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SCATTERING OF RAYLEIGH WAVES DUE TO PLANE BARRIERS IN THE SURFACE OF DEEP OCEANS

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The problem of Rayleigh wave scattering due to rigid plane vertical barriers in the surface of a deep ocean has been discussed. Deep ocean is a liquid half space ($z \geqslant 0$, $-\infty < x < \infty$) and equally spaced plane vertical barriers of small depth $H(x = ma$, $m = 0, 1, 2, ..., n)$ are erected artificially in the surface of the deep ocean. The elastic medium is homogeneous, isotropic and slightly dissipative. The Wiener-Hopf technique is the method of solution. Evaluation of the integrals along appropriate contours in the complex plane yields the reflected, transmitted and the scattered waves. The scattered waves are originating at the tips (ma, H) of the plane barriers and at their images (ma, $-$H) in the free surface. The numerical computations for the amplitude of the scattered waves have been made versus the wave number. The amplitude falls off rapidly as the wave number increases very slowly.

Key Words: Rayleigh; Wave; Scattering; Elastic Medium; Seismic Waves

Introduction

Seismic waves appear on the surface of the earth during an earthquake and let loose their energies around the inhomogeneities and irregularities on the surface of the earth. Rayleigh wave is mainly responsible for the destruction of buildings and loss of human lives. Scattering of Rayleigh waves due to surface defects results in large amplification of waves during earthquakes. The problem of scattering of Rayleigh waves at the edges of rigid plane barriers requires investigation.

The effect of vertical plane barriers fixed in an infinitely deep sea, on normally incident surface waves was first studied by Ursell[1] for the two-dimensional case. Faulkner[2] extended his solution to the three-dimensional case to study the diffraction of obliquely incident surface waves by a vertical barrier of finite depth. Deshwal and Mann[3] studied the problem of scattering of Rayleigh waves due to a rigid plane barrier in a liquid half-space. The scattered waves are found to be cylindrical waves originating at the edge of the barrier and at its image in the free surface. Mann and Deshwal[4] also discussed the scattering behaviour of Rayleigh waves due to rigid plane barrier in the surface of a shallow ocean. Deshwal and Mann[5,6] further studied the problem of Rayleigh wave scattering at a corner of a quarter space and at a coastal region.

We propose to discuss here the problem of scattering of Rayleigh waves due to the presence of a finite number of vertical plane barriers in the surface of a deep ocean. The barriers are equally spaced ($x = ma$, $m = 0, 1, 2, ..., n$) and of equal length H. They are assumed to be rigid permitting no displacement across them. The paper has application to scattering of seismic waves due to (i) artificially created plane barriers in the surface of a deep ocean or (ii) docks or (iii) patches of pack-ice.

Formulation of the Problem

The origin of co-ordinates is taken in the free surface of deep ocean with x-axis lying in the free surface and z-axis pointing vertically downward. The oceanic water is assumed to be a homogeneous, isotropic and slightly dissipative liquid half space. The $(n+1)$ rigid vertical plane barriers of small depth H are held parallel to z-axis at equal distances a in the free surface. The two-dimensional wave equation is

$$\frac{\partial^2 \bar{\phi}}{\partial x^2} + \frac{\partial^2 \bar{\phi}}{\partial z^2} = \frac{1}{c^2}\left(\frac{\partial^2 \bar{\phi}}{\partial t^2} + \varepsilon \frac{\partial \bar{\phi}}{\partial t}\right), \qquad \ldots (1)$$

where c is the velocity of wave propagation and $\varepsilon > 0$ is a damping constant. A time harmonic two-dimensional Rayleigh wave

$$\phi_i(x,z) = A_0 e^{-i\alpha_0 x - \gamma_0 z}, \ \gamma_0 = \pm(\alpha_0^2 - k^2)^{1/2}, \qquad \ldots (2)$$

α_0 being the wave number for Rayleigh waves, is incident on the barriers (Fig. 1). If the potential for a time-harmonic wave be

$$\bar{\phi}(x, z, t) = \phi(x, z) e^{-iwt}, \qquad \ldots (3)$$

then (1) reduces to

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} + k^2 \phi = 0, \ k = \sqrt{(w^2 + i\varepsilon w)}/c = k_1 + ik_2 \qquad \ldots (4)$$

The imaginary part of k is small and positive. Let the total potential be

$$\phi_t(x, z) = \phi_i(x, z) + \phi(x, z). \qquad \ldots (5)$$

Boundary Conditions

Various relations to be satisfied on the boundaries are

(i) $\phi(x, z)$ is bounded as $z - \infty$, ... (6)

