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IMPORTANCE OF MOUNTAIN WAVES IN AVIATION AND WEATHER HAZARDS ASSOCIATED WITH IT

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The paper summarises the importance of mountain waves and their impact on aviation with particular reference to the Indian scenario. The lee waves occur on different space scale. In the scale of waves ranging between 10 and 100 km, the flow is non-rotating and hydrostatic. Studies using steady state two dimensional models using perturbation technique are in general successful in explaining most of the salient features. However, nonlinear effects are to be considered for explaining the phenomena of hyraulic jump and upstream blocking, which are associated with strong down slop winds off steep mountain barriers. Work done in India and its relevance to aviation meteorology has also been discussed. The paper also summarises important factors which can be used for providing adequate guidance for safer air navigation in the air routes overflying orographic barriers.

Key Words: Mountain Waves; Hydraulic Jump; Rotor Clouds, Froude Number; Clear Air Turbulence

Introduction

Mountain waves belong to the category of atmospheric waves which interest both theoretical as well as operational meteorologists. Early aviators were largely aware of the hazards in flying over uneven terrain. From the view point of aeronautical meteorology, mountain waves account for some disastrous accidents in the past years. However, for the last four decades, a slow but steady collection of the information and observation together with adequate theoretical work have made it possible to overcome such hazards and make flying safer from this angle.

Theoretical Aspects

The atmosphere is capable of sustaining a large number of wave phenomena. We may classify them according to the nature of the displacement i.e., transverse or longitudinal or depending upon the nature of the restitutional forces i.e. gravity, rotation of the earth, inertia etc.

Mountain waves are essentially a class of gravity waves. However, the importance of the other terms like the compressibility, Coriolis force etc. are also to be considered for the appropriate scales of motion. The measure of significance of gravity is reflected if we consider the Froude Number

$$F=\frac{|U|}{\sqrt{gL}},$$

where U is the characteristic velocity, L is the characteristic length and 'g' is the acceleration due to gravity.

Historically, the origin of the problem can be traced back to the works of Rayleigh[1] and Kelvin[2]. They considered the flow of an incompressible inviscid and homogeneous fluid over an obstacle at the bottom. However, the corresponding problem in the earth's atmosphere is much more complex. This is due to thermal stratification of the atmosphere and the effect of the earths rotation.

Quenny[3] performed a sort of scale analysis of the atmospheric flow across a mountain barrier and showed that for an adiabatic perturbation the wave equation can be written as

$$\left(1-\frac{f^2}{K^2U^2}\right)\frac{\partial^2\psi}{\partial z^2}-\left(\frac{g}{C^2}+\beta\right)\frac{\partial\psi}{\partial z}+\left(\frac{g\beta}{U^2}-K^2\right)\psi=0 \qquad \ldots(1)$$

where

f is the coriolis force

ψ is the stream function

$$\beta=\frac{1}{\theta}\frac{d\theta}{dz}$$

θ is the potential temperature

C the speed of sound

and K the wave number.

He showed from physical consideration that there could be three different scales of mountain waves depending upon the value of and the other forcing terms:

$$\left.\begin{array}{lll} \text{if} & (i) & K>\sqrt{g\beta/U} \\ & (ii) & \dfrac{f}{U}<K<\sqrt{g\beta/U} \\ \text{and} & (iii) & K<\dfrac{f}{U}. \end{array}\right\} \qquad \ldots(2)$$

In the first case motion is nonhydrostatic and non-geostrophic. While in the second case the motion is hydrostatic but non-geostrophic. In the last case, the motion is hydrostatic and quasi geostrophic.

In the atmosphere all the three types of mountain waves are possible, but the scales of waves from 1 to 100 km which fall under the category (*i*) and (*ii*) are of major concern to aviation. These waves are frequently observed and reported and can be approximated well by two dimensional linear models.

Most of the early investigation using the two dimensional linear models could explain usual features like the lee wave clouds and the wave motion on the leeward side of the mountains; Scorer[4], Doos[5], Fritz[6], De[7]. Organised field observations by Kuettner[8], Holmboe and Klieforth[9], Vergeiner[10] *et al.* and Küettner and Lilly[11] and recent experiment ALPEX have shown that the air motion in a mountainous terrain is highly modified and is often complex in nature. Broadly speaking two classes of flows have been documented—

(a) Wave motion.

