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EFFECT OF BODY FORCES ON THE PROPAGATION OF SH-TYPE WAVES IN A SEMI-INFINITE TRANSVERSELY ISOTROPIC ELASTIC MEDIUM

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The effect of time-dependent body forces on the propagation of SH-type motion in a transversely isotropic elastic half-space is discussed in this paper. The surface displacement component is obtained in a closed form for two special type of crystals. Numerical results are obtained for a particular case of time dependent body force at different distances from the source for the different values of the non-dimensional time. The results are shown graphically and are found to be in good agreement with classical results.

Key Words: Body Forces; Propagation of SH-Type Waves; Semi Infinite Transversely Isotropic Elastic Medium

Introduction

In recent years, a great deal of attention has been given to problems concerned with wave propagation in homogeneous isotropic elastic solids. Much of this work has been concerned with the problems of seismological interest involving wave propagation in a single half-space. The problem of evaluating the displacement produced in an isotropic semi-infinite medium has been studied by Cagniard[1], Lapwood[2], Garvin[3], Garvin used a modified version of Cagniard's method. Mitra[4] & Nag[5], have used the same method to determine the surface displacement due to different types of body forces. Most of the authors have considered the two dimensional motion with displacement in the plane of motion. Eason and Wilson[6] have considered the effect of impulsive tortional body forces on the displacements produced in a composite infinite isotropic solid. Following the approach given by Mitra and Roy[7] and Mitra & Maiti[8] have considered the effect of sources (asymmetric and symmetric) on the generation of different pulses and waves in an isotropic elastic half-space. They have adopted Cagniard's modified method to solve the problem.

The present problem is concerned with the transversely isotropic version of the half-space problem of SH-type waves. Unlike the isotropic problem solved by different authors, the orientation of the crystallographic axes relative to the half-space surface does make a considerable difference in the complexity of the analysis for the corresponding transversely isotropic solid.

Burridge[9] has considered the problem of surface motion in a general anisotropic half-space subjected to a point impulsive normal load without any symmetrical orientation with respect to crystallographic axes of the medium. Burridge illustrates some of his results for the cubic crystal copper.

The intention in this problem is to obtain some detailed information concerning the likely effect of time dependent body forces on the disturbance of SH-mo-

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tion in a transversely isotropic elastic half-space. The results are obtained in a closed form following the method of Nag (1961). The surface displacement in a particular case is obtained numerically for different values of the depth of the source and at different distance along the surface. The results are shown graphically and compared with corresponding isotropic case.

Formulation of the Problem

Geometry of the problem is depicted in Fig. 1, where x and y axes are taken along the free surface of the semi-infinite transversely isotropic elastic medium and z-axis ($z \geqslant 0$) perpendicular to the free surface. Our main concern here is to evaluate displacement at the surface $z=0$ due to body forces; the displacement being assumed to tend to zero as $z \to \infty$. For SH-type of waves the displacement and body forces do not depend on y and if (u, v, w) be the displacement at any point $P(x, y, z)$ into the medium then $u=w=0$. The two equations of motion are identically satisfied.

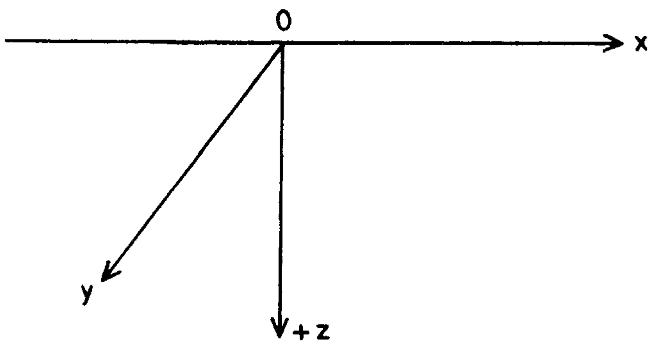

Fig 1 Geometry of the problem

The third equation of motion is

$$\rho \frac{\partial^2 v}{\partial t^2} = \rho Y + \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yz}}{\partial t}, (t \geqslant 0), \qquad \ldots (1)$$

where v is the displacement component, τ_{xy}, τ_{yz} are the stress components, Y the body force and ρ is the density of the medium. Now in a transversely isotropic elastic medium.