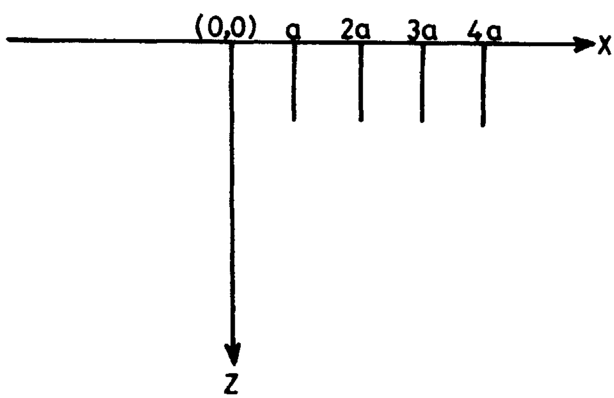

Fig 1 Vertical plane barriers in the surface of a liquid half space

(ii) $\phi_1(x, z)=0, z=0$ for all x, ... (7)

(iii) $u=\frac{\partial\phi_1}{\partial x}=0, x=ma, 0\leqslant z\leqslant H$ ... (8)

$m=0, 1, 2, ..., n$.

u is the displacement component at any point (x, z). It is assumed that for given z, $\phi(x, z)$ has the behaviour of $\exp(-d|x|)$ as $|x|\to\infty$, $d>0$. If we define the Fourier transforms

$$\bar{\phi}(\alpha, z)=\int_{-\infty}^{0}\phi(x, z)e^{i\alpha x}dx+\int_{0}^{\infty}\phi(x, z)e^{i\alpha x}dx,$$

$$\bar{\phi}(\alpha, z)=\bar{\phi}_-(\alpha, z)+\bar{\phi}_+(\alpha, z), \quad \text{... (9)}$$

then $\bar{\phi}_+(\alpha, z)$ and $\bar{\phi}_-(\alpha, z)$ are analytic in the regions $\Gamma>-d$ and $\Gamma<d$ $(\alpha=\sigma+i\Gamma)$ respectively of the complex α-plane, σ being the real-part of the complex number α. Hence $\bar{\phi}(\alpha, z)$ along with its derivatives is analytic in the strip $-d<\Gamma<d$ of the complex α-plane.

Solution of the Problem

Taking Fourier transformation of (4), we get

$$\left(\frac{d^2}{dz^2}-\gamma^2\right)\bar{\phi}(\alpha, z)=0, \ \gamma=\pm\sqrt{\alpha^2-k^2}. \quad \text{... (10)}$$

We choose that sign before the radical in (10) which makes the real part of $\gamma\geqslant 0$ for all α. The solution of the differential equation (10) is

$$\bar{\phi}(\alpha, z)=A(\alpha)e^{\gamma z}+B(\alpha)e^{-\gamma z} \quad \text{... (11)}$$

Since $\bar{\phi}(\alpha,z)$ is bounded as $z\to\infty$, therefore, $A(\alpha)=0$ and (11) leads to

$$\bar{\phi}(\alpha)=-\frac{\bar{\phi}'(\alpha)}{\gamma}, \quad \text{... (12)}$$

where $\bar{\phi}(\alpha)$, $\bar{\phi}'(\alpha)$ denote $\bar{\phi}(\alpha, H)$ and $\bar{\phi}'(\alpha, H)$ respectively. Similar notations are used for $\bar{\phi}_+(\alpha, H)$ and $\bar{\phi}_-(\alpha, H)$ and for their derivatives. Decomposition of (12) by Wiener-Hopf technique and application of Liouville's theorem gives

$$\bar{\phi}_+(\alpha)=g(\alpha)\bar{\phi}'_+(k)-\frac{\bar{\phi}'_+(\alpha)}{\gamma}, \quad \text{... (13)}$$

$$\bar{\phi}_-(\alpha)=-g(\alpha)\bar{\phi}'_+(k)-\frac{\bar{\phi}'_-(\alpha)}{\gamma} \quad \text{... (14)}$$

where $\bar{\phi}'_+(k)=\bar{\phi}'_-(-k)$ and

$$g(\alpha)=\frac{1}{\sqrt{2k(\alpha-k)}}-\frac{1}{\sqrt{2k(\alpha+k)}}. \quad \text{... (15)}$$

Similarly,

$$\bar{\phi}_{+}(-\alpha)=-g(\alpha)\,\bar{\phi}'_{+}(k)-\frac{\bar{\phi}'_{+}(-\alpha)}{\gamma} \quad \ldots (16)$$

$$\bar{\phi}_{-}(-\alpha)=g(\alpha)\,\bar{\phi}'_{+}(k)-\frac{\bar{\phi}'_{-}(-\alpha)}{\gamma}. \quad \ldots (17)$$

The Fourier transform of (4) when x varies from $-\infty$ to 0 and use of (8) leads to a differential equation whose complete solution is

$$\bar{\phi}_{-}(\alpha, z)+\bar{\phi}_{-}(-\alpha, z)=A_1(\alpha)e^{\gamma z}+A_2(\alpha)e^{-\gamma z}+\frac{2i\alpha_0 A_0 e^{-\gamma_0 z}}{\alpha^2-\alpha_0^2},\ 0\leqslant z\leqslant H. \quad \ldots (18)$$

