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FLEXURAL VIBRATION OF PYROELECTRIC SOLID CIRCULAR CYLINDER OF CRYSTAL CLASS 6

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The flexural and longitudinal wave propagations in pyroelectric solid circular cylinders of bone and ceramic have been considered. The frequency equations have been derived for solid circular cylinder whose surfaces are traction free, completely coated with electrode which are shorted and thermally insulated. Numerical work has been carried out for Barium titanate ceramic. The results are tabulated and the dispersion curves are also presented.

Key Words: Flexural Vibration; Wave Propagation; Pyroelectric Solid Circular Cylinder; Barium Titanate; Ceramic; Crystal Class 6; Bone

Introduction

Pyroelectric materials are used in information storage devices. Pyroelectric polymer (PVF_2) is used in hydrophones[1]. In measuring the thermal diffusivity of solids, high thermal conductivity pyroelectric sensors are used. In delay line and subtract signal processing techniques, pyroelectric vidicons are used. Lang[2] reported the measurement of pyroelectric property in bones and tendons. Pyroelectric responses from sensory organs control the nerve impulse rates[3]. Athenstaedt[4] even measured the pyroelectricity in the grains of wheat and rye. Hence, the study of pyroelectricity will be of much importance to the agriculturists and bio-engineers. Vibrations of piezoelectric solid cylinders are considered by the authors[5,6,7]. Out of 20 piezoelectric crystal classes 10 have pyroelectric property. So the thermal effect on piezoelectric materials has attracted attention of many researchers.

The governing equations of the vibrations of pyroelectric materials are given by Mindlin[8] and Nye[9]. The high frequency equations of crystal plates accounting of coupling of mechanical, electrical and thermal fields are presented by Mindlin[10]. Paul and Renganathan[11] studied the vibration of pyroelectric infinite plate of crystal class (6mm).

In the present analysis, both flexural and longitudinal vibrations of a pyroelectric solid circular cylinder of crystal class 6 and (6mm) are considered. The frequency equation has been derived for solid cylinder whose surfaces are traction free, completely coated with electrode which are shorted and thermally insulated. Since heat conduction coefficients and specific heat capacity of crystal class 6 are not available, we have derived the frequency equations for crystal class (6mm). Material constants are taken for $BaTiO_3$. The roots of the frequency equations are computed numerically using Newton-Raphson method. The numerical results are tabulated and the dispersion curves are also presented.

Method of Analysis

The constitutive equations governing elastic, piezoelectric and thermal behaviour are given by[8]

$$\overline{T} = cS - e^t E - \beta T,$$
$$D = eS + \varepsilon^s E + p^s T$$

and

$$\sigma = \beta^t S + (p^s)^t E + (\rho c_v / T_0) T,$$

where

$$c = \begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{12} & c_{11} & c_{13} & 0 & 0 & 0 \\ c_{13} & c_{13} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{bmatrix} \qquad \beta = \begin{bmatrix} \beta_1 \\ \beta_1 \\ \beta_3 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

in which $c_{66} = (c_{11} - c_{12})/2$

$$e = \begin{bmatrix} 0 & 0 & 0 & e_{14} & e_{15} & 0 \\ 0 & 0 & 0 & e_{15} & -e_{14} & 0 \\ e_{31} & e_{31} & e_{33} & 0 & 0 & 0 \end{bmatrix} \qquad p^s = \begin{bmatrix} 0 \\ 0 \\ p_3 \end{bmatrix} \qquad \varepsilon^s = \begin{bmatrix} \varepsilon_{11} & 0 & 0 \\ 0 & \varepsilon_{11} & 0 \\ 0 & 0 & \varepsilon_{33} \end{bmatrix}$$

and $\overline{T}$, S, D, E, σ, T are stresses, strains, electric displacements, electric fields, entropy and temperature. c_v is the specific heat capacity, T_0 is the reference temperature, ρ is the mass density and c_{ij}, e_{mn}, ε_{kl}, β_j and p_m are elastic, piezoelectric, dielectric, thermal stress coefficients and pyroelectric constants respectively.

