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FORCED VIBRATIONS OF A PIEZOELECTRIC LAYER OF (6 mm) CRYSTAL CLASS

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Forced vibrations of a piezoelectric PZT-4 ceramic layer are studied by asymptotic expansion technique. For a prescribed forcing frequency and prescribed wave number, the surface tractions are of sinusoidal dependence on in-plane coordinates and on time. The response to the surface tractions of more general spatial dependence is obtained by Fourier superposition. Time being arbitrary, the nondimensional displacement component in the thickness direction, and the dimensionless electric potential at the mid plane of the layer are computed. The results thus obtained are tabulated and discussed.

Key Words: Forcing Frequency; Asymptotic Method; Electric Potential; Crystalline Materials

Introduction

Importance of the study of vibrational problems in piezoelectric materials and their wide range of applications in ultrasonics, remote sensing, wave filters, transducers etc. are extensively discussed by Mason[1] and the references cited therein. From a detailed survey of the works of many investigators as found in Dockmeci[2,3,4,5] a few have studied the forced vibrations in piezoelectric plates. Mindlin[6] discussed the forced thickness-shear and flexural vibrations of piezoelectric crystal plates. A forced vibration problem on piezoelectric trigonal crystal plate was presented by Tiersten and Mindlin[7]. Paul and Sharma[8] studied the torsional waves generated by a circular piezoelectric infinite plate of (6 2 2) crystal class when excited by an electric potential impulse applied between one of its flat plates and the opposite face.

Anandam and Venkata Subramanyam[9] studied the vibrations of a piezoelectric layer of (6mm) class under thinfilm coatings. Recently Shimizu and Kondo[10] analysed the vibrations of axially polarized axisymmetric piezoelectric vibrators by finite element method. Achenbach[11] discussed the free and forced vibrations of an elastic layer by an asymptotic method.

In this paper, the forced vibrations of a piezoelectric layer of Hexagonal (6mm) crystal class are studied by an asymptotic expansion method. This iteration method is based on asymptotic integration of the displacement equations of motion and electric potential for small wave numbers. For a prescribed forcing frequency and prescribed wave number, the response to the surface tractions of more general spatial dependence is obtained by Fourier superposition. The numerical work is carried out in the case of piezoelectric ceramic PZT-4 pertaining to (6mm) crystal class. The results are tabulated and discussed.

Basic Equations

The frame of reference and dimensionless equations of motion, as given in reference (12) are

$$\frac{d^2 U}{dr^2} - \varepsilon^2 (\bar{c}_{11} + \bar{c}_{66}\eta^2) U - (\bar{c}_{12} + \bar{c}_{66})\varepsilon^2 \eta V - (\bar{e}_{15} + \bar{e}_{31}) \frac{d\Phi}{dr} - (1 + \bar{c}_{13})\varepsilon \frac{dW}{dr}$$

$$= -\frac{\rho\omega^2}{c_{44}} hU \qquad \ldots (1)$$

$$\frac{d^2 V}{dr^2} - \varepsilon^2 (\bar{c}_{11} + \bar{c}_{66}\eta^2) V - (\bar{c}_{12} + \bar{c}_{66})\varepsilon^2 \eta U - (\bar{e}_{15} + \bar{e}_{13})\eta\varepsilon \frac{d\Phi}{dr} - (1 + \bar{c}_{13})\varepsilon\eta \frac{dW}{dr}$$

$$= -\frac{\rho\omega^2}{c_{44}} hV \qquad \ldots (2)$$

$$\bar{c}_{33}\frac{d^2 W}{dr^2} - (1+\eta^2)\varepsilon^2 W + (1 + \bar{c}_{13})\varepsilon \frac{dU}{dr} + (1 + \bar{c}_{13})\eta\varepsilon \frac{dV}{dr} + \frac{d\Phi}{dr^2} - \bar{e}_{15}(1+\eta^2)\varepsilon^2 \Phi$$

$$= -\frac{\rho\omega^2}{c_{44}} hW \qquad \ldots (3)$$

$$(\bar{e}_{31} + \bar{e}_{15})\varepsilon \frac{dU}{dr} + (\bar{e}_{31} + \bar{e}_{15})\eta\varepsilon \frac{dV}{dr} - \bar{e}_{15}(1+\eta^2)\varepsilon^2 W$$

$$+ \frac{d^2 W}{dr^2} + k_{13}^{-2}(1+\eta^2)\varepsilon^2 \Phi - k_{33}^{-2}\frac{d^2 \Phi}{dr^2} = 0 \qquad \ldots (4)$$