(b) Hydraulic Jump type motion.

Wave Motion

Linear equations are adopted to accommodate a number of simplifying assumptions in the governing equations to obtain analytical solutions. Scorer[4], Doos[5], Palm and Foldvik[12] and Sarker[13] adopted this technique. Most of these studies used a two-dimensional adiabatic frictionless flow with the undisturbed flow being a function of the vertical co-ordinate only. By using the perturbation technique (where large amplitude and non linear effects are eliminated) they showed that a second order equation of the type

$$\frac{\partial^2 W}{\partial x^2}+\frac{\partial^2 W}{\partial z^2}+f(z)W=0 \qquad \ldots (3)$$

is obtained, where,

W is the perturbation vertical velocity and $f(z)$ is a complex function of stability and wind shear given by

$$f(z)=\frac{g(\gamma^*-\gamma)}{U^2 T}-\frac{1}{U}\frac{\mathrm{d}^2 U}{\mathrm{d}z^2}+\left(\frac{\gamma^*-\gamma}{T}-\frac{g}{xRT}\right)\frac{1}{U}\frac{\mathrm{d}U}{\mathrm{d}z}-\frac{2}{xRT}\left(\frac{\mathrm{d}U}{\mathrm{d}z}\right)^2-\left(\frac{g-R\gamma}{2RT}\right)^2.$$

where U, T are the undisturbed wind and temperature in the basic flow, γ and γ^* respectively lapse rate and the dry adiabatic lapse rate, R is the universal gas constant and $x=C_p/C_v$.

In the linear steady state models it is assumed that the perturbations are small and the product of perturbation quantities can be neglected.

Though the linear small amplitude theory is capable of explaining the "lee wave phenomenon" but important nonlinear effects like 'rotor' phenomenon and hydraulic jump type flow remain beyond its perview.

Non linear effects were introduced in the studies by Long[14,15]. He considered a stratified invicid but incompressible flow over a small barrier. Several other authors pursued studies following Long. Drazin and Moore[16], Miles[17,18], Miles and Huppert[19] and Davis[20] supported the initial results of Long. The phènomenon of upstream "blocking" and the "hydraulic jump" at very small Froude Number and for large barriers was confirmed.

Earlier Foldvik and Wurtele[21] considered the nonlinear effects by considering a time-dependent equation.

For the class of flow for an ideal fluid, the Froude number is used to discriminate the following up types of terrain induced motion.

(i) If $F >> 1$ there are no lee waves,

(ii) If $F \rightarrow 0$ upstream blocking is complete

and for the intermediate range of values, there is partial blocking on the windward side and hydraulic jumps in the lee of the barrier. Since the Froude Number depends upon the mountain height also under typical atmospheric conditions these are observed when the mountain height exceeds 0.5 to 1 km.

Scorer[22] showed that the condition for occurrence of rotors to be

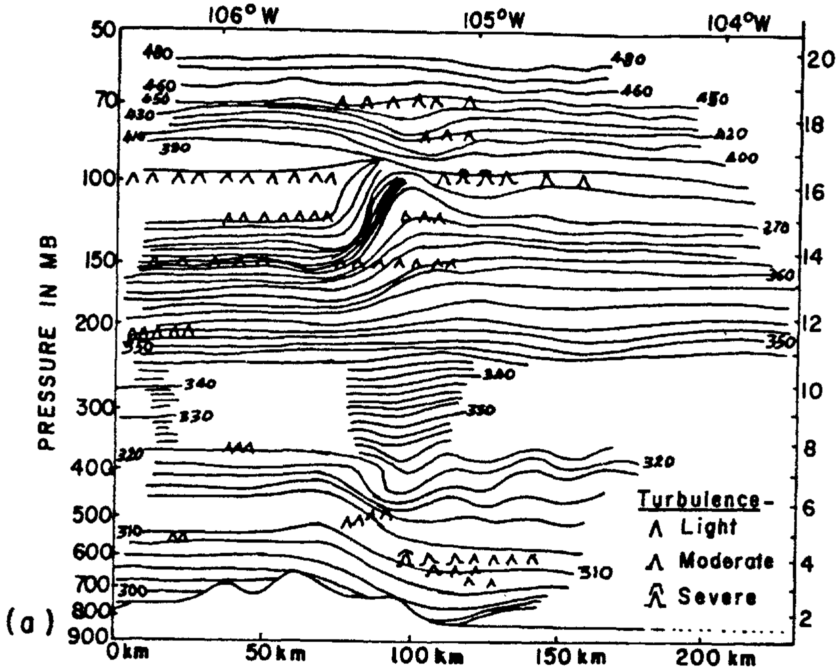
$$\frac{\partial W}{\partial z} > 1 \text{ or } < -1, \qquad \ldots (4)$$

