.$$  

This gives after necessary integrations, the result $F_\pi = 25.64 \text{ MeV}^{-1}$ corresponding to a $\pi^0 \rightarrow \gamma \gamma$ width\textsuperscript{47}

$$\Gamma(\pi^0 \gamma \gamma) = F_\pi^2 M^3 / 4\pi = 8.03 \text{ eV} \ (\text{Vs} \ 7.9 \text{ eV}) \quad \ldots(35)$$

Finally for the $\gamma \rightarrow 3\pi$ coupling there are six diagrams three of which are shown in Figs. 1(d, e, f) and the other three are obtained with the loop lines reversed in direction. The calculational details are given in ref [47] for the $\gamma \rightarrow 3\pi$ amplitude.

$$A(\gamma 3\pi) = F_{3\pi} \epsilon_{\mu \nu \rho \sigma} \epsilon_\mu \epsilon_\nu P_\rho P_\sigma$$

The coupling constant $F_{3\pi}$ works out as\textsuperscript{47}

$$F_{3\pi} = \frac{64}{27} e \left( \frac{\pi}{\beta^2} \right)^{3/4} \frac{m_\pi}{\sqrt{3}} \left[ -\frac{1}{4} M^2 \beta^{-2} \right] \left( m_\pi^2 + \frac{3}{2} \beta^2 + \frac{1}{4} M^2 \right)^{-3/2}$$

$$= 12.69 \text{ GeV}^{-3} \ (\text{Vs} \ 13 \pm .9 \pm 1.3).^{57} \quad \ldots(36)$$
One thus finds the results on the three basic quantities all in good agreement with the data, and thus \textit{a fortiori}, with the low energy theorem, eq (32). What conclusion if any can be drawn from compatibility of this $q\bar{q}$ composite model with those of the WZW theory? Indeed, the proximity of the theoretical prediction for the pion-mass (163.1 MeV) to the observed mass would lead one to suspect that our $q\bar{q}$ pion has a very "tight" structure, not far removed from a point pion ($\beta^2 \to \infty$) whose mass is "shifted" (by Chiral Symmetry breaking) in accordance with the Nambu-JonaLasino mechanism.\textsuperscript{49} A crude comparison of our $q\bar{q}$ equation for the pion suggests a consistency with the NJ-type mass prediction in the limit $\beta^2 \to \infty$. More importantly, the calculation of the pion form factor in this model give a rather small radius:\textsuperscript{58}

$$ \langle r_2^2 \rangle^{1/2} = 0.45 F_\pi \ (\text{Vs} \ 0.66 F_\pi) \quad \text{(37)}$$

Other transition amplitudes involving moderate energies which have been calculated with the "half-off-shell" pion wave function given by $\phi_2$ in eq. (27) include the following:\textsuperscript{58}

\begin{itemize}
  \item \textit{hadronic}: $\rho \to \pi\pi, f_0 \to \pi\pi, A_2 \to \rho\pi, B \to \omega\pi$
  \item \textit{c.m.}: $\omega \to \pi^0\gamma, A_2 \to \pi\gamma$
\end{itemize}

The results, most of which are in excellent agreement with data, provide strong experimental support for the applicability of this formalism at least up to moderate energies.

\textit{High Energy Tests: Structure and Fragmentation Functions}

Further extrapolation on the energy scale involves the application of the model to truly high energy processes such as structure ($F_h$) and fragmentation ($D_h$) functions of $q\bar{q}$ hadrons, with which we shall end this discussion on transition amplitudes. These two quantities are intimately related, as is brought out by Feynman's intuitive parton picture\textsuperscript{59} when applied to deep inelastic $e^-e^-$ scattering on the hadron on the one hand, and single inclusive production of that hadron in $e^+e^-$ annihilation on the other. In a more formal way, the simplest manifestation of this connection is known through the famous Gribov-Lipatov result\textsuperscript{60} obtained in perturbative QED, viz.

