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ON THE STUDY OF HEAT TRANSFER CO-EFFICIENTS ON AXI-SYMMETRIC BODY UNDER SUCTION AND BLOWING

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A mathematical model has been developed to predict heat transfer rate on axi-symmetric body under suction and blowing conditions. The momentum integral equation is solved for velocity distribution. Using boundary layer characteristics, temperature distribution is obtained from the energy integral equation which is coupled with velocity gradients. Through series expansion, heat rate is computed for suction and blowing conditions. It is observed from the results that heat transfer rate is found to be decreased in blowing and the same increases under suction conditions.

Key Words: Heat Transfer, Axi-symmetric Body; Velocity Distribution; Thermal Diffusivity

Introduction

Heat transfer associated fluid mechanics problems are· interest to wide engineering process equipment. However, the phenomena is complicated for both theory and experiments due to non-linearities and coupling with velocity gradients. In order to gain insights into thermal problems associated with suction and blowing conditions, this makes an attempt to predict heat transfer rate on an axi-symmetric body through mathematical model.

Using Mangler transformation, Howe and Mersman[1] obtained solution for heat transfer co-efficients for various rates of blowing for axi-symmetric stagnation point. Reshotko and Cohen[2] obtained solution of the heat transfer problem at the nose of a blunt nosed axi-symmetric body. Squire[3] suggested an approximate method for the solution of the thermal boundary layer on a body of arbitrary shape by integrating the energy integral equation with the aid of fourth degree velocity profile. Following Sahu & Mishra[4] approaches, a model has been developed for temperature distribution using the fourteenth degree formulation. The function representing the velocity distribution is substituted in the momentum integral which in turn becomes non-linear equation with one unknown $\delta(x)$. Pohlhausen's approximate method is applied to obtain the values of $\delta(x)$ and other boundary layer characteristics δ_2, H_{12} and l. Using these results the heat transfer co-efficient for various rates of blowing and suctions has been evolved analytically.

Problem Statement and Assumptions

It has been considered that flow past a body of revolution whose sketch is shown in Fig. 1. The boundary layer flows with rotational are three dimensional nature and involves several complexities due to non-linear and couplings.

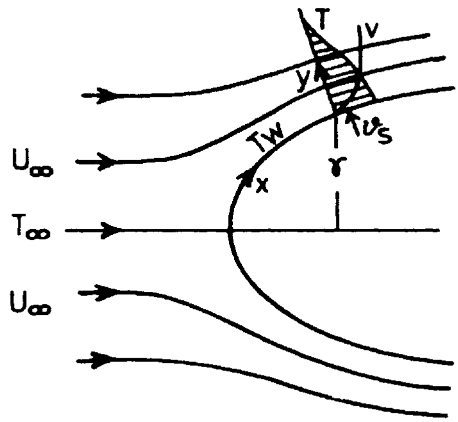

Fig 1 Flow past—a body of revolution

Hence, the axi-symmetric motion which will reduce velocity components will help to gain insights, when flow is induced to thermal environment.

In deriving the governing differential equations following assumptions are employed.

1. The fluid blown through the wall in the boundary layer is the same as the boundary layer fluid.
2. It is assumed that the thermal and velocity boundary layers which develop along the surface of the body are not affected by the development of boundary layers on any adjacent surfaces.
3. It is further assumed that in absence of all body forces the fluid is forced over the body.
4. The motion considered in this problem is laminar, stable and incompressible.
5. The velocity boundary layer or thermal boundary layer should join the conditions at infinity at the finite distance from the wall, $y = \delta(x)$ or $y = \delta_t(x)$.

Governing Equations

The equations governing the velocity and temperature distribution in a boundary layer flow past an axi-symmetric body of arbitrary shape in forced convections are, given by

Nomenclature

a	Thermal diffusivity
b	Representative length
C_p	Specific heat at constant pressure
$f(\eta)$	Function representing the velocity distribution as defined in equation (11)
$H(\Delta)$	Enthalpy thickness of thermal boundary layer as defined in equation (17)
p	Pressure