We seek solutions of eqs (1) to (4) where the dimensionless wave number ε is less than unity. $U(r)$, $V(r)$, $W(r)$ and $\Phi(r)$ are expressed as power series of ε, in the form

$$U(r) = \sum_{n=0}^{\infty} U_n(r)\,\varepsilon^n; \quad V(r) = \sum_{n=0}^{\infty} V_n(r)\,\varepsilon^n; \quad W(r) = \sum_{n=0}^{\infty} W_n(r)\,\varepsilon^n;$$

$$\Phi(r) = \sum_{n=0}^{\infty} \Phi_n(r)\,\varepsilon^n \qquad \ldots (5)$$

If the nondimensional forcing frequency Ω is defined as

$$\Omega = \left(\frac{\rho\omega^2}{c_{44}}\right)^{1/2} h \qquad \ldots (6)$$

the system of equations (1) to (4) analogous to equations (75) to (77) of Achenbach[11] are

$$\frac{d^2 U_n}{dr^2} + \Omega^2 U_n = (\bar{c}_{11} + \bar{c}_{66}\eta^2) U_{n-2} + (\bar{c}_{12} + \bar{c}_{66})\eta V_{n-2} + (1 + \bar{c}_{13})\frac{dW_{n-1}}{dr} + (\bar{e}_{15} + \bar{e}_{31})\frac{d\Phi_{n-1}}{dr} \qquad \ldots (7)$$

$$\frac{d^2 V_n}{dr^2} + \Omega^2 V_n = (\bar{c}_{12} + \bar{c}_{66})\eta U_{n-2} + (\bar{c}_{66} + \bar{c}_{11}\eta^2) V_{n-2} + (1 + \bar{c}_{31})\eta\frac{dW_{n-1}}{dr} + (\bar{e}_{15} + \bar{e}_{31})\eta\frac{d\Phi_{n-1}}{dr} \qquad \ldots (8)$$

$$\bar{c}_{33}\frac{d^2 W_n}{dr^2} + \frac{d^2\Phi_n}{dr^2} + \Omega^2 W_n = (1 + \eta^2) W_{n-2} - (1 + \bar{c}_{13})\frac{dU_{n-1}}{dr} - (1 + \bar{c}_{13})\eta\frac{dV_{n-1}}{dr} + \bar{e}_{15}(1 + \eta^2)\Phi_{n-2} \qquad \ldots (9)$$

$$\frac{d^2 W_n}{dr^2} - K_{33}^{-2}\frac{d^2\Phi_n}{dr^2} = -(\bar{e}_{31} + \bar{e}_{15})\frac{dU_{n-1}}{dr} - (\bar{e}_{31} + \bar{e}_{15})\eta\frac{dV_{n-1}}{dr} + \bar{e}_{15}(1 - \eta^2) W_{n-2} - K_{13}^{-2}(1 + \eta^2)\Phi_{n-2} \qquad \ldots (10)$$

On writing $\quad V_n = \eta\, U_n \qquad \ldots (11)$

the system of eqs (7) to (10) simplify as

$$\frac{d^2 U_n}{dr^2} + \Omega^2 U_n = \bar{c}_{11}(1 + \eta^2) U_{n-2} + (1 + \bar{c}_{13})\frac{dW_{n-1}}{dr} + (\bar{e}_{31} + \bar{e}_{15})\frac{d\Phi_{n-1}}{dr} \qquad \ldots (12)$$

$$(\bar{c}_{33} + k_{33}^2)\frac{d^2 W_n}{dr^2} + \Omega^2 W_n = -(1 + \eta^2)\left[A_2\frac{dU_{n-1}}{dr} - A_3 W_{n-2} - A_4\Phi_{n-2}\right] \qquad \ldots (13)$$

$$\frac{d^2\Phi_n}{dr^2} = k_{33}^2(1 + \eta^2)\left[(\bar{e}_{31} + \bar{e}_{15})\frac{dU_{n-1}}{dr} - \bar{e}_{15} W_{n-2} + k_{33}^{-2}\Phi_{n-2}\right] + k_{33}^2\frac{d^2 W_n}{dr} \qquad \ldots (14)$$