$$ \frac{1}{x} D_h \left( \frac{1}{x}, \xi \right) \leftrightarrow F_h(x, \xi) \quad \text{(38)}$$

but its validity in QCD is \textit{a priori} limited by unknown non-perturbative effects only to $x$-values near the maximum ($x \ll 1$) where it is possible to argue that the process is dominated by the lowest order (tree) diagrams (which obey crossing). Therefore a relationship like (38) has been ignored in favour of empirical probability distributions [$q(x)$] of the quark-partons to represent such quantities, subject to appropriate sum rules.\textsuperscript{59} So far the effect of QCD on these quantities has been restricted to its perturbative aspects (scaling violations), but it would be more interesting to ask if at least its gross non-perturbative features can be identified.
Fig 2 (a) Structure function of pion through a hypothetical Drell-Yan process; (b) Pion fragmentation function through $e^+e^-$ annihilation diagram.

and harnessed for throwing more light on such basic connections as (38), even if it means a concrete model for working out the quantitative consequences.

The two-tir BS formalism presented here provides precisely such a model candidate for a unified treatment of both the processes, through the key ingredient of the $Hq\bar{q}$ vertex function (22). For simplicity we shall illustrate only the pion case whose structure function may be deduced from a hypothetical Drell-Yan process $^{61}$ $\pi\pi \rightarrow \mu\beta X (X = q\bar{q})$ shown in Fig. 2(a), while its fragmentation function may be pictured through the process $e^+e^- \rightarrow \pi q\bar{q}$ where the leading quark ejects the pion as in Fig. 2(b). In each of these diagrams the key role is played by the $q\bar{q}$ vertex function (incorporating all the non-perturbative effects), which is "factored out" of the rest of the (otherwise perturbative looking) diagram. Thus a contact with the data on the corresponding processes will provide a direct observational test of this vertex function, and in particular about the "half-off-shell" form $\phi_2$ of the 3-dimensional wave function, eq. (27), which is (topologically) the right choice for the two processes under study.

The pion structure function $F_2(X)$ may be extracted from the D.Y. cross section for Fig. 2(a):$^{61}$

$$\frac{d^2\sigma^{(1)}}{dx dx'} = \frac{4\pi\alpha^2}{q_s} e^x F_2(x) F_2(x')$$

...(39)
(in which the #1 quark for the upper pion participates in the $\mu\bar{\mu}$ production while the #2 escapes in the wilderness). In the present model the $F_2(x)$ works out as$^{45}$

$$F_2(x) = \left( \frac{x}{1 - x} \right) \int \frac{W_1^2}{\Delta_i^2} \frac{1}{\sqrt{2}} d^2k_{2i} \left( \frac{2\pi}{3} \right)^{-3} \left( \Delta_1 + xM^2 \right)$$

where $k_{2+} = P_+^+ (1 - x)$ and $W_1$ is defined through (26). The result which is shown Fig. 3, together with the data of the NA3-collaboration$^{62}$ shows a nearly $(1 - x)^2$ behaviour for large $x$, and a distribution strongly peaked at $x \approx 0.3$, (even with our valence quark description). It follows the experimental data quite well beyond the domain of the sea effects ($x \gtrsim 0.2$), without the use of any adjustable parameters or additional assumptions. The details may be found in ref. (44).

The fragmentation function, which may be evaluated from Fig. 2(b), involves similar techniques but heavier algebra.$^{46}$ Taking $x$ as the fraction of the longitudinal momentum carried by the hadron ($P_+$), and $x_2, x_1$ as the corresponding quantities for the “same-side” quark ($k_{2\pi}$) and away-side quark ($k_{1\pi}$) respectively, the experimental quantity of interest$^{63}$ is the “scaled” cross section for pion production which is expressible in the present model as$^{46}$