which implies backward sloping streamlines. Rotors therefore tend to develop when the wave amplitude increases where the slope of the vertical profile of W is the largest.

Observational Aspects

As stated in the earlier section the effects of topography on air motion operate on a wide range of scales. There is as such an hierarchy of circulation systems associated with them. Fortuitious orientation of mountain barriers over the Indian subcontinent allows us to see all these effects in different space scales. Investigations by Das[23] refers to the planetary scale, while those by Sarker[13], De[7] and Sinha Ray[24] refer to the mesoscale. On the other hand investigations by Ghosh & Mukherjee[25] relate to the local circulations induced due to orography.

Early observations regarding mountain waves were based on visual observations of the leewave clouds and experience of pilots of sail planes and gliders. The Sierra wave project which was carried out over the Sierra Nevada Mountain ranges in 1951-52 was perhaps the most ambitious observational programme. Using radiosonde, time lapse photographs double theodolite balloon ascents and powered aircraft flights and sail planes observations a valuable data set was collected. The main results of the experiment showed that in case of powerful waves the amplitude could reach upto 2000 m at 6 km, height with vertical velocities ranging between +12.5 m to −9.5 m/sec. Often a single wave was reported in the stratosphere though the main wave activity was confined to the troposphere. During the last two decades more quantitative observations have been collected through the use of sophisticated ground base equipments in conjunction with instrumented aircraft observations. These have been summarized by Klemp and Lilly. A typical case of moderate lee wave (17 Feb. 1970) conditions as reported in their study relating to the Rocky mountains in Colorado is shown in Fig. 1. In another case of a more severe wave situation reported on 11 Jan 1972, the amplitude of the lee wave was of the order of 6,000 m and the associated vertical velocity of 20 mps was reported. The undisturbed flow conditions for this case showed presence of an inversion layer above the mountain top level and horizontal wind speed of 20 mps at the mountain top level. These facts highlight the importance of predicting such extreme situation and its impact on aviation.