In eqs. (12) to (14)

$A_1 = (\bar{e}_{31} + \bar{e}_{15})\, k_{33}^2;\quad A_2 = A_1 + (1 + \bar{c}_{13});\quad A_3 = (1 + \bar{e}_{15} k_{33}^2);$

$$A_4 = (\bar{e}_{15} - k_{33}^2 k_{13}^{-2}) \qquad \ldots (15)$$

The stress components are as given in eq. (3.4) of ref. (12)

Let T be the applied force on the faces of the layer at $r = \pm 1$.
We consider the surface tractions of the form

$$T_{zz} = \mp T \cos k_{1x} \cos k_{2y} \sin \omega t\,, \qquad \ldots (16)$$

where the negative sign holds at $r = 1$ and the positive sign at $r = -1$. The other relevent stress components are identically zero. i.e.

$$T_{zx} = 0 = T_{yz}\,. \qquad \ldots (17)$$

This system of surface tractions produce antisymmetric motions for the forced vibrations of a piezoelectric layer as required.

For the electric potential the suitable boundary condition is

$$\Phi = 0 \quad \text{at} \quad r = \pm 1\,. \qquad \ldots (18)$$

Using the stress components as given in eq. (3.4) of ref. (12) along with eq. (2.2) of the same reference. in eqs. (16) to (18) suitably and by virtue of eq. (5), the necessary boundary conditions transform to

$$\bar{c}_{33} \frac{\mathrm{d} W_n}{\mathrm{d} r} + \frac{\mathrm{d} \Phi_n}{\mathrm{d} r} + (1 + \eta^2)\, \bar{c}_{13}\, U_{n-1} = \mp \frac{Th}{c_{44}}\,, \qquad \ldots (19)$$

$$\frac{\mathrm{d} U_n}{\mathrm{d} r} - W_{n-1} - \bar{e}_{15}\, \Phi_{n-1} = 0 \qquad \ldots (20)$$

$$\text{and} \quad \Phi_n = 0 \qquad \ldots (21)$$

Iterative Solution

The governing equations as given in equations (12) to (14) are a system of coupled, inhomogeneous ordinary differential equations with boundary conditions as given in eqs. (19) to (21). As we consider only antisymmetric motions, the solutions of eqs. (12) to (14) when $n = 0$ are

$$U_0(r) = U_{0A} \sin \Omega r \qquad \ldots (22)$$

$$W_0(r) = W_{0A} \cos \beta r \qquad \ldots (23)$$

and

$$\Phi_0(r) = W_{0A}\, k_{33}^2 (\cos \beta r - \cos \beta), \qquad \ldots (24)$$

where

$$\beta^2 = \frac{\Omega^2}{\bar{c}_{33} + k_{33}^2}\,. \qquad \ldots (25)$$

Utilising eq. (22) in the boundary condition given by equation (20)

$U_{0A} = 0$

By using eqs. (23) and (24) appropriately in boundary conditions given by eqs. (19) and (21), we have

$$W_{0A} = \frac{Th}{c_{44}(\bar{c}_{33} + k_{33}^2)\,\beta \sin\beta} \qquad \dots (26)$$

and the required results overall are

$$U_0(r) = 0 \qquad \dots (27)$$

$$W_0 = W_{0A} \cos\beta r \qquad \dots (28)$$

$$\Phi_0(r) = W_{0A}\, k_{33}^3 (\cos\beta r - \cos\beta). \qquad \dots (29)$$

When $n = 1$, the solutions are

$$U_1(r) = U_{1A} \sin\Omega r - \frac{\beta A_2 W_{0A}}{\Omega^2 - \beta^2} \sin\beta r \qquad \dots (30)$$

$$W_1(r) = 0 \qquad \dots (31)$$

and

$$\Phi_1(r) = 0 \qquad \dots (32)$$

where

$$U_{1A} = W_{0A} \frac{\cos\beta}{\Omega\cos\Omega}\left(\frac{\beta^2 A_2}{\Omega^2 - \beta^2} + 1\right). \qquad \dots (33)$$

Using equation (33) in equation (30)

$$U_1(r) = W_{0A}\left[(\beta K_1 + 1)\frac{\cos\beta}{\Omega\cos\Omega}\sin\Omega r - K_1 \sin\beta r\right]. \qquad \dots (34)$$

K_1 appearing in equation (34) and all other K_i's that appear in further analysis are combinations of forcing frequency Ω and material parameters. They are all presented in the *Appendix*.

