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TERRAIN VARIANCE SPECTRA FOR INDIAN WESTERN GHATS

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The terrain height variance spectra for the Pune region, including the Western Ghats, are calculated with the available topographic map along three different cross sections. The required minimum grid size for mesoscale models to adequately represent terrain forcing, without resorting to sub-grid scale parameterization, is determined in this study Calculated values show that over 85% of the terrain variations are accounted for, if the chosen grid interval is of 4km. The terrain variances are found to vary proportional to the square of the wave length from the calculated spectra.

Key Words: Spectral Studies; Grid Resolution

Introduction

The distribution of terrain height variance with wavelength is the variance spectrum. This spectrum gives us an idea about the dominant wavelengths of terrain variation, which will be useful for finding the optimum horizontal gride size for numerical models of orographic flow. The purpose of this present study is to determine the optimum grid size for the area taking into account terrain variances. For the purpose of the study, the eastern slope of the Western Ghats comprising Poona and its environs is chosen. This area is prone to thunderstorms and mesoscale weather developments.

The results of previous investigators vary considerably depending on geomorphology of the studied region. For example, Bretherton[1] found the terrain variance, for northern Wales, to be proportional to $\lambda^{3/2}$ for $\lambda < 30$km. For West Central Virginia in the U.S.A., Pielke and Kennedy[2] have shown terrain height variance to increase as λ^2 for short wave lengths. Young and Pielke[3] found the dependence as λ^1 for three different cross sections in the Colorado area. Since terrain variations are dissimilar throughout the world, it is necessary to study each region for application in NWP models.

Based on the above studies, this study determines terrain variance spectra for the above mentioned region, using a Fast Fourier Transform (FFT) method. The methodology and data are described in the next section. In the third section, the results are discussed. Summary and conclusion form the last section.

Data

Detailed fine resolution topographic maps are usually not available. The available digitized topographic data are of coarse resolution ~ 15km, and not suitable for this study. In this study, a published topographic map with a contour interval of 250' is used (Atmanathan[4]). This is shown in Fig. 1. The terrain

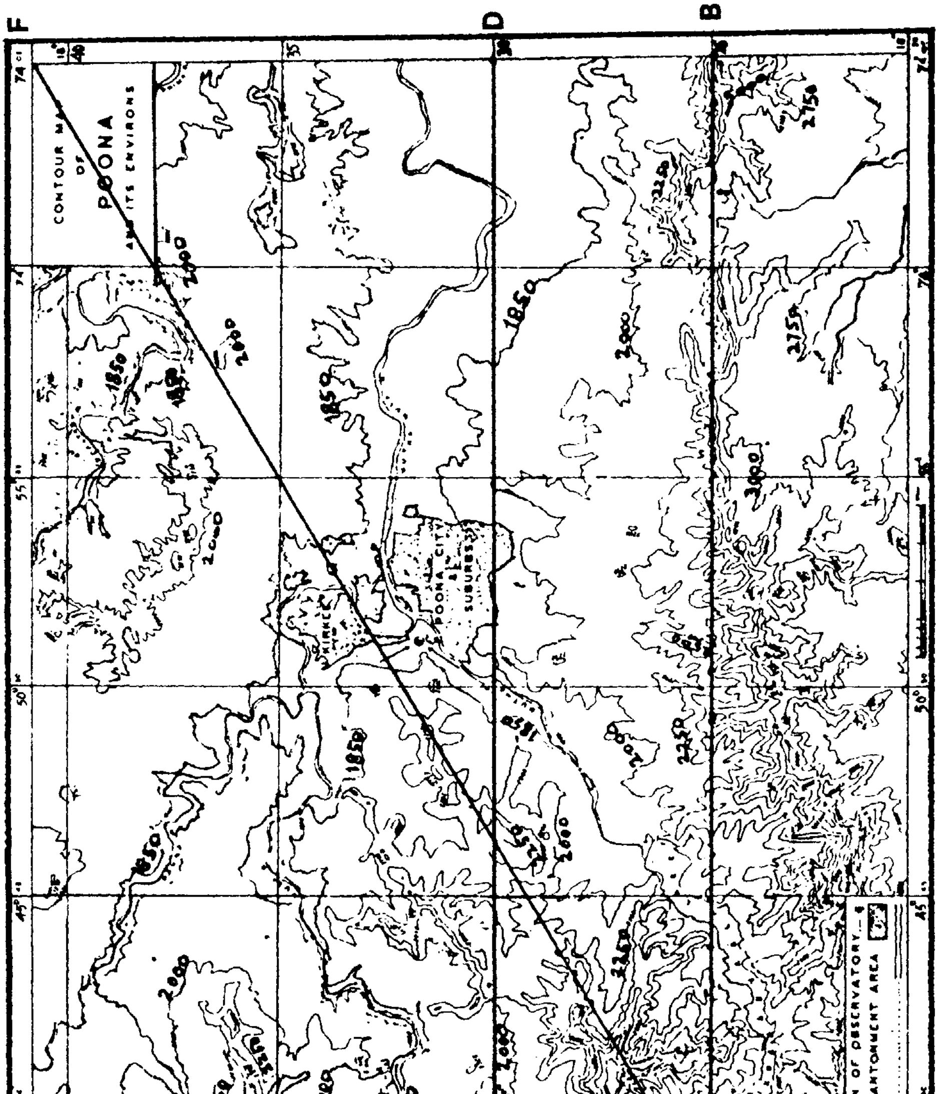

Fig 1 Elevation contour map of the study region.