$$\frac{s}{\beta} \frac{d\sigma}{dx} = 780 \cdot 24\pi a^2 \tilde{N}_H^2 \int_0^{1-x} dx_2 \frac{x_2^2}{x^2} \left[ 1 + \frac{x_1^2}{(x + x_2)^2} \right] (e_1^2 + e_2^2).$$

$$x \left( 2\beta^2 + m_\pi^2 \frac{x}{x} \right) \exp \left[ \frac{1}{2\beta^2} \left( \frac{1}{2} - \frac{x_2}{x} \right) \left( M^2 - 2m_\pi^2 \frac{x}{x_2} \right) \right]$$

$$\ldots (40)$$

where $\tilde{N}_H$ is the reduced normalization constant defined by

$$\tilde{N}_{H} = (2\pi)^{-3/2} P_+^{-1} P_{-1}^{-1} \tilde{N}_H$$

$$\ldots (41)$$

![Graph showing comparison of pion structure function $F(X)$ with data for $x \geq 0.2$.](image)
appropriate to the wave function $\phi_2$, eq. (27). A similar, but more complicated expression holds for the kaon case. Fig. 4 which gives the comparison with the data of Derrick et al.$^{63}$ for the pion and kaon fragmentation functions shows that the theoretical curves follow the data quite clearly even down to small $x$-values, without and adjustable parameters whatsoever. The small deviations (under estimation of the data) for low $x$ is not unexpected in this (highly simplified) description of the entire phenomena, which neglects non-leading quark effects. The details may be found in ref (45) and in a more recent write up$^{64}$ which includes the production of other $q\bar{q}$ hadrons ($\rho$, $K^*$, $\eta$, $\phi$ etc.).

**BEYOND QUARKS. WHAT?**

In terms of the theoretical picture of today, quarks and leptons are regarded as the closest candidates for "elementary particle" status, up to a distance $\lesssim 10^{-16}$cm representing the probing limits of modern experimental technology. However, the rapid proliferation of both these varieties (reaching the halfcentury mark without supersymmetry (SuSY) and the century mark with supersymmetry) has understandably caused strong speculation about a "fourth generation" of particle structure in the making. This is despite the influence of certain "all-time" theories which, by their very nature, must decide once for all in favour of "elementarity" of particles at a certain preassigned level and "refuse" to entertain any further possibilities of compositeness in their own structures. Such a decision must be made in advance, whether at the level of quark-leptons or even of "preons" but once made, the frameworks of these theories are such as to render irrelevant the very questions of further substructures of these building blocks. This kind of philosophy may be illustrated by the Bootstrap theories of the early sixties$^{65}$ wherein all strongly interacting particles were postulated to be bound states of all others ("nuclear democracy"), something akin to the Mach Principle. In more modern times a similar philosophy is discernible in GUT's$^{66}$ and more especially in string and superstring field theories,$^{12}$ wherein the theoretical ques-
tion of their internal self-consistency is so overwhelmingly dominant as to render irrelevant the parallel need for contact with data.

The alternative point of view of successive substructures, on the other hand, is not only in accord with History (which has already seen three such generations), but is much less rigid in the conceptual formulation, leaving enough scope for further substructures, if need be, in response to subsequent observations. In principle one could even conceive of an unending process of successive shell structures (much like the peeling of the onion skins), but for the limitations imposed by experimental technology of the day which puts effective "brakes" on such speculations beyond observational feasibility. At the present state of the experimental art, the window just seems to be opening up for a "peep" into a possible fourth stage of elementarity, though it is premature to make a firm judgement on this score, except for support from the general lessons of history. In other words, the observational signatures of quark-lepton compositeness are much too few and much too thin at this stage for basing such conclusion on experimental facts alone. Yet certain broad spectrum features of the observed quark-lepton families are perhaps not incompatible with a composite picture. These are the similarities in the characteristics of their successive "recurrences" as depicted by the following series

\[ (e_v e_d u); (\mu, \nu, \tau); (\tau \nu, b t (?)) \]

