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COMMENT ON VISCOUS FLOW THROUGH A POROUS MEDIUM PAST AN OSCILLATING PLATE IN A ROTATING SYSTEM

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There is an error in the paper of Kumar and Varshney[1] and a correct solution is given here. Asymptotic solution of the problem is also obtained. It is observed that in the initial stages there is no inertial oscillations while at large time the steady state is reached through decay of inertial oscillations.

Key Words : Viscous Flow; Porous Medium; Oscillating Plate; Rotating System

MATHEMATICAL ANALYSIS

CONSIDER the Equation (19) of Kumar and Varshney[1]

$$\bar{q}(z, p) = \frac{1}{p - i\sigma} \exp\left[-\frac{z}{2}\left\{S + \sqrt{S^2 + 4\left(\frac{1}{k} + \frac{pE}{2} + iE\right)}\right\}\right], \quad \ldots(1)$$

where $\bar{q}(z, p)$ is the Laplace transform of q, defined by

$$\bar{q}(z, p) = \int_0^\infty e^{-pt}\, q(z, t)\, \mathrm{d}t; \quad \ldots(2)$$

S, the suction parameter, k, the permeability parameter; E, the rotation parameter and σ, the frequency parameter.

The inversion of equation (1), given by Kumar and Varshney[1] in equation (20) by inverse Laplace transform as

$$q(z, t) = \exp\left[i\sigma t - \frac{Sz}{2} - z(a_1 + ib_1)\right], \quad \ldots(3)$$

is wrong whereas its correct form is as follows (McLachlan[2]) :

$$q = \frac{1}{2}\left[\exp i\sigma t - \frac{Sz}{2}\right]\left[\exp\,(a_1 + ib_1)\,z \right.$$
$$\times \operatorname{erfc}\left\{\frac{z}{2}\sqrt{\frac{E}{2t}} + (a_1 + ib_1)\sqrt{\frac{2t}{E}}\right\} + \exp\left[-(a_1 + ib_1)\,z\right]$$
$$\left.\times \operatorname{erfc}\left\{\frac{z}{2}\sqrt{\frac{E}{2t}} - (a_1 + ib_1)\sqrt{\frac{2t}{E}}\right\}\right], \quad \ldots(4)$$

where

$$a_1, b_1 = \frac{1}{\sqrt{2}}\left[\left\{\left(\frac{S^2}{4}+\frac{1}{k}\right)^2+\frac{E^2}{4}(\sigma+2)^2\right\}^{1/2}\pm\left(\frac{S^2}{4}+\frac{1}{k}\right)\right]^{1/2} \quad ...(5, 6)$$

This solution represents the general features of the unsteady boundary layer flows in a rotating porous system including the effect of suction or injection according as $S > 0$ or $S < 0$.

Using the asymptotic expansion of the complementary error function with complex argument ζ of the form

$$\text{erfc}(\zeta) \simeq \exp\frac{(\zeta^2)}{\zeta\sqrt{\pi}} \text{ as } |\zeta| \to \infty \quad ...(7)$$

and with the result erfc $(-\zeta) = 2-$erfc (ζ), it turns out that the asymptotic solution for large time has the form

$$\begin{aligned} q(z, t) \simeq & \exp\left\{i\sigma t - \frac{S}{2}z - (a_1 + ib_1)z\right\} - \frac{z}{2}\sqrt{\frac{E}{2\pi t}} \\ & \times \exp\left\{-\frac{S}{2}z - \frac{z^2E}{8t} - \left(\frac{S^2}{2}+\frac{2}{k}\right)\frac{t}{E} - 2it\right\} \\ & \times \left\{2(a_1+ib_1)^2\frac{t}{E} - \frac{z^2E}{8t}\right\}^{-1}. \end{aligned} \quad ...(8)$$

The first term of the solution (8) represents the steady state boundary layer flow having thickness of the order

$$\delta = \left(\frac{S}{2} + a_1\right)^{-1}. \quad ...(9)$$

It is clearly seen from this expression that the boundary layer thickness δ decreases with increase in either suction parameter S or rotation parameter E and increases with increase in permeability parameter k. On the other hand, the most remarkable feature of the asymptotic solution is that the second term of the solution (8) represents the existence of the inertial oscillations of the frequency 2. It is interesting to note that these shear-oscillations persist even when $\sigma = 0$.

In order to study the generation and propagation of the diffused hydrodynamic waves in the rotating fluid, it is necessary to introduce some properties of the complementary error function due to strand.[3] For any complex number $c = x + iy$,

$$\overline{\text{erfc}(c)} = \text{erfc}(\bar{c})$$

and

$$\text{erfc}(c) = \exp(-2ixy)\, g(x, y), \qquad x > 0, y \geqslant 0, \quad ...(10)$$

where $g(x, y)$ is the complex function given by

$$g(x, y) = \sum_{n=0}^{\infty} (xy)^{2n}\{f_n(x) - i(n+1)f_{n+1}(x)\}, \quad ...(11)$$

which tends to zero as $x \to \infty$ and

$$f_{n+1}(x) = \frac{2}{2n+1}\left\{\frac{\exp(-x)^2}{\pi^{1/2}(n+1)!\,x^{2n+1}} - \frac{f_n(x^2)}{n+1}\right\},\ n = 0, 1, 2 \quad ...(12)$$

where

$$f_0(x) = \text{erfc}\,(x).$$

In view of these results, the solution (4) can be put into the form

$$q(z, t) = \frac{1}{2}\exp\left\{i\,\sigma\,t - \frac{S}{2}z - i\,(2+\sigma)\,t\right\}$$
$$\times\left[e^{a_1 z}\,g\left(\frac{z}{2}\sqrt{\frac{E}{2t}} + a_1\sqrt{\frac{2t}{E}},\ b_1\sqrt{\frac{2t}{E}}\right)\right.$$
$$\left.+\ e^{-a_1 z}\,\bar{g}\left(\frac{z}{2}\sqrt{\frac{E}{2t}} - a_1\sqrt{\frac{2t}{E}},\ b_1\sqrt{\frac{2t}{E}}\right)\right], \quad ...(13)$$

when $z > 4a_1t/E$ and $\bar{g}(x, y)$ is the complex conjugate of $g(x, y)$;

$$q(z, t) = \exp\left\{i\sigma t - \frac{S}{2}z - (a_1 + ib_1)\,z\right\}$$
$$+\frac{1}{2}\exp(i\sigma t) - \frac{S}{2}z - i(2+\sigma)\,t\left[e^{a_1 z}\,g\left(\frac{z}{2}\sqrt{\frac{E}{2t}} + a_1\sqrt{\frac{2t}{E}},\right.\right.$$
$$\left.\left. b_1\sqrt{\frac{2t}{E}}\right) - e^{-a_1 z}\,g\left(a_1\sqrt{\frac{2t}{E}} - \frac{z}{2}\sqrt{\frac{E}{2t}},\ b_1\sqrt{\frac{2t}{E}}\right)\right], \quad ...(14)$$

when $z < 4a_1t/E$.

The structure of the terms involving g in (14) shows the generation and propagation of diffused waves travelling outwards from the plate with a velocity equal to $4a_1U/E$. It may clearly be seen from (5) that it increases with increase in S and decreases with increase in k.

References

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3 O N Strand *Math Comp* **19** (1965) 127