r	Radius of the suction at right angles to the axis of symmetry
T	Temperature on any arbitrary scale
T_w	Temperature of the wall
u	Velocity component in x-direction
U(x)	Potential velocity along x-direction
U_∞	Free stream velocity
v	Velocity component in y-direction
v_s	Velocity normal to the wall
x	Distance along the surface of the body
γ	Finite height in the direction normal to the surface
δ	Boundary layer thickness
δ_1	Velocity displacement thickness
δ_2	Momentum displacement thickness
δ_t	Thermal boundary layer thickness
Δ	Ratio of thermal and flow boundary layer thickness
η	Dimensionless distance from the wall in the velocity distribution field
η_t	Dimensionless distance from the wall in the temperature distribution field
ρ	Density of fluid
μ	Dynamic viscosity
K	Thermal conductivity
ν	Kinematic viscosity
θ_1	Temperature distribution
Re	Reynold's number
P_r	Prandtl number
Nu	Nusselt number

$$\frac{\partial(ru)}{\partial x} + \frac{\partial(rv)}{\partial y} = 0 \qquad \ldots (1)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U\frac{dv}{dx} + \nu\frac{\partial^2 u}{\partial y^2} \qquad \ldots (2)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{K}{\rho C_p}\frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho C_p}\left(\frac{\partial u}{\partial y}\right)^2 \qquad \ldots (3)$$

With the boundary conditions

$$y = 0: u = 0,\ v = v_s\ (v_s = \text{velocity normal to wall})\ T = T_w(x) \text{ or } \frac{\partial T}{\partial y} = 0$$

$$y = \infty: u = U(x),\ T = T_\infty \qquad \ldots (4)$$

equations (1-2) after integration over y = 0, $\delta(x)$ with the following relations.

$$\delta_1 = \int_0^\delta \left(1 - \frac{u}{v}\right) dy$$

$$\delta_2 = \int_0^\delta \frac{u}{v}\left(1 - \frac{u}{v}\right) dy$$

Eqn 5 contd

$$H_{12} = \frac{\delta_1}{\delta_2}$$

$$1 = \frac{\delta_2}{v}\left(\frac{\partial u}{\partial y}\right)_{y=0}$$

$$\lambda = \frac{\delta_2^2}{\nu}\frac{du}{dx}$$

$$\lambda_1 = \frac{v_s\delta_2}{\nu} \quad \text{and}$$

$$Z = \frac{\delta_2^2}{\nu}\cdot\frac{U}{r}\frac{dr}{dx} \qquad \ldots (5)$$

leads to

$$\frac{d}{dx}\left(\frac{\delta_2^2}{\nu}\right) = \frac{2}{v}\left[1-(2+H_{12})\lambda+\lambda_1-Z\right] \qquad \ldots (6)$$

In view of assumption (4), the energy equation in integral form is written as

$$\frac{d}{dx}\int_0^{\delta_t}\frac{u}{v}(T-T_\infty)dy = -\frac{a}{v}\left(\frac{\partial T}{\partial y}\right)_{y=0} \qquad \ldots (7)$$

Governing Equations in Non-Dimensional Quantities

Introducing the following dimensionless quantities in (6) and (7)

$$\bar{x}=\frac{x}{b}, \bar{y}=\frac{y}{b}, \bar{u}=\frac{u}{V_\infty}, \bar{v}=\frac{v}{U_\infty}, t^*=\left(\frac{\delta_2}{b}\right)^2\frac{V_a b}{\nu}$$

$$\lambda = \frac{\delta_2^2}{\nu}\frac{dv}{dx} = t^*\frac{d\bar{u}}{d\bar{x}}, \quad \bar{v}_s = \frac{v_s}{v_\infty}$$

$$\bar{U} = \frac{U(x)}{u_\infty}, \frac{u}{V} = \frac{\bar{u}}{\bar{U}}$$

$$\lambda_1 = \frac{v_s\delta_2}{\nu} = \bar{v}_s t^{*1/2}$$

$$\bar{r} = \frac{r}{b}$$

$$\theta_1 = \frac{T - T_\infty}{T_w - T_\infty}$$

We get

$$\frac{dt^*}{d\bar{x}} = \frac{2}{\bar{x}}\left[1-(2+H_{12})t^*\frac{d\bar{u}}{d\bar{x}} + \bar{v}_s t^{*1/2} - t^*\frac{\bar{U}}{\bar{r}}\frac{d\bar{r}}{d\bar{x}}\right] \quad \ldots (8)$$

and

$$\frac{d}{d\bar{x}}\int_0^{\delta_t}\frac{\bar{u}}{\bar{U}}\,\theta_1\,d\bar{y} = -\frac{a}{\bar{U}}\left(\frac{\partial\theta_1}{\partial\bar{y}}\right)_{\bar{y}=0} \quad \ldots (9)$$