The Fourier transform of (7) between $-\infty$ and 0 gives

$$\bar{\phi}_{-}(\alpha, o)+\bar{\phi}_{-}(-\alpha, 0)=\frac{2iA_0\alpha_0}{\alpha^2-\alpha_0^2},\ \alpha\neq\pm\alpha_0. \quad \ldots (19)$$

From (18) and (19), we get

$$\bar{\phi}_{-}(\alpha, z)+\bar{\phi}_{-}(-\alpha, z)=2A_1(\alpha)\sinh \alpha z+\frac{2i\alpha_0 A_0 e^{-\gamma_0 z}}{\alpha^2-\alpha_0^2} \quad \ldots (20)$$

Elimination of $A_1(\alpha)$ between (20) and its derivative with respect to z when $z=H$ leads to

$$\bar{\phi}_{-}(\alpha)+\bar{\phi}_{-}(-\alpha)=\frac{\tanh\gamma H}{\gamma}\left[\bar{\phi}'_{-}(\alpha)+\bar{\phi}'_{-}(-\alpha)+\frac{2i\alpha_0\gamma_0 A_0 e^{-\gamma_0 H}}{\alpha^2-\alpha_0^2}\right]$$
$$+\frac{2i\alpha_0 A_0 e^{-\gamma_0 H}}{\alpha^2-\alpha_0^2}. \quad \ldots (21)$$

Intergrating (4) from $x=0$ to $x=a$, $x=a$ to $x=2a$, ..., $x=(n-1)a$ to $x=na$, after multiplying it by $e^{i\alpha x}$ and adding, we get

$$\left(\frac{d^2}{dz^2}-\gamma^2\right)\bar{\phi}_a(\alpha, z)=-\left(\frac{\partial\phi}{\partial x}\right)_{x=na}e^{ina\alpha}+\left(\frac{\partial\phi}{\partial x}\right)_{x=0}$$
$$+i\alpha(\phi)_{x=na}\,e^{ina\alpha}-i\alpha(\phi)_{x=0}, \quad \ldots (22)$$

$0\leqslant z\leqslant H$ and $\bar{\phi}_a(\alpha, z)=\int_0^{na}\phi(x, z)\,e^{i\alpha x}\,dx.$

The right hand side of (22) is obtained by using the boundary conditions (7-8) and the result that $\phi_t=0$ on (o, H) and (ma, H). Adding (22) to the new result obtained by changing α to $-\alpha$ in it, we get the differential equation whose solution is

$$\bar{\phi}_a(\alpha, z)\,e^{-ina\alpha}+\bar{\phi}_a(-\alpha, z)e^{ina\alpha}=C_1(\alpha)\,e^{\gamma z}+C_2(\alpha)\,e^{-\gamma z}$$

$$-\frac{2iA_0e^{-\gamma_0 z}}{\alpha^2-\alpha_0^2}\left[\alpha_0\cos(na\alpha)-i\alpha\sin(na\alpha)-\alpha_0e^{-ina\alpha_0}\right]. \qquad \ldots (23)$$

Likewise, (7) is integrated on $z=0$ to get

$$\bar{\phi}_a(\alpha, 0)\, e^{-ina\alpha}+\bar{\phi}_a(-\alpha, 0)\, e^{ina\alpha}=-\frac{2iA_0}{\alpha^2-\alpha_0^2}\left[\alpha\cos(na\alpha)-i\alpha\sin(na\alpha)-\alpha_0e^{-ina\alpha_0}\right]$$

$$\ldots (24)$$

Using (24) and (23), we obtain

$$\bar{\phi}_a(\alpha, z)e^{-ina\alpha}+\bar{\phi}_a(-\alpha, z)e^{ina\alpha}=C_2(\alpha)\sinh\gamma z-\frac{2iA_0e^{-\gamma_0 z}}{\alpha^2-\alpha_0^2}$$

$$\times\left[\alpha_0\cos(na\alpha)-i\alpha\sin(na\alpha)-\alpha_0e^{-i\alpha_0 na}\right] \qquad \ldots (25)$$

Elimination of $C_2(\alpha)$ between (25) and its derivatives when $z=H$, yields to

$$\bar{\phi}_a(\alpha)\, e^{-ina\alpha}+\bar{\phi}_a(-\alpha)\, e^{ina\alpha}=\frac{\tanh\gamma H}{\gamma}\Big[\bar{\phi}'_a(\alpha)\, e^{-ina\alpha}+\bar{\phi}'_a(-\alpha)\, e^{ina\alpha}$$

$$-\frac{2iA_0\gamma_0e^{-\gamma_0 H}}{\alpha^2-\alpha_0^2}\left(\alpha_0\cos(na\alpha)-i\alpha\sin(na\alpha)-\alpha_0e^{-ina\alpha_0}\right)\Big]$$

$$-\frac{2iA_0e^{-\gamma_0 H}}{\alpha^2-\alpha_0^2}\left(\alpha_0\cos(na\alpha)-i\alpha\sin(na\alpha)-\alpha_0e^{-ina\alpha_0}\right)$$