... (42)

reminiscent of the early multiplet classifications of mesons and baryons which had lead to the eventual quark picture. These groupings which have been variously termed as successive generations, horizontal symmetry etc. have led to group theoretical assignments of different types, such as SU(4) of Pati-Salam and SU(5) of Georgi-Glashow among others, and inevitably composite models carrying these and allied group signatures have emerged, all with the common feature that the proton would no longer enjoy the status of a "stable" particle, as hitherto believed, but rather would decay into lighter hadrons (pions) and leptons (e, u,...). This paradoxical result follows from the circumstance that such energetically favourable decays are no longer forbidden by selections rules preventing transitions between quark and lepton species, since both now have a common underlying substructure. On the other hand, the failure of the proton to decay so far within observable time limits has helped rule out certain types of models, leaving fewer ones in the fray, and putting strong restrictions on their actual dynamical formulations.

The Harari-Shupe Model

As an illustrative example of one of the "admissible" models of composite quarks and leptons, it is useful to consider a simple model which has shown the capacity to incorporate many of the observed quark-lepton patterns within its basic tenets in a very natural way. This is the two-component (t, V) preon model of Harari-Shupe,71 whose SU(2) structure (charge content \( \frac{1}{2}, 0 \)) helps it satisfy the so-called 't Hoof anomalies trivially (an important self-consistency requirement
for a composite model), and gives a simple 3-preon characterization to each quark-lepton quartet as

\[ e^+ = ttt, \; ve = VVV; \; u = tvV; \; \bar{d} = VVt. \]  \hspace{1cm} \ldots(43) \]

and similarly for the higher sets in (42). As for the color attribute, the original effort\textsuperscript{71} was to simulate this property combinatorially through constructions based on permutation (S\(_3\)) symmetry which prima facie yield color-signlet (unique) assignments to \( e \) and \( v \), \( 3 \) to \( u \) and \( 3^* \) to \( \bar{d} \). Taken literally this would imply that conservation of color would have to be contingent on an exact satisfaction of certain formal S\(_3\)-symmetry requirements on the wave functions of 3 identical \((t, V)\) preons in a broad quantum mechanical sense. Such a requirement need not, in principle, be incompatible with the more accepted principle of color-gauge invariance at the quark-gluon level, if it is remembered that the operation of the gauge principle at "composite" particle levels (quarks and gluons are now composites of preons) cannot be directly compared with the corresponding statement for "elementary" quark interactions with the gauge field of (elementary) gluons. In other words the operation of the usual QCD gauge principle has to be viewed in the larger perspective of composite dynamics not only for the fermions (quarks) but also for the field (gluons are now \( t \bar{t} \) and/or \( VV \) composites). The QED gauge invariance in this model would still be governed by an elementary photon field but with composite electrons. The question of compatibility of S\(_3\)-symmetry at the composite level with usual color gauge invariance was not pursued further by the originators of the preon model reverting\textsuperscript{73} to the conventional gauge picture at the cost of a three-fold increase (color triplets) in the number of preon constituents. However, this attractive possibility deserves a closer look.

For a dynamical understanding of the detailed structure of the successive families listed in (42), within the Harari-Shupe model,\textsuperscript{71} it is first necessary to identify their main observational features as follows:

(A) Spin-\( \frac{1}{2} \) for all the numbers (without exception) with no trace of spin \( \frac{3}{2} \) — a most unusual coincidence for a 3-body composite made up to only \( \frac{1}{2} \)—spin constituents.

(B) Relative stability of successive generations against mutual transitions (e.g., \( \mu \rightarrow e\gamma \)) which presumably calls for a rather special kind of quantum number to characterize them.

(C) Steeply rising mass pattern of the successive generations. A possible mechanism\textsuperscript{74} to deal with these features is outlined in the next section.