For the flow in the vicinity of the stagnation point of an axi-symmetric body, the velocity in dimensionless form is given by

$$r(x) = \bar{x}$$

and

$$\bar{U}(x) = \bar{x} \quad \ldots (10)$$

Under equation (10), equation (8) and (9) are transformed to

$$\frac{dt^*}{d\bar{x}} = \frac{2}{\bar{x}}\left[1-(3+H_{12})t^* + \bar{v}_s t^{*1/2}\right] \quad \ldots (11)$$

$$\frac{d}{d\bar{x}}\int_0^{\delta_t}\frac{\bar{u}}{\bar{U}}\,e_1\,d\bar{y} = -\frac{a}{\bar{U}}\left(\frac{\partial\theta_1}{\partial\bar{y}}\right)_{\bar{y}=0} \quad \ldots (12)$$

which are solved subjected to

$\bar{y} = 0;\ \bar{u} = 0,\ \bar{v} = \bar{v}_s,\ \theta_1 = 1$

$\bar{y} = \delta_t;\ \bar{u} = \bar{U}(x),\ \theta_1 = 0$

Solution for Displacement Thickness

To solve the momentum integral equation (11), we choose a function in η satisfying all the conditions of axi-symmetric stagnation point flow in the form

$$\frac{u}{V} = f(\eta) = 1-(1-\eta)^{12}\left[1 + 5.448\eta + 11.631\eta^2\right] \quad \ldots (13)$$

At the stagnation point $\bar{x}=0$, the momentum integral equation (11) exhibits singularity. If $(dt^*/d\bar{x})$ is to remain finite at the stagnation point, implies

$$1-(H_{12}+3)t^*+\bar{v}_s t^{*1/2}=0 \qquad \dots (14)$$

Solution of equation (14) gives

$$t^{*1/2} = \frac{\bar{v}_s \pm \sqrt{\bar{v}_s^2+4(3+H_{12})l}}{2(3+H_{12})}$$

Substituting

$\delta_2 = 0.05078$

$H_{12} = 2.2744$

$l = 0.3324$

$$t^* = \left(\frac{\delta_2}{b}\right)^2 \frac{U_\infty b}{\nu} = \delta_2^2 \frac{U_\infty}{\nu b} = \delta_2^2 \frac{\frac{U}{\bar{U}}}{\frac{\nu x}{\bar{\nu}}} = \delta_2^2 \frac{U}{\nu x}$$

in it gives

$$\delta = \frac{\bar{v}_s \pm \sqrt{\bar{v}_s^2+7.0128}}{0.5348} \cdot \sqrt{\frac{\nu x}{U}} \qquad \dots (15)$$

The energy integral equaion (12) in terms of enthalpy is written as

$$\frac{d}{d\bar{x}}\left[\delta_t H(\Delta)\right] = \frac{6.551a}{\bar{u}v_\infty \delta_t} \qquad \dots (16)$$

$$\text{where} \quad H(\Delta) = \int_0^1 f(\eta)\theta_1 d\eta_t \qquad \dots (17)$$

$$\text{and} \quad \theta_1 = \left(1-\frac{\eta P_r^{1/3}}{1.804}\right)^{12}\left[1+\frac{5.448P_r^{1/3}}{1.804}\cdot\eta+\frac{11.631P_r^{2/3}}{(1.804)^2}\eta^2\right]$$

θ_1 is computed for different P_r and are plotted in Fig. 2. Integration of (14) and (15) give rise to

$$\delta_t^2 H^2(\Delta) = \frac{2.6551ab}{U}\int_0^{\bar{x}} H(\Delta)d\bar{x} \qquad \dots (18)$$

and $H(\Delta) = 0.091\,\Delta - 0.0882\Delta^2 - 0.0064\Delta^4 + 0.0654\Delta^5$ ... (19)

Taking the value of Δ as Δ = constant, the solution of (18) gives

$$\Delta^2 H(\Delta) = \frac{13.102\,ab\bar{x}}{U\delta^2}$$

Substituting $H(\Delta) = 0.091\,\Delta$, obtained from (19) for small value of $\cdot\Delta$ and the vlaue of δ from (15) in it, we get

$$\Delta = \left[\frac{13.102}{0.091}\left\{\frac{0.5348}{\bar{v}_s \pm \sqrt{\bar{v}_s^2 + 7.0128}}\right\}^2 \cdot \frac{1}{P_r}\right]^{1/3} \qquad \ldots (20)$$