$$\ldots (26)$$

In the same way, the Fourier transform of (4) as x varies from na to ∞ gives us

$$\bar{\phi}_{+a}(\alpha)\, e^{-ina\alpha}+\bar{\phi}_{+a}(-\alpha)\, e^{ina\alpha}=\frac{\tanh\gamma H}{\gamma}\Big[\bar{\phi}'_{+a}(\alpha)\, e^{-ina\alpha}+\bar{\phi}'_{+a}(-\alpha)\, e^{ina\alpha}$$

$$-2iA_0\alpha_0\gamma_0\, e^{-ina\alpha_0-\gamma_0 H}\Big]-\frac{2i\alpha_0A_0e^{-ina\alpha_0-\gamma_0 H}}{\alpha^2-\alpha_0^2} \qquad \ldots (27)$$

where

$$\bar{\phi}_{+a}(\alpha, z)=\int_{na}^{\infty}\phi(x, z)\, e^{i\alpha x}\, dx\, .$$

Adding (26) and (27), it is obtained that

$$\bar{\phi}_{+}(\alpha)\, e^{-ina\alpha}+\bar{\phi}_{+}(-\alpha)\, e^{ina\alpha}=\frac{\tanh\gamma H}{\gamma}\Big[\bar{\phi}'_{+}(\alpha)\, e^{-ina\alpha}+\bar{\phi}'_{+}(-\alpha)\, e^{ina\alpha}$$

$$-\frac{2iA_0\gamma_0e^{-\gamma_0 H}}{\alpha^2-\alpha_0^2}\left(\alpha_0\cos(na\alpha)-i\alpha\sin(na\alpha)\right)\Big]$$

$$-\frac{2iA_0e^{-\gamma_0H}}{\alpha^2-\alpha_0^2}(\alpha_0\cos(na\alpha)-i\alpha\sin(na\alpha)). \qquad \ldots (28)$$

Using (13) and (16) in (28), we get

$$(\bar{\phi}'_+(\alpha)e^{-ina\alpha}+\bar{\phi}'_+(-\alpha)e^{ina\alpha})\frac{e^{\gamma H}}{\gamma\cosh\gamma H}=\frac{2iA_0\gamma_0e^{-\gamma_0H}}{\alpha^2-\alpha_0^2}\cdot\frac{e^{\gamma H}}{\gamma\cosh\gamma H}$$

$$[\alpha_0\cos(na\alpha)-i\alpha\sin(na\alpha)]-2ig(\alpha)\sin(na\alpha)\,\bar{\phi}'_+(k)$$

$$+\frac{2iA_0e^{-\gamma_0H}}{\alpha^2-\alpha_0^2}[\alpha_0\cos(na\alpha)-i\alpha\sin(na\alpha)] \qquad \ldots (29)$$

Similarly, from (14), (17) and (21), we get

$$[\bar{\phi}'_-(\alpha)+\bar{\phi}'_-(-\alpha)]\frac{e^{\gamma H}}{\gamma\cosh\gamma H}=\frac{-2i\alpha_0\gamma_0A_0e^{-\gamma_0H}}{\alpha^2-\alpha_0^2}\cdot\frac{e^{\gamma H}}{\gamma\cosh\gamma H}$$

$$-\frac{2i\alpha_0A_0e^{-\gamma_0H}}{\alpha^2-\alpha_0^2} \qquad \ldots (30)$$

Factorization and Decomposition

Let us now factorize $\cosh\gamma H\exp(-\gamma H)$. We write

$$\exp(-\gamma H)=\exp[-T_+(\alpha)-T_-(\alpha)] \qquad \ldots (31)$$

where

$$T_+(\alpha)=\gamma H\pi^{-1}\cos^{-1}(\alpha/k)\sim i\alpha H\pi^{-1}\log(2\alpha/k) \qquad \ldots (32)$$

as $|\alpha|\to\infty$ and

$$T_-(\alpha)=T_+(-\alpha) \qquad \ldots (33)$$

The factorization of $\cosh\gamma H\exp(-\gamma H)/\gamma H$ as an infinite product is (Noble (1958))

$$L(\alpha)=L_+(\alpha)L_-(\alpha)=\frac{\exp(-\gamma H)\cosh\gamma H}{\gamma H}$$

$$=\frac{\exp[-T_+(\alpha)-T_-(\alpha)]}{H(\alpha+k)^{1/2}(\alpha-k)^{1/2}}\prod_{n=1}^{\infty}[1-k^2b_{n-1/2}^2+\alpha^2b_{n-1/2}^2] \qquad \ldots (34)$$

where

$$L_-(\alpha)=\frac{\exp[X(\alpha)-T_-(\alpha)]}{[H(\alpha-k)]^{1/2}}\prod_{n=1}^{\infty}[(1-k^2b_{n-1/2}^2)^{1/2}+i\alpha b_{n-1/2}]\exp\left(\frac{-i\alpha H}{\pi(n-1/2)}\right), \qquad \ldots (35)$$