*A Dynamical Basis for the H-S Model*\textsuperscript{74}

By analogy with quark-gluon confinement, it is natural to suppose that preons are confined by "hyper-color" interactions. Since little is known about them, only an effective description is possible through a suitable ansatz on the
overall spin and momentum dependence of the preon interaction kernel, say, within a BSE type of framework. In other words, hypercolor as an attribute must be a "hidden" degree of freedom and manifests only through the effective preonic interactions for which a direct three-preon force was suggested as a possible alternative scenario to the beaten track of a 2-body type interaction. Within the 3-body kernel, a spin-dependence of the form\textsuperscript{74}

\[
\Sigma^a = \sigma^{(1)}_\mu \sigma^{(2)}_{\nu\lambda} \sigma^{(3)}_{\lambda\mu}
\]

was suggested as a possible mechanism to keep out \( S = \frac{2}{3} \) states \( \chi^* \) from the influence of this operator. This is most easily seen from its space-like from \( \sigma_1, \sigma_2 \times \sigma_3 \) which gives zero on a spin \(-3/2\) function \( \chi^* \), thus making such states "force-free" and hence devoid of physical interest. On the other hand, \( \Sigma^a \) converts the two standard spin \(-\frac{1}{2}\) states \( \chi' \) and \( \chi'' \) into each other, so that the complex combinations\textsuperscript{74}

\[
\sqrt{2} \chi^\pm = \chi^* \pm i\chi'
\]

of spin \(-\frac{1}{2}\) are eigenstates of \( \Sigma^a \):

\[
\sigma_1, \sigma_2 \times \sigma_3 (\chi^\pm; \chi^*) = \pm 2 \sqrt{3} (\chi^\pm; 0)
\]

How can such an effective \( \Sigma^a \)-structure arise in a field theoretic model? It was suggested\textsuperscript{74} that an \textit{intrinsically} antisymmetric hypergluon field \( F_{\mu\nu} = -F_{\nu\mu} \) can couple to a preon field \( \psi \) in the Pauli form \( \psi \sigma_{\mu\nu} F_{\mu\nu} \psi \). And if the \( F_{\mu\nu} \) field has a self-coupling of the form \( F_{\mu\nu} F_{\lambda\sigma} F_{\lambda\mu} \), these two couplings together can generate a \textit{three way}, Mercedes-Benz type diagram for a direct 3-preon interaction, as in Figs. 5(a, b).

While eq. (5.5) account quite qualitatively for feature (A) on the absence of \( S = \frac{3}{2} \) states, the \( \Sigma^a \)-dynamics also bear on the other two features (A) and (B), though in a less obvious way. Qualitatively, a harmonic spring force simultaneously in each of the hypergluon lines produce a strongly confining effect with a very steeply rising "potential". This provides the basis for a steeply rising mass spectrum for the successive generations as listed in item (C). Item (B) is more subtle and depends on a certain form of the 3-preon BSE. It may be shown that under certain

Fig 5 A 3-way hypergluonic mechanisms\textsuperscript{74} for \textit{ttt} and \textit{ttt} couplings in the Harari-Shup model.\textsuperscript{71}
general conditions this BSE for almost massless preons may be cast in the form of a six-dimensional Schrödinger-like equation in \((\xi, \eta)\) where the roles of the coordinates and momenta are interchanged

\[(\Delta_{\xi} + \Delta_{\eta} - \alpha \rho^{\frac{1}{2}} + b) \theta(\xi, \eta) = 0 \quad \ldots(47)\]

where \(\rho^2 = \xi^2 + \eta^2\) and \(\theta\) is a spatial function of the appropriate \(S_3\)-symmetry associated with the spin-\(\frac{1}{2}\) functions \((\lambda', \lambda'')\) to produce the correct representation of \(S_3\) in the combined space of spin and orbital degrees of freedom. It is easily seen that (47) corresponds to an \(R^6\) confinement after due conversion to the co-ordinate representation. As to the item (B), the general structure of eq. (47) offers the following insight into this question. The spatial function \(\theta(\xi, \eta)\) for orbitally unexcited states \((L = 0)\) can be expressed in terms of 3 independent scalars:

\[\xi^2 + \eta^2 = \rho^2, \quad \xi^2 - \eta^2 = -\gamma \cos \lambda, \quad 2\xi \eta = \gamma \sin \lambda, \quad \ldots(48)\]