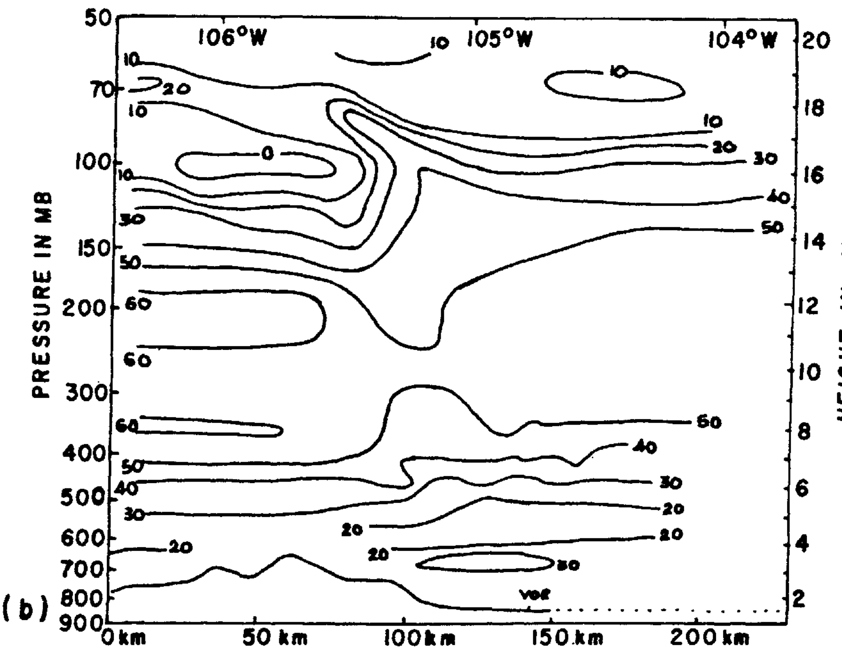


Fig 1 (*a*) Potential temperature cross section for 17 February 1970. Solid lines are isentropes (*K*) (*b*) Westerly wind component cross section for 17 February 1970. Isotachs are in ms^{-1}

Theoretical and observational studies carried out in India have indicated that strong mountain waves occur under favourable condition during winter over the Assam-Burma hills, the Western and Central Himalayas and over the Western Ghats. The observed wavelength of the lee waves using satellite imagery for the Assam Burma hills ranged between 17-34 km. Lee waves for the Himalayas ranged between 10.2 to 22.3 km. in wavelength. Theoretical computations using linear two dimensional mountain wave model support these observations. For the Western Ghats wave length during winter ranged from 25.1 km to 78.5 km and during the southwest monsoon season between 19.2 to 31.7 km. Though the direct verification for the Western Ghats is not available yet aircraft reports on similar situations by and large support these figures.

The associated vertical velocities for the Assam Burma hills were of the order of 0.2 $m.sec^{-1}$ to 4.3 m sec^{-1} during winter. Over the Western Ghats it ranged from 0.4 to 5.7 m sec^{-1}. These observations clearly indicate the need for a closer study of the phenomenon over India. Major air routes which exist in these areas and any future expansion of air routes should take into account this problem and its impact on the routine operations.

Factors Influencing the Airflow Over the Orographic Barriers

Airflow over mountains in the horizontal scale of 1-100 km has a significant influence on the weather in the area. Aviators are thus most interested in these scales of motions which are directly induced by orography. In the text that follows an attempt has been made to highlight the salient features relating to this aspect of problem. The behaviour of the airflow in this spatial domain depends on the following factors:

(*i*) the vertical wind profile; (*ii*) the stability structure; and (*iii*) the shape and size of the obstacle.

From the point of view of aviation the vertical currents associated with the waves are of crucial importance. These waves can produce severe turbulence in which the aircraft is tossed up and down. On December 18, 1952, a Viking aircraft approaching the mountain barrier north of Madrid first fell by 2.7 km and subsequently rose by 4.3 km as it crossed the barrier. The sequence was repeat-

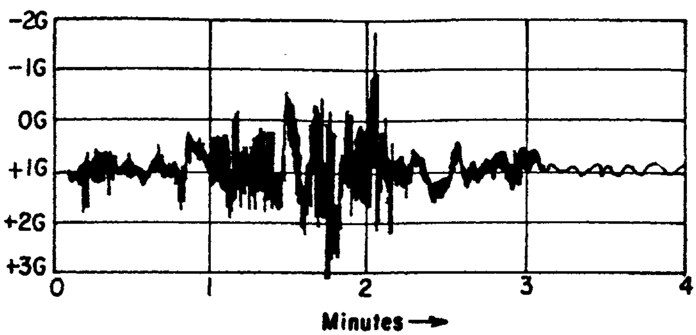


Fig 2 Flight recorder trace of vertical accelerations experienced during a severe turbulence encountered in clear-air turbulence (CAT) by a four-engine turbojet (B-747). This turbulence was generated by a severe mountain wave off the rocky mountains east of Helena, Montana

ed three times on the leeward side indicating that the aircraft was caught in powerful lee wave. In another case a four engine Turbojet (B-747) experienced severe turbulence due to mountain wave off the Rocky mountains east of Helena. The fluctuations in the vertical acceleration were more than ± 1g in a matter of one minute, as shown in Fig. 2. Over the Indian region the areas where mountain waves may be frequently encountered are Assam, the Western Ghats, J& K Hills and western Himalayas during winter and the South West Monsoon season.