$$\tau_{xy} = \frac{(c_{11} - c_{12})}{2} \frac{\partial y}{\partial x}$$

and $\qquad \ldots (2)$

$$\tau_{yz} = c_{44} \frac{\partial y}{\partial z},$$

where $\frac{(c_{11} - c_{12})}{2} > 0$, $c_{44} > 0$, are real elastic parameters. Putting relation (2) in (1), the equation of motion becomes

$$\rho \frac{\partial^2 v}{\partial t^2} = \rho Y + L \frac{\partial^2 v}{\partial x^2} + N \frac{\partial^2 v}{\partial z^2} \qquad \ldots (3)$$

For simplicity, we use L & N in places of $\frac{(c_{11} - c_{12})}{2}$ & c_{44} respectively.

Therefore,

$$\frac{\partial^2 v}{\partial t^2} = \rho Y + \beta_1^2 \frac{\partial^2 v}{\partial x^2} + \beta_2^2 \frac{\partial^2 v}{\partial z^2} \qquad \ldots (4)$$

$\beta_1 = \sqrt{(L/\rho)}$ $\beta_2 = \sqrt{(N/\rho)}$ and these correspond to shear wave velocities in the medium.

Solution of the Problem

To solve eq. (4) the following transforms will be applied

$$f_1(x, z, p) = \int_0^\infty e^{-pt} f(x, z, t)\, dt, \qquad \ldots (5)$$

$(f_1(p) \doteqdot f(t)$ and $f(t) \doteqdot f_1(t)$,

$$f_2(\xi, z, p) = \int_{-\infty}^{\infty} e^{i\xi x} f_1(x, z, p)\, dx \qquad \ldots (6)$$

and

$$f_3(\xi, q, p) = \int_0^\infty e^{-qz} f_2(\xi, z, p)\, dz, \qquad \ldots (7)$$

p being real and positive.

Eqs (5) and (7) define Laplace transform while eq. (6) defines a Fourier transform.

It will be assumed that Y_3 regarded as a function of q, has no poles with positive real part and that $(Y_3 e^{qz}) \to 0$ as $R \to \infty$, where $q = Re^{i\theta}$, $\pi/2 < \theta < 3\pi/2$ and $z \to \infty$.

The initial condition and boundary conditions of the problem are

$$v(x, z, t) = \frac{\partial}{\partial t} v(x, z, t) = 0 \text{ at } t = 0. \qquad \ldots (8)$$

and

$$\tau_{yz} = 0 \text{ at } z = 0. \qquad \ldots (9)$$

Now applying transforms (5) to (7) in succession to eq. (4), we get

$$p^2 v_3 = Y_3 + v_3(q^2\beta_2^2 - \xi^2\beta_1^2) - q\beta_2^2 v_{20} - \beta_1^2 v'_{20} \qquad \ldots (10)$$

$$\text{or } v_3[q^2 - (\phi^2\xi^2 + p^2/\beta_2^2)]\,\beta_2^2 = -Y_3 + \beta^2(q^2 v_{20} + \phi v'_{20}), \qquad \ldots (11)$$

$$\phi^2 = \beta_1^2/\beta_2^2,$$

where $v_{20} = v_2(\xi, 0, p)$ and $v'_{20} = \left[\frac{\partial v_2}{\partial z}(\xi, z, p)\right]_{z=0}$

Applying the transform (5) and (6) to (9), we get

$v'_{20} = 0$.

Thus from (11)

$$v_3 = \frac{-Y_3}{\Delta} + \frac{\beta_2^2 q v_{20}}{\Delta} \qquad \dots (12)$$

and

$$\Delta = \beta_2^2(q^2 - \eta_s^2),\ \eta_s^2 = \phi^2\xi^2 + p^2/\beta_2^2 \qquad \dots (13)$$

Now inverting (13), we get

$$v_2 = \frac{1}{2\pi i} \int_{\sigma - i\infty}^{\sigma + i\infty} e^{qz}(-Y_3/\Delta)\, \mathrm{d}q + v_{20} \cosh z\eta_s \qquad \dots (14)$$