$b_{n-1/2} = H/(n-1/2)\pi$ and $X(\alpha)$ is an arbitrary function to give a suitable behaviour of $L_-(\alpha)$ as $|\alpha| \to \infty$. The behaviour of $L_-(\alpha)$ as $|\alpha| \to \infty$ is given by

$$L_-(\alpha) = \frac{\exp[X(\alpha) + i\alpha H\pi^{-1}\log(-2\alpha/k)]}{[H(\alpha-k)]^{1/2}} \prod_{n=1}^{\infty}\left[1 + \frac{i\alpha H\pi^{-1}}{n-1/2}\right]\exp\left(-\frac{i\alpha H\pi^{-1}}{n-1/2}\right). \quad \ldots (36)$$

The infinite product in (36) is approximated by the result[9] (ex. 10, p. 41)

$$\prod_{n=1}^{\infty}\left[1 + \frac{\alpha}{n-1/2}\right]\exp\left(-\frac{\alpha}{n-1/2}\right) \sim \exp(\alpha + 1/2 - C_1\alpha)2^{-2\alpha} \quad \ldots (37)$$

where $C_1 = 0.5772$ is Euler's constant. Therefore,

$$L_-(\alpha) \sim \frac{\exp[X(\alpha) + i\alpha H\pi^{-1}(1 - C_1 + \log(\pi/2kH)) - \alpha H/2]}{[H(\alpha-k)]^{1/2}} \quad \ldots (38)$$

is asymptotic to $(\alpha)^{-1/2}$ as $\alpha \to \infty$, if

$$X(\alpha) = -i\alpha H\pi^{-1}(1 - C_1 + \log(\pi/2kH)) + \alpha H/2. \quad \ldots (39)$$

Using (34) in (30) and decomposing the resulting equation, we obtain

$$\frac{\bar{\phi}'_-(\alpha)}{H(\alpha-k)L_-(\alpha)} + \frac{2i\alpha_0\gamma_0 A_0 e^{-\gamma_0 H}}{H(\alpha+\alpha_0)}\left[\frac{1}{(\alpha-k)(\alpha-\alpha_0)L_-(\alpha)} - \frac{1}{2(\alpha_0+k)\alpha_0 L_-(-\alpha_0)}\right]$$

$$+ \frac{\bar{\phi}'_-(-\alpha_m)}{H(\alpha-k)L_-(\alpha)} + \frac{iA_0 e^{-\gamma_0 H}(\alpha_0+k)L_+(\alpha_0)}{\alpha-\alpha_0}$$

$$= \frac{-1}{H(\alpha-k)L_-(\alpha)}[\bar{\phi}'_-(-\alpha) - \bar{\phi}'_-(-\alpha_m)]$$

$$- \frac{i\gamma_0 A_0 e^{-\gamma_0 H}}{H(\alpha+\alpha_0)(\alpha_0+k)L_-(-\alpha_0)} - \frac{2i\alpha_0 A_0 e^{-\gamma_0 H}}{(\alpha-\alpha_0)}\left[\frac{L_+(\alpha)(\alpha+k)}{\alpha+\alpha_0}\right.$$

$$\left. - \frac{L_+(\alpha_0)(\alpha_0+k)}{2\alpha_0}\right] \quad \ldots (40)$$

where $\alpha_m = k$, $\alpha_n \cdot \alpha_n$ are zeros of $L_-(\alpha) = 0$. By analytic continuation and Liouville's theorem, each member is zero and we have

$$\bar{\phi}'_-(\alpha) = \frac{-2i\alpha_0\gamma_0 A_0 e^{-\gamma_0 H}(\alpha-k)L_-(\alpha)}{\alpha+\alpha_0}\left[\frac{1}{(\alpha-k)(\alpha-\alpha_0)L_-(\alpha)}\right.$$

$$-\frac{1}{2(\alpha_0+k)\alpha_0 L_-(-\alpha_0)}\Bigg]-\bar{\phi}'_-(-\alpha_m)$$

$$-\frac{iA_0 e^{-\gamma_0 H}(\alpha_0+k)L_+(\alpha_0)H(\alpha-k)L_-(\alpha)}{(\alpha-\alpha_0)} \qquad \ldots (41)$$

Similarly decomposing (29), we get

$$\bar{\phi}'_+(\alpha)=\bar{\phi}'_+(\alpha_m)\,e^{ina(\alpha-\alpha_m)}+\frac{iA_0\gamma_0 G_1(-\alpha_0)(\alpha-k)L_-(\alpha)}{(\alpha+\alpha_0)(\alpha_0+k)\alpha_0 L_-(-\alpha_0)}e^{ina\alpha-\gamma_0 H}$$

$$-2ig(\alpha)\sin(na\alpha)\,\bar{\phi}'_+(k)(\alpha+k)(\alpha-k)\,H\,L(\alpha)$$