When $n = 2$, the solutions are

$$U_2(r) = 0 \qquad \dots (35)$$

$$W_2(r) = -\frac{(1+\eta^2)\,W_{0A}}{\bar{c}_{33} + k_{33}^2}\left[\left(\frac{A_2(\beta K_1 + 1)}{\beta^2 - \Omega^2}\frac{\cos\beta}{\cos\Omega}\right)\cos\Omega r + K_2 \frac{r\sin\beta}{2\beta} + A_4 k_{33}^2 \frac{\cos\beta}{\beta^2}\right]$$

$$+ W_{2A}\cos\beta r \qquad \dots (36)$$

and

$$\Phi_2(r) = -k_{33}^2(1+\eta^2)\, W_{0A}[K_3(\cos \Omega r - \cos \Omega) + K_4(\cos \beta r - \cos \beta) + K_5] + W_{2A}\, k_{33}^2(\cos \beta r - \cos \beta), \quad \ldots (37)$$

where

$$W_{2A} = \frac{(1+\eta^2)\, W_{0A}}{(\bar{c}_{33} + k_{33}^2)\, \beta \sin \beta}[K_6 \Omega \cos \Omega - K_7 \cos \beta + K_8 \sin \beta] + W_{0A}\,. \quad \ldots (38)$$

When $n = 3$, $U_{3A}(r)$ is given by

$$U_{3A}(r) = U_{3A} \sin \Omega r - W_{0A}(1+\eta^2)[K_9 \cos \Omega r + K_{10} r \cos \beta r + K_{11} \sin \beta r] - W_{2A} K_1 \sin \beta r \quad \ldots (39)$$

On utilising the boundary condition given by eq. (20)

$$U_{3A} = W_{0A}(1+\eta^2)\left[K_9\left(\frac{1}{\Omega} - \tan \Omega\right) + K_{12}\frac{\cos \beta}{\Omega \cos \Omega} - K_{13}\frac{\sin \beta}{\Omega \cos \Omega} - \frac{\bar{e}_{15} K_5}{\Omega \cos \Omega}\right] + W_{2A}(A_2 \beta + 1)\frac{\cos \beta}{\cos \Omega} \quad \ldots (40)$$

In forced frequency problems, Ω the external forcing frequency is an important parameter and it determines to a large extent the range of ε for which the present asymptotic method is valid. If an error of r% is acceptable, the criterion is

$$W_{2a}(0)\varepsilon^2 < 0.01\, r\, W_{0A}$$

In the present case

$$1 + \frac{(1+\eta^2)}{2(\bar{c}_{33} + k_{33}^2)}\left[\frac{K_8}{\beta} - K_7\frac{\cos \beta}{\sin \beta} - \frac{A_2(\beta K_1 + 1)}{\beta^2 - \Omega^2}\frac{\cos \beta}{\cos \Omega} - A_4 k_{33}^2 \frac{\cos \beta}{\beta^2} - K_6 \frac{\Omega \cos \Omega}{\beta \sin \beta}\right]\varepsilon^2 < 0.01\, r. \quad \ldots (41)$$

In general, if n terms of $W_{nA}(r)$ are evaluated, a dimensionless wave number $\varepsilon^*(n, \Omega)$ can be computed such that the expansion is valid in the range $0 \le \varepsilon \le \varepsilon^* < 1$.

So far, in the analysis a general discussion was carried out without any specifications on surface tractions. Due to the nature of the problem considered, Fourier superposition can be used to compute the response to the surface tractions of arbitrary dependence on time and spatial coordinates x and y, when the response of the layer to time harmonic surface tractions with sinusoidal dependence on x and y is known.

Following Achenbach, the surface tractions are assumed to be symmetric with respect to x and y.