elevations were evaluated manually from Fig. 1 along three different cross sections as shown by the lines AB, CD and EF. For each cross section 80 equally spaced intervals were taken along these lines to evaluate the terrain heights. A one-dimensional variance spectra for the data set were computed using the FFT algorithm. For computational purposes, the deviations of terrain elevations from the average value along each cross sections are obtained. The resulting height series was transformed to wave number space (k) and the terrain variances were calculated. The calculated distribution of variance as a function of wavelength (λ) or wave number (k) are presented in Figs 2 to 5. Following

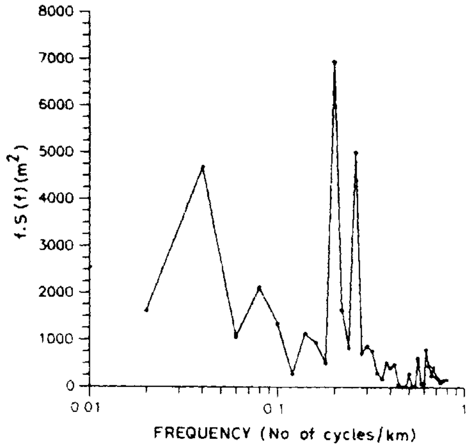

Fig 2 Spectral density (S) multiplied with its frequency (f-cycles/km) is plotted as a function of frequency and wavelength (λ), for the cross section AB

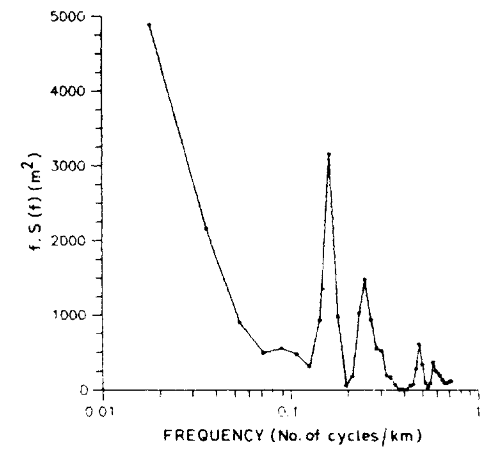

Fig 3 Same as fig 2 but for EF.

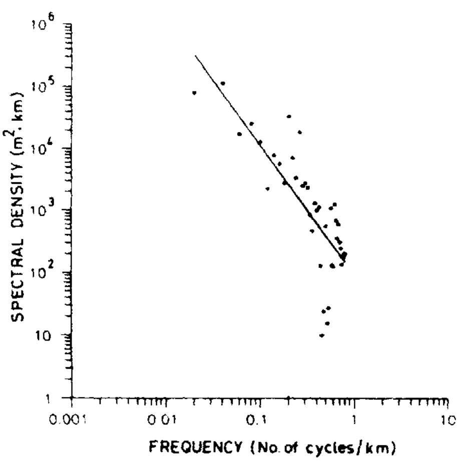

Fig 4 Terrain height variance spectra for the cross section AB. Spectral density (S) is plotted as a function of frequency and wavelength in log-log scale

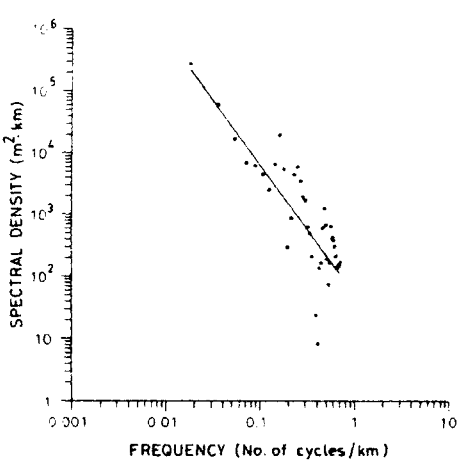

Fig 5 Same as Fig 4 but for EF.

Young and Pielke[3], a linear trend was also calculated for the terrain height series by the method of least squares and then subtracted from the terrain height series for each cross section. The calculated spectra for this series yielded similar results as those obtained with the series by subtracting the average value of the terrain series. For brevity, the calculated spectra using the first method are shown here.

Results and Discussion

The major use of terrain spectra is for determining the required spatial resolution in mesoscale numerical models. In such simulations the smallest atmospheric circulation that can be represented has a horizontal wave length equal to twice the minimum grid resolution in the model. Over rough terrain, this characteristic of the model also states that the smallest topographic feature which can be resolved is equal to twice the minimum grid interval. Thus, for example, if the grid spacing in the model is 10km, but 90% of the terrain irregularities are 5km or smaller, there is little hope of correctly simulating the atmospheric response to this topography. But, on the other hand, if the grid spacing was at 2km intervals and 90% of the variations are larger than 4km, the model should be capable of realistically representing the situation.