$$+\frac{2iA_0 e^{-\gamma_0 H}e^{ina\alpha}H(\alpha-k)L_-(\alpha)}{(\alpha-\alpha_0)}\Bigg[\frac{G_1(\alpha)(\alpha+k)L(\alpha)}{(\alpha+\alpha_0)L_-(\alpha)}$$

$$-\frac{G_1(\alpha_0)(\alpha_0+k)L(\alpha_0)}{(2\alpha_0)L_-(\alpha_0)}\Bigg] \qquad \ldots (42)$$

where

$$G_1(\alpha)=\alpha_0\cos(na\alpha)-i\alpha\sin(na\alpha) \qquad \ldots (43)$$

Adding (41) and (42), we get

$$\bar{\phi}'(\alpha)=\bar{\phi}'_+(\alpha_m)\,e^{ina(\alpha-\alpha_m)}+\frac{iA_0\gamma_0 G_1(-\alpha_0)(\alpha-k)L_-(\alpha)}{(\alpha+\alpha_0)(\alpha_0+k)\alpha_0 L_-(-\alpha_0)}e^{ina\alpha_0-\gamma_0 H}$$

$$-2ig(\alpha)\sin(na\alpha)\,\bar{\phi}'_+(k)(\alpha+k)(\alpha-k)H\,L(\alpha)$$

$$+\frac{2iA_0 e^{-\gamma_0 H}e^{ina\alpha}H(\alpha-k)L_-(\alpha)}{(\alpha-\alpha_0)}\Bigg[\frac{G_1(\alpha)(\alpha+k)L(\alpha)}{(\alpha+\alpha_0)L_-(\alpha)}$$

$$-\frac{G_1(\alpha_0)(\alpha_0+k)L(\alpha_0)}{(2\alpha_0)L_-(\alpha_0)}\Bigg]-\bar{\phi}'_-(-\alpha_m)$$

$$-\frac{iA_0 e^{-\gamma_0 H}(\alpha_0+k)L_+(\alpha_0)H(\alpha-k)L_-(\alpha)}{\alpha-\alpha_0}$$

$$-\frac{2i\alpha_0\gamma_0 A_0 e^{-\gamma_0 H}(\alpha-k)L_-(\alpha)}{(\alpha+\alpha_0)}\Bigg[\frac{1}{(\alpha-k)(\alpha-\alpha_0)L_-(\alpha)}$$

$$-\frac{1}{2(\alpha_0+k)\alpha_0 L_-(-\alpha_0)}\Bigg] \qquad \ldots (44)$$

Reflected and Transmitted Waves

The potential function $\phi(x, z)$ is obtained by the inverse Fourier transform

$$\phi(x, z)=\frac{1}{2\pi}\int_{-\infty+i\Gamma}^{\infty+i\Gamma}\frac{\bar{\phi}'(\alpha)e^{-\gamma(z-H)}e^{-i\alpha x}}{\gamma}d\alpha \quad \ldots (45)$$

To evaluate the integral (45), the contour is taken along the line $\Gamma=\mathrm{Im}(\alpha_0)$ as shown in Fig. 2, avoiding the points $\alpha=\pm\alpha_0$. $\alpha=\pm k$ are the branch points. The condition $\mathrm{Re}(\gamma)=0$ on the branch cut as discussed by Ewing and

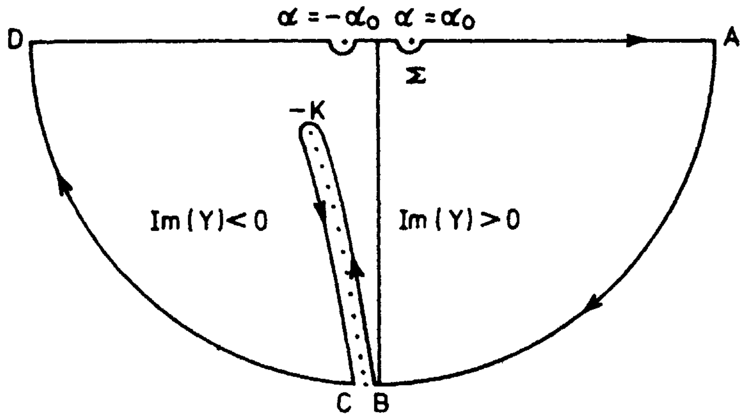

Fig 2 Contour of integration in lower half of the complex plane

Press[7] gives the points of hyperbola to be used as branch cuts with $\alpha=\pm k$ as branch points. The presence of the factor $\exp(-i\alpha x)$ makes the integral vanish along the infinite circular arcs AB and CD. The contribution of identations are

$$\phi_1(x, z)=A_0e^{-i\alpha_0 x}\quad e^{-\gamma_0 z}(1+e^{-\gamma_0 H}\cosh\gamma_0 H),\ \alpha=\alpha_0,\ x<0 \quad \ldots (46)$$

$$\phi_2(x, z)=A_0e^{i\alpha_0(x-2na)}e^{-\gamma_0 z}\cdot(1+e^{-\gamma_0 H}\cosh\gamma_0 H),\ \alpha=-\alpha_0,\ x>0 \quad \ldots (47)$$

In (46), we have waves transmitted to the region $x<0$ and in (47) we have waves reflected from the barriers in the region $x>0$.