The contribution to the integral in (14) which becomes unbounded as $z \to \infty$ is from the pole at $q = \eta_s$. Now evaluating the residue at $q = \eta_s$, it is seen that the term in v_2 which becomes unbounded at $z \to \infty$ is

$$\frac{v_{20}}{2} e^{z\eta_s} - \frac{e^{\eta_s z}}{2\beta_2^2 \eta_s} Y_3(\xi, \eta_s, p) \qquad \dots (15)$$

Since a bounded solution for v is required the co-efficient of $e^{\eta_s z}$ in above expression (15) should vanish,

$$\text{i.e., } v_{20} = \frac{Y_3(\xi, \eta_s, p)}{\beta_2^2 \eta_s} \qquad \dots (16)$$

the inverse Fourier transform of (17) yields

$$v_1(x, 0, p) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{Y_3(\xi, \eta_s, p)}{\beta_2^2 \eta_s} e^{-\xi x}\, \mathrm{d}\xi \qquad \dots (17)$$

The displacement component on the free surface will now be obtained for different forms of Y by the method given by Garvin, the functions and their derivatives being taken in the distortional sense. Now onwards the suffixes will not indicate the transformed quantities except v_1 and Y_3.

Time-Dependent Body Forces

To determine Y_3 in (17) we now consider the following three different forms of Y.

Case I

Let Y represent a line force at $(0, 0, h)$ i.e., assume

$$Y = P_0\, H(t)\, \delta(x)\, \delta(z - h) \qquad \dots (18)$$

where $H(t)$, $\delta(x)$, $\delta(z-h)$ are respectively Heaviside's unit function and Dirac's delta function of argument t, x, $(z-h)$ respectively and P_0 is a constant. Therefore applying the transforms (5) to (7) in (18), we have

$$Y_3 = P_0\, e^{-qh}/p \qquad \ldots (19)$$

Eq. (17) now becomes

$$v_1(x, 0, p) = \frac{P_0}{2\pi} \int_{-\infty}^{\infty} \frac{e^{-(\eta_s h + i\xi x)}}{p\,\beta_2^2\, \eta_s}\, d\xi$$

$$= \frac{P_0}{\pi\beta_2^2\, p}\, Re \int_0^{\infty} \frac{e^{-(\eta_s h + i\xi x)}}{\eta_s}\, d\xi \qquad \ldots (20)$$

Putting $\zeta = \xi\beta_2/p$, we get

$$\eta_s = P(1+\zeta^2\phi^2)^{1/2}/\beta_2$$

Eq. (20) then becomes

$$v_1(x, 0, p) = \frac{p_0}{\pi\beta_2^2 p}\, Re \int_0^{\infty} \frac{e^{-p\{i\zeta x} + h\sqrt{i+\xi^2\phi^2}\}/\beta_2}{p\sqrt{(1+\zeta^2\phi^2)}/\beta_2}\, \frac{p}{\beta_2}\, d\zeta$$

$$= \frac{p_0}{\pi\beta_2^2\, p}\, Re \int_0^{\infty} \frac{e^{-p\{i\zeta x} + h\sqrt{1+\zeta^2\phi^2})/\beta_2}{\sqrt{(1+\zeta^2\phi^2)}}\, \frac{p}{\beta_2}\, d\zeta \qquad \ldots (21)$$

The integrand in (21) has branch points at $\zeta = \pm i$.

Now introduction of change of variable given by

$$t = (i\zeta/x + h\sqrt{1+\phi^2\zeta^2})/\beta_2$$

$$\text{or } \zeta(t) = \frac{\beta_2}{(x^2+h^2\phi^2)}\left[-itx + h\sqrt{t^2\phi^2 - \frac{x^2+h^2\phi^2}{\beta_2^2}}\right] \qquad \ldots (22)$$

will be made next. The mapping of the ζ-plane into the t-plane has been considered by Garvin and then by Nag and is shown in Fig. 2.

In brief we can use the method of Garvin and for understanding the problem we can repeat the same.

As ζ moves along the real axis from 0 to ∞, t moves along the curve AB to infinity where AB asymptotically approaches the straight line $t=(h+ix)\,\zeta/\beta_2$. As ζ changes from i through $[-ix/\sqrt{(\phi^2h^2+x^2)}]$ to $-i$ along the imaginary axis, t increases along the real axis from $-x/\beta_2$ to $\sqrt{[(\phi^2h^2+x^2)]}/\beta_2$ and again comes back through real axis to x/β_2. The region outside $[i, -i]$ of the imaginary axis is mapped on the curve C_1 and C_2.