The results are presented in two different ways. In Figs 2 and 3 the contributions of variance are plotted as a function of wavelength. This shows the horizontal spacing of most typical terrain features, as well as the amount of variance. In an alternate representation, the terrain height variances are plotted against wavelength (λ) or wavenumber (k) in a logarithmic scale. A power law relationship between the variables such as $S = a \cdot \lambda^b$ was fitted by the method of least squares. This analytic form is dictated by the nature of the distribution of points in the diagonal. The power law dependence for atmospheric circulations is also well known from earlier studies (Vander Hoven[5]). In Figs 2 and 3 by plotting $f \cdot S(f)$ against $\log(f)$, (f is frequency with units No. of cycles per km, and $S(f)$ is the spectral density with units m^2 km associated with the frequency f). The low frequency portions of the spectra are expanded along the abscissa. Also, the ordinate for the high frequency portions are enhanced because the spectral density is multiplied by frequency. Another excellent quality is that the area under any portion of the curve continues to the porportional to the variance. Further, it gives us an idea about the dominant wavelengths towards the contribution of variance in the calculated spectra. From Fig. 2, the dominant wavelengths are 25km, 5km, and 4km over the cross section AB. Similar results were obtained for the cross section CD. Over cross section EF, from Fig. 3, the dominant wavelengths are 56km, 6km and 4km.

In Figs 4 and 5 the spectral density is plotted as a function of wavenumber on a logarithmic scale. The power law relationship between the variables (i.e.) spectral density (S) and wavelength (λ) in the form of $S = a \cdot \lambda^b$ was fitted by the method of least squares. The results are tabulated in Table I.

Table I

Parameters of the least squares best fit relation of the form $S = a \cdot \lambda^b$ for three terrain height variance spectra considered over Poona

[The tabulated parameters are coefficient a, the exponent b, the correlation coefficient (γ) between $\log(f)$ and $\log(S)$ and standard.deviation S_b of the exponent.]

Data set	a	b	γ	S_b
Cross section AB	96.28	2.08	0.80	0.25
Cross section CD	12.00	2.02	0.85	0.20
Cross section EF	53.72	2.07	0.75	0.29

The value of the exponent b was used to discuss the choice of horizontal grid spacing for mesoscale models. From Table I, the value of the exponent b is almost same in all the three cross sections considered, where as the coefficient a varies between cross sections because of differing geographic coverage.

Confidence intervals may be computed for the true value of the exponent b using $F = \frac{(B-b)^2}{S_b^2}$, where F has the degrees of freedom 1 and n-2, n being the number of harmonics in the spectra (Kreyszig[6]). A value of $\alpha = 0.05$ was selected for the computation of confidence intervals. The confidence intervals for the true value of the exponent were $B = 2.08 \pm 0.5$ for the cross section AB, $B = 2.02 \pm 0.4$ for the cross section CD and $B = 2.07 \pm 0.58$ for the cross section EF.

Similar studies on other regions of the world indicate that the spectral densities varies proportional to λ^1 or λ^2 with corresponding variations in the coefficient a (Pielke and Kennedy[2] and Young and Pielke[3]). However, our study has shown that the terrain variance varies as λ^2 as shown in Table I. Since the terrain heights and the configuration are dissimilar throughout the world, the obtained values are reasonable and provides a quantitative measure on terrain smoothness.

For nonlinear mesoscale models, using a terrain following co-ordinate system, Pielke[7] has shown the term containing subgrid scale terrain fluctuations can be neglected in grid volume averaged conservation equations if the subgrid scale terrain variance is small compared to grid resolvable terrain variance. We can find a maximum grid size for the model, so that the subgrid scale terrain variance can be neglected. For terrain height variance spectra of the form $S = a \cdot \lambda^2$, the ratio of the subgrid terrain height variance to model resolved terrain height variance is

$$\int_{1/2\delta x}^{1/2\Delta x} ak^{-2}\mathrm{d}k \Big/ \int_{1/2\Delta x}^{1/n\Delta x} ak^{-2}\mathrm{d}k,$$

where $k = 1/\lambda$, $2\delta x$ is the shortest wavelength in the measured spectra, $2\Delta x$ is the shortest wavelength that can be resolved by the numerical model and $n\Delta x$ is the model domain length. This ratio turns out to be 0.15 for a 50km domain, (from the spectra) $2\delta x = 1.25$km and model grid size 4km. Thus, numerical models of mesoscale orographic flows over Poona region, requires a minimum horizontal grid spacing of 4km to resolve adequately the terrain without resorting subgrid scale terrain variance parameterization.

Conclusions

One dimensional terrain height spectra for three different cross sections over a region in and around Poona with western ghats were calculated. The calculated results show that a horizontal gride spacing of 4km is necessary to resolve the majority of terrain height variations without requiring a subgrid scale parameterization for the terrain height variances. A power law fit for the obtained spectra show that the terrain variances are proportional to λ^2. The presented results may serve as a guide for choice of grid interval in Numerical Weather Prediction models.

Acknowledgement

The computations were made on an ICL-3980 computer system installed at IIT-Delhi.

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