Scattered Waves

We now evaluate the integral (45) along the branch cut $L_\alpha\cdot L_-(\alpha)$ being analytic in the lower half plane does not change its value on two sides of branch cut. $\mathrm{Im}(\gamma)$ has different signs on the opposite sides of branch cut. The main contribution comes from the neighbourhood of the branch point $\alpha=-k$, then. $\alpha=-k-iu$, u is small, since $\mathrm{Re}(\gamma)=0$ on the branch cut, therefore,

$$\begin{aligned}\gamma&=\pm\sqrt{\alpha^2-k^2}=\pm\sqrt{(k+iu)^2-k^2}=\pm\sqrt{2i(k_1+ik_2)u-u^2}\\&=\pm\sqrt{-(2k_2u+u^2)},\ k_1=0\\&=\pm i\gamma_1,\ \gamma_1=\sqrt{(2k_2u+u^2)}.\end{aligned} \quad \ldots (48)$$

Integrating (45) along two sides of the branch cut, we get

$$I(x, z)=\frac{ie^{-k_2x}}{2\pi}\int_0^\infty\left[\left(\frac{\overline{\phi}'(\alpha)e^{-\gamma(z-H)}}{\gamma}\right)_{\gamma=i\gamma_1}-\left(\frac{\overline{\phi}'(\alpha)e^{-\gamma(z-H)}}{\gamma}\right)_{\gamma=-i\gamma_1}\right]e^{-ux}du$$

$$=-\frac{e^{-k_2x}}{\pi}\int_0^\infty\left[\frac{H_1(u)\cos\gamma_1(z-H)}{\gamma_1}+2H_2(u)\cos\gamma_1H\,\sin\gamma_1z\right.$$

$$\left.-2H_3(u)\frac{\cos\gamma_1H\cos\gamma_1z}{\gamma_1}\right]e^{-ux}\,du \qquad \ldots (49)$$

Expanding $H_i(u)$ around $u=0$,

$$H_i(u)=H_i(o)+uH_i'(o)+\frac{u^2}{2!}H_i''(o)+\ldots. \qquad \ldots (50)$$

and retaining $H_i(o)$ only, we have

$$I(x, z)=-\frac{e^{-k_2(x-na)}}{\pi}\int_0^\infty\left[\frac{H_1(o)\cos\sqrt{(2k_2u+u^2)}\,(z-H)}{\sqrt{(2k_2u+u^2)}}\right.$$

$$+H_2(o)\,(\sin\sqrt{(2k_2u+u^2)}\,(z+H)+\sin\sqrt{(2k_2u+u^2)}\,(z-H))$$

$$\left.-\frac{H_3(o)\,(\cos\sqrt{(2k_2u+u^2)}\,(z+H)+\cos\sqrt{(2k_2u+u^2)}\,(z-H))}{\sqrt{(2k_2u+u^2)}}\right]e^{-ux}\,du$$

$$\ldots (51)$$

where

$$H_1(o)=-\overline{\phi}'_+(\alpha_m)e^{-ina\alpha_m}-\frac{2A_0\gamma_0G_1(-\alpha_0)\,k_2L_-(-ik_2)e^{-\gamma_0H}}{(-ik_2+\alpha_0)\,(\alpha_0+ik_2)\alpha_0L_-(-\alpha_0)}$$

$$+\frac{2A_0e^{-\gamma_0H}\,Hk_2L_-(ik_2)G_1(-\alpha_0)\,(\alpha_0+ik_2)L_+(\alpha_0)}{(-ik_2-\alpha_0)\alpha_0}$$

$$+\frac{\overline{\phi}'_-(-\alpha_m)}{e^{nak_2}}+\frac{2A_0e^{-\gamma_0H}(\alpha_0+ik_2)L_+(\alpha_0)Hk_2L_-(-ik_2)}{(-ik_2-\alpha_0)\,e^{nak_2}}$$

$$+\frac{2\alpha_0\gamma_0A_0e^{-\gamma_0H}\,k_2L_-(-ik_2)}{(-ik_2-\alpha_0)\,e^{nak_2}}\left[\frac{1}{ik_2(ik_2+\alpha_0)L_-(-ik_2)}\right.$$

$$\left.-\frac{1}{(\alpha_0+ik_2)\,\alpha_0L_-(-\alpha_0)}\right], \qquad \ldots (52)$$

$$H_2(o) = -\frac{iA_0 e^{-\gamma_0 H} G_1(-ik_2)}{(k_2^2+\alpha_0^2)} + \frac{\sinh(nak_2)\overline{\phi}'_+(ik_2)}{2k_2}, \qquad \ldots (53)$$