Thus from (20) with the help of (22), we get

$$I = \frac{v_1(x, 0, p)\,\pi\beta_2^2}{P_0} = \frac{1}{p}\, Re \int_A^B e^{-pt}\, F_1[\zeta(t)]\, \frac{d\zeta(t)}{dt}\, dt, \qquad \ldots (23)$$

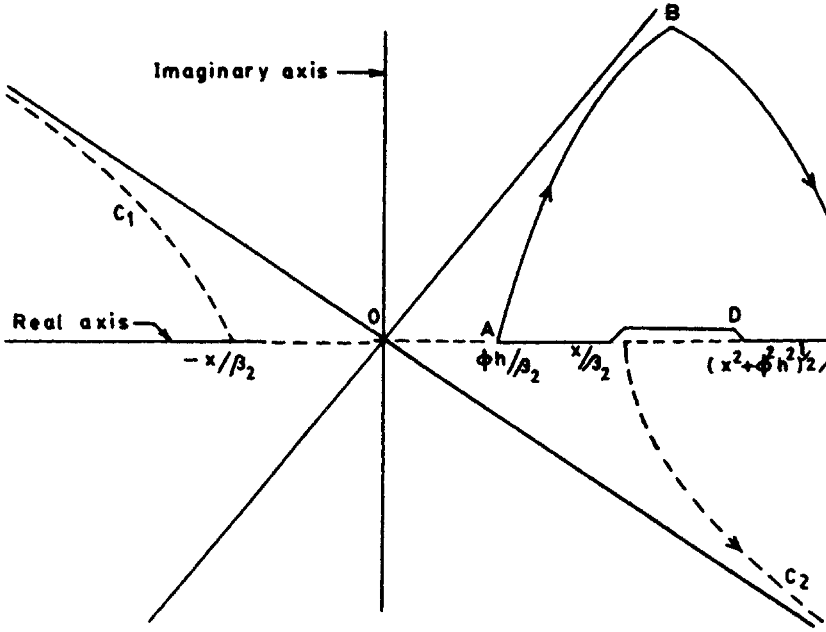

Fig 2 t-plane showing the contour mapping

where B tends to infinity along AB and

$$F_1[\zeta(t)] = \frac{1}{\sqrt{1+\zeta^2(t)\,\phi^2}}\,. \qquad \ldots (24)$$

As the integrand has no poles within the contour $ABCDA$ (the portion of AD of the contour is approached from the upper half-plane), and as

$\int_B^C e^{-pt} F_1[\zeta(t)] \frac{\mathrm{d}\zeta(t)}{\mathrm{d}t}\,\mathrm{d}t \to 0$ as $BC \to \infty$, we get

$$I = (1/p)\, Re \int_{h/\beta_2}^{\infty} e^{-pt} F_1[\zeta(t)] \frac{\mathrm{d}\zeta(t)}{\mathrm{d}t}\,\mathrm{d}t \qquad \ldots (25)$$

so that $Re \int_{h/\beta_2}^{\infty} F_1[\zeta(t)] \frac{\mathrm{d}\zeta(t)}{\mathrm{d}t} e^{-pt}\,\mathrm{d}t$ is the Laplace transform of $G_1(t)$ where

$$G_1(t) = 0,\ t < h/\beta_2$$

$$= Re\, F_1[\zeta(t)] \frac{\mathrm{d}\zeta(t)}{\mathrm{d}t},\ t > h/\beta_2,$$

For $h/\beta_2 < t < (\phi^2 h^2 + x^2)^{1/2}/\beta_2$, $F_1[\zeta(t)] \frac{\mathrm{d}\zeta(t)}{\mathrm{d}t}$ is imaginary. Hence, we can replace

h/β_2 by $(\phi^2h^2+x^2)^{1/2}/\beta_2$ in (26).