The stress components T_{ij}, electric displacements D_m and the entropy σ satisfy the following equations for hexagonal symmetry

$$T_{rr,r} + r^{-1} T_{r\theta,r} + T_{rz,z} + r^{-1}(T_{rr} - T_{\theta\theta}) = \rho u_{,tt}$$
$$T_{\theta z,z} + r^{-1} T_{\theta\theta,\theta} + T_{r\theta,r} + 2r^{-1} T_{r\theta} = \rho v_{,tt}$$
$$T_{zz,z} + r^{-1} T_{\theta z,\theta} + T_{rz,r} + r^{-1} T_{rz} = \rho w_{,tt}$$
$$r^{-1}[rD_{r,r} + D_r + D_{\theta,\theta}] + D_{z,z} = 0$$

and

$$k_{11} \overline{\nabla}^2 T + k_{33} T_{,zz} = T_0 \sigma_{,t},$$

where $\overline{\nabla}^2 = \partial^2/\partial r^2 + r^{-1}\partial/\partial r + r^{-2}\partial^2/\partial\theta^2$.

The equations of motion, Gauss's equation and the entropy equation in cylindrical polar coordinates r, θ, z for class 6 are

$$c_{11}[u_{,rr} + r^{-1}u_{,r} + r^{-2}u] + c_{66}r^{-2}u_{,\theta\theta} + c_{44}u_{,zz} + [c_{66} + c_{12}]r^{-1}v_{,r} - [c_{66} + c_{11}]r^{-2}v_{,\theta}$$
$$+ [c_{44} + c_{13}]w_{,rz} + [e_{31} + e_{15}]V_{,rz} - e_{14}r^{-1}V_{,\theta z} - \beta_1 T_{,r} = \rho u_{,tt}$$
$$[c_{66} + c_{12}]r^{-1}u_{,r\theta} + [c_{66} + c_{11}]r^{-2}u_{,\theta} + c_{66}[v_{,rr} + r^{-1}v_{,r} - r^{-2}v] + c_{11}r^{-2}v_{,\theta\theta} + c_{44}v_{,zz}$$
$$+ [c_{44} + c_{13}]r^{-1}w_{,\theta z} + e_{14}V_{,rz} + [e_{31} + e_{15}]r^{-1}V_{,\theta z} - r^{-1}\beta_1 T_{,\theta} = \rho v_{,tt}$$
$$[c_{44} + c_{13}][u_{,rz} + r^{-1}[u_{,z} + v_{,\theta z}]] + c_{44}[w_{,rr} + r^{-1}w_{,r} + r^{-2}w_{,\theta\theta}] + c_{33}w_{,zz}$$
$$+ e_{15}[V_{,rr} + r^{-1}V_{,r} + r^{-2}V_{,\theta\theta}] + e_{33}V_{,zz} - \beta_3 T_{,z} = \rho w_{,tt} \quad \ldots (1)$$

(*equations contd*)

(*equations contd*)

$$\varepsilon_{11}[V_{,rr}+r^{-1}V_{,r}+r^{-2}V_{,\theta\theta}]+\varepsilon_{33}V_{,zz}-[e_{31}+e_{15}][u_{,rz}+r^{-1}[u_{,z}+v_{,\theta z}]]$$
$$+e_{14}[r^{-1}[u_{,\theta z}-v_{,z}]-v_{,rz}]-e_{15}[w_{,rr}+r^{-1}w_{,r}+r^{-2}w_{,\theta\theta}]$$
$$-e_{33}w_{,zz}-p_3T_{,z}=0$$

and

$$k_{11}[T_{,rr}+r^{-1}T_{,r}+r^{-2}T_{,\theta\theta}]+k_{33}T_{,zz}-T_0dT_{,t}=T_0[\beta_1[u_{,rt}+r^{-1}u_{,t}+r^{-1}v_{,\theta t}]$$
$$+\beta_3w_{,zt}-p_3V_{,zt}]$$

where k_{ii} is heat conduction coefficient, $d=\rho c_v/T_0$, u, v and w are the displacements along r, θ, z direction, V is the electric potential, ρ is the mass density and t is the time. Partial differentiation with respect to an independent variable is denoted by a comma followed by the corresponding independent variable as subscript. We seek the solutions for (1) in the following form:

$$u(r,\theta,z,t)=[\phi_{,r}+r^{-1}\psi_{,\theta}]\exp i(kz+pt),$$
$$v(r,\theta,z,t)=[r^{-1}\phi_{,\theta}-\psi_{,r}]\exp i(kz+pt),$$
$$w(r,\theta,z,t)=i(W/a)\exp i(kz+pt),$$
$$V(r,\theta,z,t)=i(c_{44}/e_{33})(V/a)\exp i(kz+pt) \qquad \ldots(2)$$

and

$$T(r,\theta,z,t)=(c_{44}/\beta_3)(T/a^2)\exp i(kz+pt),$$

where ϕ, ψ, W, V and T are functions of r and θ only, k is the wave number, p is the angular frequency and $i=(-1)^{1/2}$.

Now we introduce the dimensionless radial coordinate $x=r/a$ and the dimensionless wave number $\varepsilon=ka$, where a is the radius of the cylinder.

Using (2) we can rewrite (1) in the following form:-

$$\partial/\partial x\{[\bar{c}_{11}\nabla^2-\varepsilon^2+(ca)^2]\phi-(1+\bar{c}_{13})\varepsilon W-(\bar{e}_{31}+\bar{e}_{15})\varepsilon V-\bar{\beta}T\}$$
$$+(1/x)\partial/\partial\theta\{[\bar{c}_{66}\nabla^2+(ca)^2-\varepsilon^2]\psi+\bar{e}_{14}\varepsilon V\}=0$$
$$(1/x)\partial/\partial\theta\{[\bar{c}_{11}\nabla^2-\varepsilon^2+(ca)^2]\phi-(1+\bar{c}_{13})\varepsilon W-(\bar{e}_{31}+\bar{e}_{15})\varepsilon V-\bar{\beta}T\}$$
$$-\partial/\partial x\{[\bar{c}_{66}\nabla^2-\varepsilon^2+(ca)^2]\psi+\bar{e}_{14}\varepsilon V\}=0 \qquad \ldots(3)$$
$$(1+\bar{c}_{13})\varepsilon\nabla^2\phi+[\nabla^2-c_{33}\varepsilon^2+(ca)^2]W+(\bar{e}_{15}\nabla^2-\varepsilon^2)V-\varepsilon T=0$$
$$(\bar{e}_{31}+\bar{e}_{15})\varepsilon\nabla^2\phi+(\bar{e}_{15}\nabla^2-\varepsilon^2)W-(k_{13}^{-2}\nabla^2-k_{33}^{-2}\varepsilon^2)V-\bar{e}_{14}\varepsilon\nabla^2\psi+\bar{p}\varepsilon T=0$$

and

$$\bar{\beta}\nabla^2\phi-W\varepsilon+\bar{p}\varepsilon V+(\bar{d}-i\bar{k}_3\varepsilon^2+i\bar{k}_1\nabla^2)T=0,$$

where

$$\nabla^2=\partial^2/\partial x^2+x^{-1}\partial/\partial x+x^{-2}\partial^2/\partial\theta^2$$

$\bar{c}_{ij}=c_{ij}/c_{44}$, $\qquad \bar{e}_{ij}=e_{ij}/e_{33}$, $\qquad \bar{d}=(\rho c_v c_{44})/(\beta_3^2 T_0)$,

$\bar{\beta}=\beta_i/\beta_3$, $\qquad c^2=\rho p^2/c_{44}$, $\qquad \bar{p}=(p_3c_{44})/(e_{33}\beta_0)$,

$k_{i3}^{-2}=(\varepsilon_{ii}c_{44})/e_{33}^2$, $\qquad k_i=(\rho c_{44})^{1/2}k_{ii}/(\beta_3^2\,aT_0)$, $\qquad \bar{k}_i=k_i/(ca)$

The first and second of eqs. (3) can be rewritten as

$$[\bar{c}_{11}\nabla^2-\varepsilon^2+(ca)^2]\phi-(1+\bar{c}_{13})\varepsilon W-(\bar{e}_{31}+\bar{e}_{15})\varepsilon V-\bar{\beta}T=0$$
$$[\bar{c}_{66}\nabla^2+(ca)^2-\varepsilon^2]\psi+\bar{e}_{14}\varepsilon V=0.$$