Of these, the variables \((\rho, \gamma^2)\) are carriers of full \([3]\) symmetry, while \(\lambda\) which is akin to an "angle" (not Euler angles) carries the information on the mixed \([2, 1]\) symmetry content in the spatial wave function. This last is expressed in terms of the eigenvalues and eigenfunction of the operator \(-i \frac{\partial}{\partial \lambda}\):

\[-i \frac{\partial}{\partial \lambda} \theta_N = \frac{N}{2} \theta_N ; \quad \theta_N \sim \gamma^{N/2} \exp \left( \pm \frac{i}{2} N\lambda \right) \quad \ldots(49)\]

where \(N\) can be one of the three possible forms

\[N = 3n, \quad 3n \pm 1; \quad (n = 0, 1, 2, \ldots) \quad \ldots(50)\]

Eq. (49) also expresses the associated \(\gamma\)-dependence of \(\theta_N\) (analogous to centrifugal barrier effects in \(L\)-excitation) which suggests a singular behaviour for \(N = -1\), but which is certainly "allowed" for a wave function in six-dimensional space.

In this picture therefore, the 3 classes of \(N\) are sought to be identified with the three horizontal generations whose lowest configurations are the 3 sets given in (43) and correspond to \(n = 0\), i.e., \(N = -1, 0, +1\) respectively. The higher values of \(n\) correspond to "vertical" excitations of these 3 horizontal generations. Other kinds vertical excitations for each horizontal generation are:

(i) Radially excited states (\(\rho\)-dependent) and (ii) orbitally excited states of \(L > 0\) (dependent on 3 Euler angles). Such a proliferation of vertical excitations for each horizontal generation is a necessary consequence of any composite model whose eventual success if any must depend on the details of dynamics, an essential condition being that the first (vertically) excited state of the first horizontal \((N = -1)\) species, viz., \(e^+\), should lie well above the ground state of the third horizontal species \((N' = +1)\), viz., \(\tau^+\). In between these two, the middle species has a ground state \(N = 0\), corresponding to \(\mu^+\).

Note the qualitative difference in the highly isotropic structure of the \(\mu^+\) state \((N = 0)\) from those of the \(e^+\) and \(\tau^+\) states \((N = \mp 1)\) which have mixed symmetry.
and are akin to each other. This could be a qualitative reason for the lack of radiative transition between \( \mu^\pm \) and \( e^\pm \). The corresponding selection rules need not be as rigid for radiative transition between \( \tau^\pm \) and \( e^\pm \), which may nevertheless be suppressed for other reasons (e.g., large energy denominators). For completeness, a more quantitative statement on the steeply rising mass spectrum (item C) which, as already noted, is a general consequence of the three-way confining mechanism, Fig. 5, is possible on the basis of the general eq. (47) which effectively incorporates this 3-way mechanism. The mass spectral prediction of eq. (47) for the lepton series was found to be expressible in units of the electron mass as

\[
\frac{M}{m_e} = \left( \frac{\beta_r}{\beta_0} \right)^{21},
\]

where \( \beta_r (r = 0, 1, 2, \ldots) \) is a function of \( N \) and the \( r \)th zero of the Airy function. The steeply rising nature of the kernel is reflected in the huge power dependence of \( M \) which, e.g., predicts

\[
\begin{align*}
M_\mu/m_e &= 196.7 \text{ (vs 206.7).} \\
M/m_\mu &= 14.3 \text{ (vs 16.9).}
\end{align*}
\]

Very similar results also hold for the other members of the quark-lepton families. The difference between the force mechanisms involved in the lepton VS quark structures is schematically indicated by the comparison of Fig. 5(a) (with 3 equal string lengths) for leptons \((e, \nu)\) and Fig. 5(b) (with only 2 equal string lengths) for quarks \((u, \bar{d})\).