De[26], Sinha Ray & De[27], Sinha Ray[24] and Tyagi and Madan[28] have extensively reported these through the use of satellite cloud imageries. Aircraft reports from the flights Madras-Bombay (Dec. 1964) and Colombo-Karachi (Dec. 1964) have reported turbulence associated with lee waves on the lee of the western ghats at roughly 8-10 km levels, Sarker[29]. A typical vertical velocity profile is shown in Fig. 3.

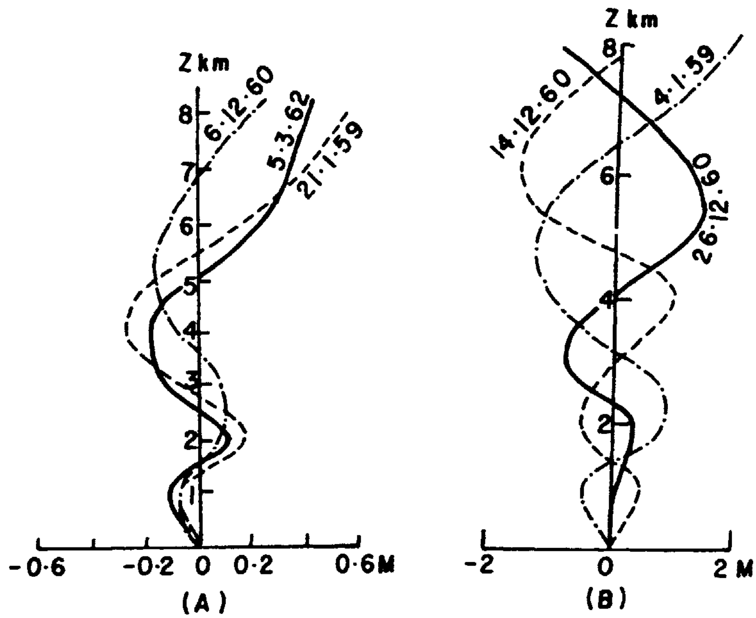


Fig 3 Variation of wave vertical velocity with height (Western Ghats)
(A) For short wavelengths and (B) For long wavelengths

Mukherjee[30] *et al.* have studied the occurrence of Clear Air Turbulence (CAT) reported by aircraft of the period of 1974-75. In this study they observed cases of CAT just east of Vishakhapatnam during winter months believed to be associated with orographic effects of eastern ghats.

The next important factor from the point of view of aviator is the amplitude of any lee wave. However, from an operational point it is difficult to predict it accurately. For a two-dimensional linear steady state model in x-z plane. Following Döös[5,31], the lee wave amplitude is given by

$$\zeta(x,z) = -\frac{U_0}{U_z}\sqrt{\frac{\rho_0}{\rho_z}}\, 2\pi ab e^{-ak} \operatorname{Sin} kx \cdot \frac{J_m\left(2 f_0^{1/2} e^{-\lambda z/2}\right)}{\frac{d}{dk} J_m\left(2\frac{f_0^{1/2}}{\lambda}\right)}, \qquad \ldots (5)$$

where $l^2 = f(z) = f_0\, e^{-\lambda z}$ ($f_0 < \lambda$ being constants) a is the half width and b the height of symmetrical bell shaped barrier and J is the Bessel function of the first kind.

Thus it depends on both the characteristics of the airstream as well as the features of the topography. In this respect, the half width of the mountain is a rather important factor. The dependence of the lee wave amplitude on the horizontal scale may be regarded as a "Resonance effect". If the power spectrum of the natural topography possesses a maxima near the natural wavelength of the air stream then the amplitude of the lee wave will be greater than otherwise. One can say if the horizontal scale of the barrier roughly coincides with the lee wavelength, the amplitude will be much larger than both, for broader or narrower mountains.

Descending currents on the leeward side of the mountain barrier close to the mountain top are a potential cause for accident if not adequately cared for. These descending currents create a cloud free zone due to adiabatic warming and an experienced pilot must avoid this pitfall. As mentioned in the earlier sections the occurrence of rotor zone due to closed circulation close to the ground (as a result of large tilt in the streamlines with height) is yet another source potential hazard to aircraft.