In case of solid cylinder, the Bessel function of the second kind will be absent. Therefore, the solutions for equations (3) for solid cylinder are

$$\phi = \sum_{j=1}^{5} A_j J_n(\alpha_j a x) \exp(in\theta),$$

$$\psi = \sum_{j=1}^{5} c_j A_j J_n(\alpha_j a x) \exp(in\theta),$$

$$W = \sum_{j=1}^{5} d_j A_j J_n(\alpha_j a x) \exp(in\theta),$$

$$V = \sum_{j=1}^{5} e_j A_j J_n(\alpha_j a x) \exp(in\theta)$$

and

$$T = \sum_{j=1}^{5} h_j A_j J_n(\alpha_j a x) \exp(in\theta),$$

where

$(\alpha_j a)^2$ are the five roots of the equation

$$\begin{vmatrix} -\bar{c}_{11}(\alpha a)^2 + (ca)^2 - \varepsilon^2 & 0 & -(1+\bar{c}_{13})\varepsilon & -(\bar{e}_{31}+\bar{e}_{15})\varepsilon & -\bar{\beta} \\ 0 & -\bar{c}_{66}(\alpha a)^2 + (ca)^2 - \varepsilon^2 & 0 & \bar{e}_{14}\varepsilon & 0 \\ -(1+\bar{c}_{13})\varepsilon(\alpha a)^2 & 0 & (ca)^2 - \bar{c}_{33}\varepsilon^2 - (\alpha a)^2 & -\bar{e}_{15}(\alpha a)^2 - \varepsilon^2 & -\varepsilon \\ -(\bar{e}_{15}+\bar{e}_{31})\varepsilon(\alpha a)^2 & \bar{e}_{14}\varepsilon(\alpha a)^2 & -\bar{e}_{15}(\alpha a)^2 - \varepsilon^2 & k_{13}^{-2}(\alpha a)^2 + k_{33}^{-2}\varepsilon^2 & \bar{p}\varepsilon \\ -\bar{\beta}(\alpha a)^2 & 0 & -\varepsilon & \bar{p}\varepsilon & \bar{d} - i\bar{k}_3\varepsilon^2 - i\bar{k}_1(\alpha a)^2 \end{vmatrix} = 0 \quad \ldots (4)$$

c_j, d_j, e_j and h_j can be computed using the following relations

$[-\bar{c}_{11}(\alpha_j a)^2 + (ca)^2 - \varepsilon^2] - (1+\bar{c}_{13})\varepsilon d_j - (\bar{e}_{31} + \bar{e}_{15}) e_j + \bar{\beta} h_j = 0,$

$[-\bar{c}_{66}(\alpha_j a)^2 + (ca)^2 - \varepsilon^2 c_j] + \bar{e}_{14}\varepsilon e_j = 0,$

$-(1+\bar{c}_{13})\varepsilon(\alpha_j a)^2 + [(ca)^2 - \bar{c}_{33}\varepsilon^2 - (\alpha_j a)^2] d_j - [\bar{e}_{15}(\alpha_j a)^2 + \varepsilon^2] e_j - \varepsilon h_j = 0$

and

$-(\bar{e}_{31} + \bar{e}_{15})\varepsilon(\alpha_j a)^2 + \bar{e}_{14}\varepsilon(\alpha_j a)^2 c_j - (\bar{e}_{15}(\alpha_j a)^2 + \varepsilon^2) d_j + [k_{13}^{-2}(\alpha_j a)^2$
$+ [k_{33}^{-2}\varepsilon^2] e_j + \bar{p}\varepsilon h_j = 0$

The stresses T_{rr}, $T_{r\theta}$, T_{rz} for class 6 are

$$T_{rr} = (-c_{44}/a^2) \sum_{j=1}^{5} A_j f_{ij}(x) \exp i(n\theta + kz + pt),$$

$$T_{rz}=(ic_{44}/a^2)\sum_{j=1}^{5}A_j f_{2j}(x)\exp i(n\theta+kz+pt)$$

and

$$T_{r\theta}=(-ic_{66}/a^2)\sum_{j=1}^{5}A_j f_{3j}(x)\exp i(n\theta+kz+pt),$$

where

$$f_{1j}(x)=[\bar{c}_{11}(\alpha_j a)^2-2\bar{c}_{66}(n/x)^2+\bar{c}_{13}\varepsilon d_j+\bar{e}_{13}\varepsilon\bar{e}_j+\bar{\beta}h_j]J_n(\alpha_j ax)+[-J_{n+1}(\alpha_j ax)+(n/\alpha_j(\alpha_j ax)J_n(\alpha_j ax)](2\bar{c}_{66}/x)+2\bar{c}_{66}(n/x)c_j[J_{n+1}(\alpha_j ax)+[(1/x)-n/\alpha_j ax]J_n(\alpha_j ax)]$$