The foregoing is only an illustration of the nature of the "issues" involved in attempting a dynamical understanding of the central features of the quark-lepton families within a composite description. The H-S model chosen in this context is merely one of many possible models of a certain general kind in which the constituents (preons) are recognized to have an existence independent of the attributes (color, flavour, generation index) they are supposed to possess. in contrast to another class of models in which the preons are considered to be almost a "paraphrase" for these very attributes. (The dynamical scenario, too, is very different for this class of models).

**Conclusion**

Physics has come a long way since quarks were first postulated as basic building blocks of matter nearly 25 years ago. Quarks have never been "detected" as such, but their color and flavour signatures have been firmly established. Their "invisibility" has been turned around as a signature of a relatively new principle in physics, viz., "confinement", a property expected to be possessed by certain types of (non-abelian) gauge fields, much like Renormalizability is a property (albeit more universal) which should be possessed by all "good" field theories. In association with leptons, quarks are the nearest candidates for elementary particle status, whose interactions (strong and electroweak) are believed to be governed by the *Gauge Principle*. Unfortunately, QCD has not yet established confinement within its
own framework, causing this principle to be more "believed" than (formally) "proved".

This circumstance has (inadvertently) lent a degree of pseudo (?) respectability to "effective theories" of confinement whose credibility would have to be judged by the depth and range of their correlative power in relation to experiment. Such theories (as distinct from narrow, adhoc models) which are ideally characterised by a modest dynamical framework (incorporating Lorentz and Gauge invariance), and a few "key" constants, have a vital role as "Bridges" between more formal theories of the future and data based models. Their correlative power, if successfully tested by key sectors of the data, also makes them ideally suited as valuable beacons to unexplored regions where their predictions would be unambiguous. This should be especially true for theories which allow arbitrary extrapolation on the energy scale not only for meaningful interpretation of data at high energies but also for reliable guidance on a-priori expectations on such phenomena. One such type of theory has been sketched in this article, together with the results of comparison with data in certain key sectors (from low energy spectroscopy to high energy partons) within a single integrated framework.

At the other extreme one sees evidence of much more ambitious theories of all particles (including quarks !) with the central aim of unification of all types of interactions.$^{12}$ In particular, the aim to unify gravity has required extrapolation all the ways to Planck's Mass ($\sim 10^{19}$ GeV), leading to two distinct kinds of developments. On the theoretical side it had involved a huge programme of self-consistent formulations to the minutest details, opening up new frontiers of mathematical machinery but at the cost of virtual loss of all contact with the data, at least for the foreseeable future.

On the experimental side, the emphasis has been almost entirely towards higher and higher energy accelerator programmes designed to probe matter to smaller and smaller distances. The general expectation of a quark-gluon plasma in nuclear matter when two heavy nuclei collide "head-on" at high enough energy, constitutes the basis of high energy experimental programmes of one kind (heavy ion physics), which has the potential to throw light on the issue of quark confinement.$^{75}$ (So far the experimental outcome is negative, and confinement continues to be an abstract concept). The bulk of experimental programmes, on the other hand, are aimed towards the exploration of new physics in the so-called "Desert" region (above $10^{4}$GeV) including some acid tests (Higgs, top etc.) of the Standard Model predictions. Other programmes (e.g., Searches for SuSy partners) are greatly handicapped by the lack of adequate "theoretical" guidance. And the theories of strings and superstrings which are supposed to be replete with predictions above the TeV region (when ready !), are so much bogged down in the formulative details, that little theoretical guidance is likely to come from that end at the moment.