At $z = \pm h, \quad T_{zz} = \mp T(x_1, x_2) \sin \omega t$ ... (42)

A double Fourier Transform

$$T(x,y) = \frac{4}{\pi^2} \int_0^\infty \cos(\eta_1 x)\, d\eta_1 \int_0^\infty T^*(\eta_1, \eta_2) \cos(\eta_2 y)\, d\eta_2 \quad \ldots (43)$$

$$\text{where, } T^*(\eta_1, \eta_2) = \int_0^\infty \int_0^\infty T(x,y) \cos(\eta_1 x) \cos(\eta_2 y)\, dx\, dy \quad \ldots (44)$$

is utilised.

For the displacement in z-direction

$$W = \frac{4 \sin \omega t}{\pi^2 h^2} \int_0^\infty \varepsilon \cos(\varepsilon \bar{x})\, d\varepsilon \int_0^\infty T^*(\varepsilon, \eta)\, W(r, \varepsilon, \eta) \cos(\varepsilon \eta \bar{y})\, d\eta . \quad \ldots (45)$$

In eq. (45) $\bar{x} = \frac{x}{h}; \quad \bar{y} = \frac{y}{h}$ ... (46)

If we consider a distribution whose cosine transform vanishes identically outside a prescribed interval, the surface load can be taken as

$$T(x_1, x_2) = h^2 T \frac{\sin m_1 x}{x} \frac{\sin m_2 y}{y} . \quad \ldots (47)$$

By this, $T^*(\varepsilon, \eta) = h^2 T \frac{\pi^2}{4}$, ... (48)

where $\varepsilon < m_1 h$ and $\varepsilon \eta < m_2 h$

Using eq. (48) in the superposition integral given by eq. (45), nondimensional displacement in the z-direction is

$$W^* = \int_0^{m_1 h} \varepsilon \cos(\varepsilon \bar{x})\, d\varepsilon \int_0^{m_2 h} W^1(r, \varepsilon, \eta) \cos(\varepsilon \eta \bar{y})\, d\eta . \quad \ldots (49)$$

In equation (49) $m_1 \mathrm{h} < \varepsilon^*$, the upper limit for the range of wave numbers as computed from eq. (41). Further

$$W^* = \frac{c_{44} W(r,x,y)}{T h \sin \omega t}; \quad w^1(r, \varepsilon, \eta) = \frac{c_{44} W(r, \varepsilon, \eta)}{h T}$$

in which $W(r,\varepsilon,\eta) = W_{0A} + W_{2A}\varepsilon^2 + O(\varepsilon^4)$. ... (50)

From equations (29) and (37), when $r = 0$, an expression similar to eq. (50) is obtained in the form

$$\Phi^1 = \Phi_0(0) + \Phi_2(0)\varepsilon^2 + O(\varepsilon^4) \quad \ldots (51)$$

Numerical Analysis

The computational work is carried out in the case of piezoelectric ceramic PZT-4 of Hexagonal (6mm) crystal class. Its elastic, piezoelectric and dielectric constants are given in Berlincourt *et al.*[13] The upper limit of the range of wave numbers ε^* is computed with an acceptable error of $r\%$ as in eq. (41). Such an ε^* is obtained as 0.5731. When $\bar{x} = \bar{y} = 1$, the forcing frequency Ω is in the range $0.5 \leq = \Omega \leq 3$ and the nondimensional wave number ε is in the range $0.1 \leq \varepsilon \leq 0.5$, various values for the nondimensional displacement W^* in the z-direction and the dimensionless electric potential $\Phi^* = \Phi^1 \times k_{33}^{-2} \times 10^2$ at the mid plane of the layer are computed. While computing W^* as given in eq. (49), Gaussian Quadrature formula is utilised. Various values of the nondimensional

Table I

Nondimensional displacement W^*

Ω/ε	0.1	0.2	0.3	0.4	0.5
0.5	0.0298	0.1167	0.2056	0.2961	0.3763
1	0.1018	0.2015	0.2968	0.4102	0.4867
1.5	0.2639	0.3824	0.4533	0.5369	0.6036
2	0.3487	0.4586	0.5575	0.6403	0.7219
2.5	0.4765	0.5888	0.6907	0.7842	0.8745
3	0.5844	0.6979	0.8006	0.8950	0.9893