$$f_{2j}(x)=(d_j+\varepsilon+\bar{e}_{15})[-J_{n+1}(\alpha_j ax)+(n/\alpha_j ax)J_n(\alpha_j ax)]+[\varepsilon c_j-ie_j\bar{e}_{14}](n/x)J_n(\alpha_j ax)$$

$$f_{3j}(x)=(1/x^2)\{n[xJ_{n+1}(\alpha_j ax)+[1-xn/\alpha_j ax]J_n(\alpha_j ax)]+[(\alpha_j ax)^2-2n^2]c_jJ_n(\alpha_j ax)+2xc_j[-J_{n+1}(\alpha_j ax)+(n/\alpha_j ax)J_n(\alpha_j ax)]\}$$

Frequency Equation

In this analysis, the pyroelectric solid circular cylinder of class 6 is considered whose surfaces are traction free, coated with electrode which are shorted and thermally insulated. The mechanical, electrical and thermal boundary conditions are

$$T_{rr}|_{x=1}=T_{rz}|_{x=1}=T_{r\theta}|_{x=1}=0$$

$$V|_{x=1}=T_{,r}|_{x=1}=0$$

The frequency equation for crystal class 6 can be obtained by applying the above boundary conditions and eliminating the arbitrary constants A_j. Since the heat conduction coefficients and specific heat capacity of crystal class 6 are not available we are analyzing numerically the frequency equation for class (6mm). For hexagonal crystal class (6mm). $e_{14}=0$. The equation (4) can be rewritten as two equations

$$(\alpha_5 a)^2=[(ca)^2-\varepsilon^2]/\bar{c}_{66} \qquad \ldots(6)$$

and

$$(\alpha a)^8+P(\alpha a)^6+Q(\alpha a)^4+R(\alpha a)^2+S=0, \qquad \ldots(7)$$

where

$$P=(1/E)[\bar{\beta}^2\varepsilon(B_{10}+B_9\bar{e}_{15})-B_{12}(B_1E_1-\bar{\beta}E_2+\bar{\beta}\bar{e}_{15}E_3)-i\bar{k}_1(B_1E_4-B_2E_1+\bar{\beta}E_5-B_3E_2+B_4E_3+\bar{\beta}\bar{e}_{15}E_7)]$$

$$Q=(1/E)[\varepsilon\bar{\beta}[B_{11}\bar{\beta}-B_4B_9-B_7\varepsilon(\bar{e}_{15}+\bar{p})-B_8\bar{\beta}\bar{e}_{15}-B_{10}]+B_1\varepsilon^2(\bar{p}B9-B_{10})+B_{12}[B_1E_4-B_2E_1+E_5\bar{\beta}-B_3E_2+B_4E_3-\bar{\beta}\bar{e}_{15}E_7]-i\bar{k}_1[B_2E_4+B_1E_6-B_3E_5+B_4E_7]]$$

$$R=(1/E)[\bar{p}\varepsilon^2[B_2B_9-B_1B_8+B_3B_7]+\bar{\beta}\varepsilon[B_4B_8-B_3B_{11}]+\varepsilon^2[B_4B_7-B_1B_{11}-B_2B_{10}]+B_{12}[B_2E_4+B_1E_6-B_3E_5+B_4E_7]-i\bar{k}_1B_2E_6]$$

$S=(1/E)[B_{12}B_2E_6-B_2\varepsilon^2[B_{11}+B_8\bar{p}]]$

in which

$B_1=[\bar{\beta}(1+\bar{c}_{13})-\bar{c}_{11}]\varepsilon,$ $B_2=[(ca)^2-\varepsilon^2]\varepsilon,$