$$H_3(o) = \sinh(nak_2)\overline{\phi}'_+(ik_2). \qquad \ldots (54)$$

To evaluate the integral in (51), we use results by Oberhettinger[8], i.e.,

$$e^{-k_2 x - na} \int_0^\infty \frac{\cos((2k_2u+u^2)^{1/2} z)}{(2k_2u+u^2)^{1/2}} \exp(-ux)du = K_0(k_2r) \qquad \ldots (55)$$

$$e^{-k_2 x - na} \int_0^\infty \sin((2k_2u+u^2)^{1/2}z) \exp(-ux)du = \frac{k_2 z}{(r)^{1/2}} K_1(k_2r) \qquad \ldots (56)$$

where $K_n(x)$ is the modified Hankel function of order n. Using (55) and (56) in (51), we get

$$I(x, z) = -\frac{1}{\pi}\left[H_1(o) K_0(k_2r_1) + H_2(o)\left(\frac{k_2(z+H)}{(r_2)^{1/2}} K_1(k_2r_2)\right.\right.$$

$$\left.\left. + \frac{k_2(z-H)}{(r_1)^{1/2}} K_1(k_2r_1)\right) - H_3(o)\left(K_0(k_2r_2) + K_0(k_2r_1)\right)\right] \qquad \ldots (57)$$

where

$$r_1^2 = (x-na)^2 + (z-H)^2,\ r_2^2 = (x-na)^2 + (z+H)^2. \qquad \ldots (58)$$

Conclusions

The transmitted waves in (46) are independent of the distance between the barriers as well as number of barriers but the reflected waves in (47) are found to depend on the distance and number of barriers. The scattered waves are obtained in (57). For small values of r, $K_0(k_2r) \sim (\log z - \log k_2r - C)$ and for large r, $K_0(k_2r) \sim \exp(-k_2r)/\sqrt{r}$. The scattered waves behave as a decaying cylindrical wave at distant points originating at the tips (na, H) of the barriers and at their images (na, $-$H) in the free surface. Close to the tips, when r_1 and r_2 are small, the scattered field possesses a logarithmic singularity implying very large amplitude close to the scatterer. The amplitude of the scattered wave increases exponentially if the number of barriers is increased. If the integral is evaluated along the branch cut at $\alpha = k$, similar scattered waves are obtained.

If we put a = 0 or n = 0 in (57), we get

$$\frac{\exp(-\gamma_0 H - i\pi/2)}{(2k_2\pi x)^{1/2}}\left[H_1(o)\, e^{-k_2r_1} + H_2(o)\,(e^{-k_2r_2} + e^{-k_2r_1})\right.$$

$$\left. -\frac{k_2}{x} H_3(o)\left\{(z+H)\, e^{-k_2r_2} + (z-H)\, e^{-k_2r_1}\right\}\right]$$

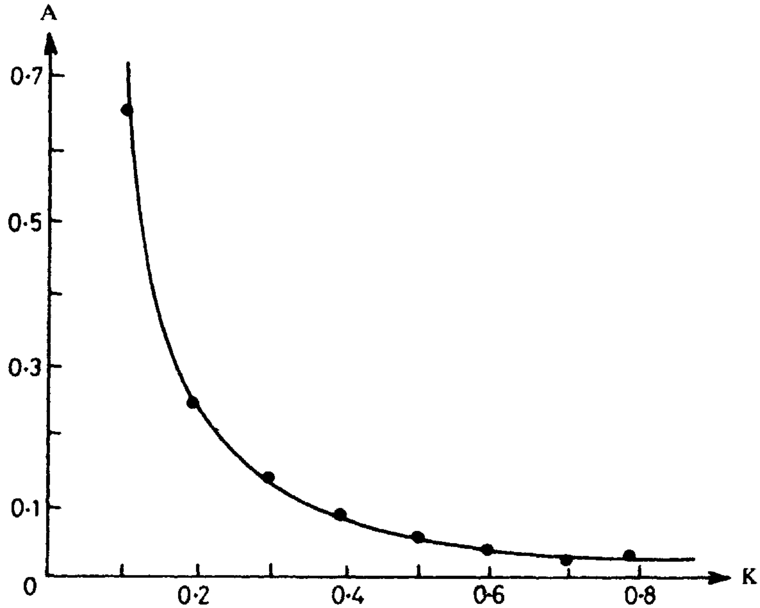

Fig 3 Amplitude of scattered wave *vs* the wave number

which is same as (97) obtained by Deshwal and Mann[3]. Thus, if $a=0$ or $n=0$, the problem reduces to one discussed by Deshwal and Mann[3] in the case of a single plane rigid barrier in the surface of the deep ocean. If $n\to\infty$, then the problem of an infinite number of plane barriers in the surface can be deduced as a particular case of the problem.

The numerical calculations for the amplitude of the scattered waves have been obtained for $n=10$, $a=0.01$km, $r_1=0.1$km, $r_2=12$km, $z\doteq H$ and $H=6$km. The amplitude of the scattered wave (Fig. 3) has been plotted versus the wave number k. The amplitude decreases rapidly as the wave number increases very slowly.

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