Within this twin scenario of theoretical and experimental developments of today, Quark physics finds itself at the cross roads after almost 25 years of its
"colourful" existence. In association with Leptons, quarks as elementary particles todate, face as Hobson's choice between two possibilities:

(I) To get subsumed in some "all-time" theory which would render irrelevant all further claims to elementarity for all time to come;

(II) As the third stage of a long process of unfolding of successive shell structures (like onion skins) which may continue *ad infinitum*, subject only to experimental limitations.

The distinction between the two alternatives is not so sharp in practice as might appear in principle, since experimental limitations have the effect of rounding off their contours. The second alternative makes no claims at exactness, as it always leaves open for *future test* the possibility of further levels of compositeness, and by its very nature is designed to stay close to experiment. This first alternative can be *exact* in principle, but *necessarily* with an element of *arbitrariness* in its precincts which may not be easily proved or disproved, since by its very nature it is not designed to stay close to experiment. [The GUT's which were patterned after the alternative I, soon after the success of electroweak unification, had exhibited this kind of arbitrariness in the choice of the group structure and this was reflected in their respective prediction on proton decay]. Alternative II on the other hand, is more modest in its conceptual scope and has the advantage of having History on its side. The preon models which are supposed to represent the fourth stage of elementarity fall in this category, but here again experimental limitations might have already proved formidable against bringing out the detailed facets of compositeness of quark-leptons in any simple and convincing manner, (And the barriers against still further shell explorations will presumably increase exponentially). Nothing short of experimental breakthroughs of major dimensions can possibly hope to bring the two alternatives from philosophical heights to the level of down-to-earth physics.

**References**

1. G Zweig *CERN Rep Non* 8182/TH401 (1963)
4. See e.g S Vand der Meer and C Rubbia *Rev mod Phys* 57 (1985) 689
   A Salam In: *Elementary Practicele Physics* (Nobel Symp No. 8) (Eds Svartholm *et al*)
   Stockholm (1986)
17. E Eichten *et al.* *Phys Rev Lett* **34** (1975) 369; C Quigg and J Rosner *Phys Rept C** 56 (1979) 167
18. E E Salpeter and H A Bethe *Phys Rev* **84** (1951) 1232; M Cell Mann and E F Low *Phys Rev D** 4 (1951) 350
21. L J Reindl H Rubinstein and Syazaki *Phys Rept D** 127 (1985)
23. R Karplus and A Klein *Phys Rev D** 7 (1952) 848
27. A De Rujula H Georgi and S L Glashow *Phys Rev D** 12 (1975) 147
33. P A M Dirac *Rev mod Phys* 21 (1949) 392
34. Sec e g SBrodsky and G P Lepage *Proc Banff Summer Inst Particles and Fields II* (Ed A Copri and A N Karnal) Plenum N Y (1983)
35. I E Tamm *J Phys (USSR)* 9 (1945) 449; S M Dancoff *Phys Rev 78* (1950) 382
40. E E Salpeter *Phys Rev D** 8 (1952) 328
42. A Mittal and A N Mitra *Phys Rev Lett* **57** (1986) 290


50. A N Mitra and A Mittal *Pramana* **22** (1984) 221, *see also* this paper for the references to SO(2), 1) algebra

51. J T Londergan and A N Mitra *Few-Body System* **2** (1987) 55; *see also* this paper for references to some "instant" models.


57. Yu M Antipov *Phys Rev Lett* **56** (1986) 796; *also see* this paper for reference to "1. Low energy Theorm"


60. V N Gribov and L N Lipatov *Sov J nucl Phys* **15** (1972) 675


64. A N Mitra and A Pagnamenta *UIC-Preprint* (1987)


70. For a perspective review of subquark approaches, see e.g H Terazawa *Phys Rev D12* (1980) 184

71. H Harari *Phys Lett* **B86** (1979) 83; M Shupe *ibid* **B86** (1979) 87

72. G't Hoof In: *Recent Developments in Gauge Theories* (Ed: G't Hoof *et al.*) Plenum (1980)