$B_3=(1+\bar{c}_{13}-\bar{\beta}\bar{c}_{33})\varepsilon^2+\bar{\beta}(ca)^2,$ $B_4=(\bar{e}_{31}+\bar{e}_{15}-\bar{\beta})\varepsilon^2,$

$B_5=(1+\bar{c}_{13})e,$ $B_6=(ca)^2-\bar{c}_{33}\varepsilon^2,$

$B_7=[\bar{p}(1+\bar{c}_{13})+\bar{e}_{15}+\bar{e}_{31}]\varepsilon,$ $B_8=\bar{p}(ca)^2-(1+\bar{p}\bar{c}_{33})\varepsilon^2,$

$B_9=\bar{p}+\bar{e}_{15},$ $B_{10}=k_{\bar{13}}^{-2}-\bar{p}\bar{e}_{15},$

$B_{11}=k_{\bar{33}}^{-2}-\bar{p},$ $B_{12}=\bar{d}-i\bar{k}_3\varepsilon^2,$

$E_1=B_{10}+B_9\bar{e}_{15},$ $E_2=B_5B_{10}+B_7\bar{e}_{15},$

$E_3=B_7-B_5B_9,$ $E_4=B_6B_{10}+B_8\bar{e}_{15}-B_9\varepsilon^2-B_{11},$

$E_5=B_5B_{11}+B_7\varepsilon^2,$ $E_6=B_6B_{11}+B_8\varepsilon^2,$

$E_6=B_6B_{11}+B_8\varepsilon^2,$ $E=i\bar{k}_1[B_1E_1-\bar{\beta}E_2+\bar{\beta}\bar{e}_{15}E_9].$

Using the boundary conditions we obtain the frequency equation for crystal class (6mm) as

$$|P_{kl}|=0 \qquad (k,\ l=1,2,\cdots 5), \qquad \ldots(7)$$

where

$P_{1i}=[\bar{c}_{11}(\alpha_ia)^2-2\bar{c}_{66}n^2+\bar{c}_{13}\varepsilon d_i+e_{13}\varepsilon e_i+\bar{\beta}h_i]J_n(\alpha_ia)+2\bar{c}_{66}[-J_{n+1}(\alpha_ia)$
$\quad+(n/\alpha_ia)J_n(\alpha_ia)]$

$P_{15}=2\bar{c}_{66}n[J_n(\alpha_5a)+J_{n+1}(\alpha_5a)-(n/\alpha_5a)J_n(\alpha_5a)]$

$P_{2i}=(d_i+\varepsilon+\bar{e}_{15}e_i)[-J_{n+1}(\alpha_ia)+(n/\alpha_ia)J_n(\alpha_ia)]$

$P_{25}=n\varepsilon J_n(\alpha_5a)$

$P_{3i}=2n[J_n(\alpha_ia)+J_{n+1}(\alpha_ia)-(n/\alpha_ia)J_n(\alpha_ia)]$

$P_{35}=((\alpha_5a)^2-n^2)J_n(\alpha_5a)+2[-J_{n+1}(\alpha_5a)+(n/\alpha_5a)J_n(\alpha_5a)]$

$P_{4i}=e_iJ_n(\alpha_ia)$

$P_{45}=0$

$P_{5i}=h_i[-J_{n+1}(\alpha_ia)+(n/\alpha_ia)J_n(\alpha_ia)]$

$P_{55}=0$ in which $i=1,2,3$ and 4.

For Longitudinal vibration the frequency equation can be deduced from (7) by putting $n=0$. The frequency equation can also be obtained without much difficulty when the surface of the cylinder is clamped, shorted and thermally insulated.

Numerical Results

Due to the presence of thermal field, the frequency equation (7) becomes complex. In the present case, simple Birge-Vieta method does not work for finding the roots of algebraic equation (6). Obtaining the two roots of the equation (6) by Birge-Vieta method, the roots are corrected for desired accuracy using Newton-Raphson method. By following the same procedure, the other two roots are obtained. This combination has overcome the difficulty in finding the roots of the equation (6). The elastic, piezoelectric, dielectric and pyroelectric constants are taken[12,13] for $BaTiO_3$. In the present analysis, the radius of the cylinder is taken as 0.03m. The dispersion curves in Fig. 1 are drawn for flexural vibration and longitudinal vibration by fixing (ca) and varying ε. In Fig. 2 curves are drawn by fixing ε and varying (ca). The curves with continuous line correspond to the flexural vibration. The curve with dotted line corresponds to the longitudinal vibration. The results are presented in Figs 1 and 2. The dispersion results are obtained by fixing real ε and varying (ca) for longitudinal and flexural vibrations and presented in Table I to highlight the damping.