Table II

Dimensionless electric potential Φ^*

Ω/ε	0.1	0.2	0.3	0.4	0.5
0.5	0.2316	0.3464	0.4427	0.5365	0.6147
1	0.3268	Q.4379	0.5348	0.6272	0.7038
1.5	0.4377	0.5406	0.6390	0.7189	0.7904
2	0.5409	0.6513	0.7462	0.8256	0.8983
2.5	0.6529	0.7665	0.8658	0.9431	1.7482
3	0.7634	0.8748	0.9737	1.0536	1.7750

displacement component W^* in the z-direction and the dimensionless electric potential Φ^* are presented in Table I and Table II respectively. The advantage of the asymptotic technique lies in the accuracy of the solutions which can be estimated by inspection of the solutions at every stage and the relative ease in computational work.

Conclusions

Forced vibrations of a piezoelectric layer of Hexagonal (6mm) crystal class are studied by an asymptotic integration method. When the external surface tractions are of sinusoidal dependence of the inplane coordinates and on time, the amplitudes are computed. The response to the surface tractions of more general spatial dependence is obtained by Fourier superposition.

The numerical work is carried out in the case of PZT-4, a piezoelectric ceramic of (6mm) crystal class. The upper limit of the wave number is obtained with restricted error percentage. For different values of forcing frequency Ω, nondimensional displacement in z-direction with arbitrary time dependence and the dimensionless electric potential at the mid plane of the layer are computed. The results thus obtained are tabulated. Increase in both the displacement in z-direction and dimensionless electric potential at the mid plane of the layer are noted, true to the nature, for increase in forcing frequency and wave number.

Appendix

$$K_1 = \frac{\beta A_2}{\Omega^2 - \beta^2}$$

$$K_2 = A_3 - \beta K_1 A_2 - A_4 k_{33}^2$$

$$K_3 = (\beta K_1 + 1)\frac{\cos\beta}{\cos\Omega}\left[\frac{A_2}{(\beta^2 - \Omega^2)(\bar{c}_{33} + k_{33}^2)} + \Omega^2(\bar{e}_{31} + \bar{e}_{15})\right]$$

$$K_4 = \beta^2[1 - \bar{e}_{15} + K_1\beta(\bar{e}_{31} + \bar{e}_{15})]$$

$$K_5 = \frac{A_4 k_{33}^2}{\bar{c}_{33} + k_{33}^2}\,\frac{\cos\beta}{\beta^2}$$

$$K_6 = (\beta K_1 + 1)\frac{\cos\beta}{\cos\Omega}\left[\frac{\bar{c}_{13}}{\Omega^2} + \frac{\bar{c}_{33}A_2}{(\beta^2 - \Omega^2)(\bar{c}_{33} + k_{33}^2)}\right] + K_3 k_{33}^2$$

$$K_7 = \frac{K_2}{2(\bar{c}_{33} + k_{33}^2)}$$

$$K_8 = \beta k_{33}^2 K_4 - K_1 \bar{c}_{13} - \frac{K_7}{\beta}$$

$$K_9 = (\beta K_1 + 1)\frac{\cos\beta}{2\cos\Omega}\left[\frac{\bar{c}_{11}}{\Omega^2} + \frac{(1+\bar{c}_{13})A_2}{(\Omega^2-\beta^2)}\right] + \frac{A_1 K_3}{2}$$

$$K_{10} = \frac{(1+\bar{c}_{13})K_2}{2(\bar{c}_{33}+k_{33}^2)}\frac{1}{\Omega^2-\beta^2}$$

$$K_{11} = \frac{2K_{10}\beta}{\Omega^2-\beta^2} + \bar{c}_{11}K_1 + \frac{(1+\bar{c}_{13})K_2}{2(\bar{c}_{33}+k_{33}^2)\beta}$$

$$K_{12} = K_{10} + K_{11} - \frac{A_3(\beta K_{11})}{(\bar{c}_{33}+k_{33}^2)} - \frac{A_4 k_{33}^2}{\beta^2}$$

$$K_{13} = K_{10}\beta + \frac{K_2}{2\beta(\bar{c}_{33}+k_{33}^2)}$$

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