These should be avoided by aircraft descending or climbing near the leeward face of the mountains, when conditions for occurrence of lee waves are satisfied. These zones being close to the ground are often visible in the form of "Rotor Clouds" as shown schematically in Fig. 4. But if the air is not very moist such visible "alert" signs are missing thus making it more difficult to avoid them.

The occurrence of lee waves depends very crucially on the airstream characteristics. Use of Scorer's parameter l^2 to identify potential situation has been quite satisfactory provided the mountain top level wind ranges at least between 8-13 mps and it has a substantial component normal to the ridge line i.e., a southerly wind will be favourable for a East-west extending ridge and a westerly flow will be favourable for a north-south extending ridge. Scorer showed that no lee wave could occur if l^2 is constant with height. Introducing the concept of layered atmosphere he proved that for the occurrence of lee waves in a two layer model

$$l_1^2 - l_2^2 \geqslant \frac{\pi^2}{4h^2},$$

where l_1^2 and l_2^2 are the values of l^2 in the lower and upper layer respectively and h is the depth of the lower layer. In addition, an inversion layer or isother-

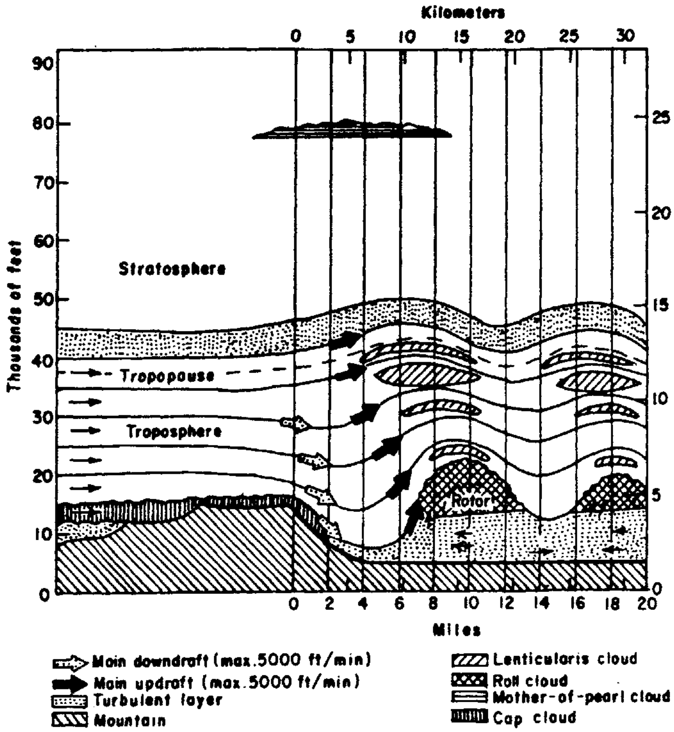


Fig 4 Idealized cross section of a mountain wave. Wind flow is left to right

mal layer in the lower or middle troposphere with a Jet stream (less stable) layer at 200-250 hPa are known to favour occurrence lee waves.

In a stratified atmosphere vertically propagating gravity waves occur when the intrinsic frequency is less than the Brunt Vaisala frequency. These waves propagate energy upward from the ground. On the other hand trapped lee waves occur down stream of the barrier when there is either (*a*) decrease in stability with height or (*b*) an increase in the wind speed with height or both.

Diurnal variation of wave length occur due to change of atmospheric stability between morning and evening. Changes in the synoptic pattern also affect the wave length. A rough estimation of lee wave length can be made by

$$L = 2\pi \bar{U}\left[\frac{T}{g(\gamma^* - \gamma)}\right]^{1/2}$$

if the lapse rate is taken as 5°C/km

$$L = 0.5\bar{U}.$$

The presence of an inversion layer is not a necessary condition for the occurrence of these waves; though the presence of an inversion layer produces a condition for occurrence maximum amplitude in a narrow zone of high static stability. Observations have indicated that the wave intensity decreases rapidly above the inversion layer. This fact is of considerable use for the forecasters to advise the pilots in choosing the flight level in a forecast—mountain wave situation.

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