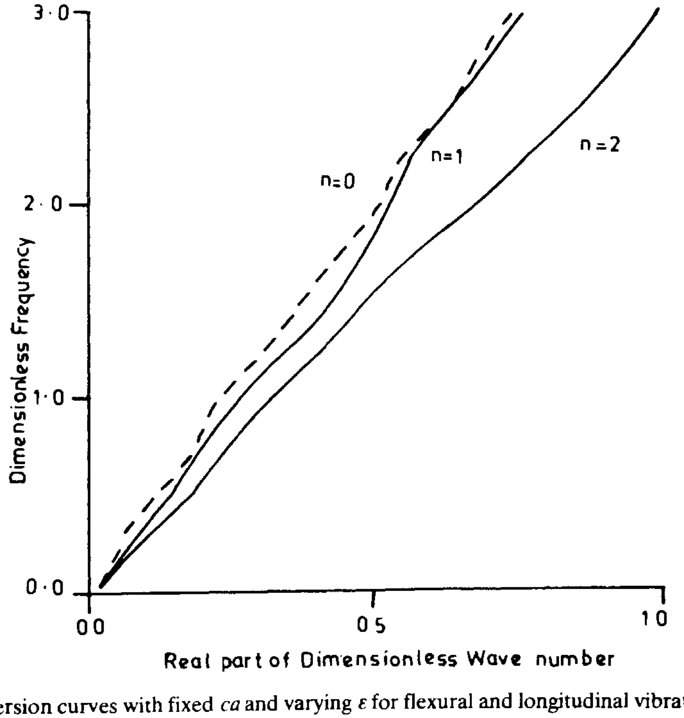

Fig 1 Dispersion curves with fixed ca and varying ε for flexural and longitudinal vibrations

Table I

Dimensionless wave number for different dimensionless frequencies for $n=0$ and $n=1$

$(ca)\rightarrow$ ε	$n=0$	$n=0$
0.05	$0.26284 + i\,0.01142$	$0.23000 + i\,0.01000$
0.25	$1.20000 + i\,0.01000$	$1.10003 + i\,0.01001$
0.50	$2.22507 + i\,0.02231$	$1.80017 + i\,0.01165$
0.75	$3.12537 + i\,0.02268$	$3.00027 + i\,0.01266$
1.00	$4.30000 + i\,0.01000$	$4.20025 + i\,0.01249$

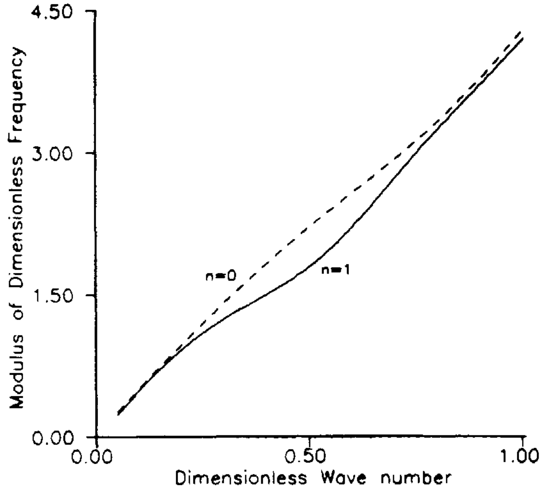

Fig 2 Dispersion curves with fixed ε and varying ca for flexural and longitudinal vibrations

Discussion

In the present analysis, the exact frequency equations for the flexural and longitudinal vibration have been derived for the solid circular cylinder whose surfaces are traction free, completely coated with electrode which are shorted and thermally insulated. The frequency equation has been analyzed numerically using Newton-Raphson method. Since the frequency equation is complex, it is difficult to compare the dispersion curve of pyroelectric cylinder with that of piezoelectric cylinder[6,7]. Dispersion curves can be obtained for crystal class 6 if the heat conduction coefficients, specific heat capacity for crystal class 6 are known.

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