The local Nussett number is defined as

$$Nu(x) = -\left(\frac{\partial \theta_1}{\partial \eta_t}\right)_{\eta_t = 0} \cdot \frac{x}{\delta_t}$$

Noting $\left(\frac{\partial \theta_1}{\partial \eta_t}\right)_{\eta_t = 0} = -6.551$

$$Nu(x) = \frac{6.551\,x}{\Delta\delta} \qquad \ldots (21)$$

Table I

Values of $Nu(x)\,P_r^{-1/3}\,Re_x^{-1/2}$ obtained with the use of formula (22) for various rates of suction and blowing

$\bar{v}_s$	$Nu(x)P_r^{-1/3}Re_x^{-1/2}$	
−2.5	0.9706	Suction
−2.0	0.9252	Suction
−1.5	0.8778	Suction
−1.0	0.8293	Suction
−0.5	0.7806	Suction
0.0	0.7331	
0.2	0.7151	Blowing
0.5	0.6888	Blowing
0.7	0.6721	Blowing
0.8	0.6640	Blowing
1.0	0.6484	Blowing

Obtaining values of δ and Δ from equation (15) and (20), equation (21) can be written as:

$$Nu(x) = 1.2499\left[\frac{0.5348}{\bar{v}_s \pm \sqrt{\bar{v}_s^2 + 7.0128}}\right]^{1/3} P_r^{1/3} Re_x^{1/2} \qquad \ldots (22)$$

Using equation (22), the values of $Nu(x)P_r^{-1/3}\ Re_x^{-1/2}$ have been calculated for various rates of suction and blowing and have been tabulated in Table I. For the case of $\bar{v}_s = 0$, the value of $Nu(x)Re_x^{-1/2}\ P_r^{-1/3}$ is calculated and found equal to 0.7333. Reshotko and Cohen approximated the value of $Nu(x)\ Re^{-1/2}\ P_r^{-0.4}$ to be 0.76. Value of $Nu(x)\ Re_x^{-1/2}$ for the case $\bar{v}_s = 0$ and $P_r = 0.7$ is calculated as 0.6511 which is very close to the value of 0.664 obtained by Howe and Mersman.

Conclusion

When P_r and Re_x are constant, the Nusselt numbers are computed for different values of $\bar{v}_s$. The increase in the rate of suction Nusselt number is found to be increased. Study of Fig. 2 shows that with decrease in the value of P_r the temperature distribution becomes flat.

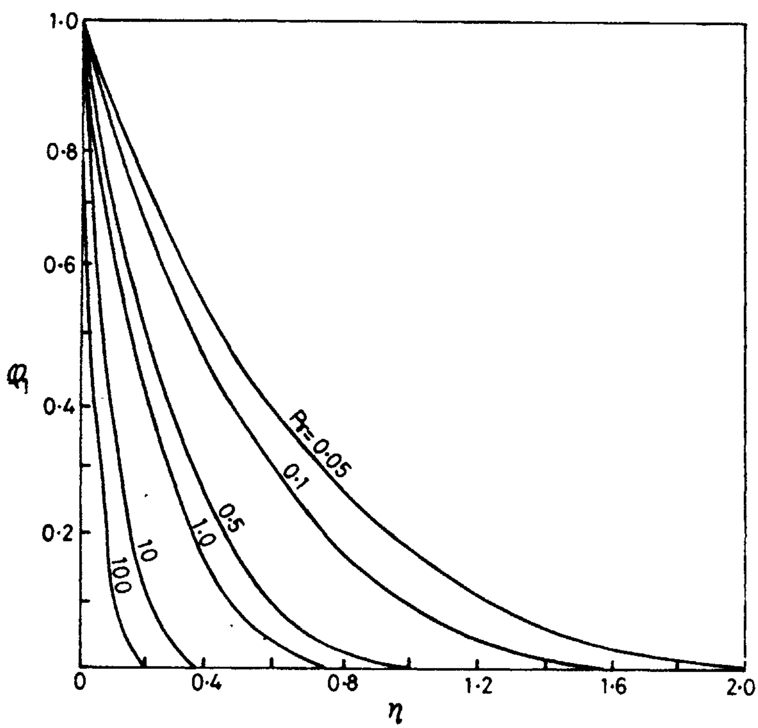

Fig 2 Curves of Q_1 for various values of Pr

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