Therefore, $G_1[\zeta(t)] = \dfrac{H[t-(\phi^2h^2+x^2)^{1/2}/\beta_2]}{\{t^2-(\phi^2h^2+x^2)^{1/2}/\beta_2\}^{1/2}}$ ... (27)

As $1/p$ is the Laplace transform of $H(t)$, we get by convolution theorem of Laplace transform

$$\frac{v(x,0,t)\,\pi}{P_0} = \frac{1}{\beta_2^2}\int_0^t G_1[\zeta(\lambda)]\,d\lambda$$

$$= (1/\beta_2^2)\,H(t-(\phi^2h^2+x^2)^{1/2}/\beta_2)\cosh^{-1}\frac{t\beta_2}{\sqrt{(x^2+h^2\phi^2)}} \quad \text{... (28)}$$

Case II

Force acting over two opposite ends of a rectangular region

Let $Y = P_0H(t)[\delta(x+a)-\delta(x-a)], |z-h| \leqslant b, b<h$ ... (29)

$= 0, |z-h| > b$

Then $Y_3(\xi,\eta_s,p) = \dfrac{-4i\,P_0\,e^{-\eta_s h}\sinh\eta_s b\sin\xi a}{p\,\eta_s}$ ... (30)

Equation (17) becomes in this case

$$v_1(x,0,p) = \frac{P_0}{2\pi\beta_2^2 p}\int_{-\infty}^{\infty}\frac{-4i\sinh\eta_s b\sin\xi a\,e^{-\eta_s h-i\xi x}}{\eta_s^2}\,d\xi$$

$$= -\frac{P_0}{\pi\beta_2^2 p}\,Re\int_0^{\infty}\frac{\{e^{-i\xi(x-a)}-e^{-i\xi(x+a)}\}\{e^{-\eta_s(h-b)}-e^{-\eta_s(h+b)}\}}{\eta_s^2}\,d\xi \quad \text{... (31)}$$

Now let $x-h = x_1 = x_2;\ x+a = x_3 = x_4;$

$h-b = h_1 = h_3;\ h+b = h_2 = h_4$ . ... (32)

and $\zeta_1 = \xi\beta_2/p,\ t = [i\zeta_1x_1 + h_1\sqrt{(1+\phi^2\zeta_1^2)}]/\beta_2$,

Therefore, $\zeta_1(t) = \dfrac{\beta_2}{\phi^2h_1^2+x_1^2}\left[-itx_1 + h_1\sqrt{(\phi^2t^2-\dfrac{x_1^2+\phi^2h^2}{\beta_2^2}}\right]$ ... (33)

Taking the first term of (31), we get as in *Case I* by replacing x and h by x_1 and h_1,

$$I_1\frac{-P_0}{\pi\beta_2^2p}\,Re\int_0^{\infty}\frac{e^{-(\eta_s h+i\xi x_1)}}{\eta_s^2}\,d\xi$$

$$= \frac{-P_0}{\pi\beta_2^2p^2}\,Re\int_{h/\beta_2}^{\infty}\frac{e^{-pt}(d\zeta_1(t)/dt)}{1+h_1^2\phi^2}\,dt \quad \text{... (34)}$$

and

$$\text{i.e.,} \frac{-\pi\beta_2 p^2 I_1}{P_0} = G_2[\zeta_1(t)], \qquad \ldots (35)$$

$$\text{where } G_2[\zeta_1(t)] = 0,\ t < (\phi^2 h_1^2 + x_1^2)^{1/2}/\beta_2 \qquad \ldots (36)$$

$$= Re\left[\frac{1}{1+h_1^2\phi^2}\frac{d\zeta_1}{dt}\right],\ t > (\phi^2 h_1^2 + x_1^2)^{1/2}/\beta_2$$

As $1/p^2$ is the Laplace transform of $tH(t)$, using the product theorem of Laplace transforms and taking the other terms of (31) we have

$$\frac{v(x,0,t)\pi\beta_2}{P_0} = \int_0^t (t-\lambda)\{-G_2[\zeta_1(\lambda)] + G_2[\zeta_2(\lambda)] + G_2[\zeta_3(\lambda)] - G_2[\zeta_4(\lambda)]\}\, d\lambda, \qquad \ldots (37)$$

$$\text{where } G_2[\zeta_n(t)] = H\left(t - \frac{\sqrt{x_n^2 + \phi h_n^2}}{\beta_2}\right) Re\left[\frac{1}{1+\phi^2 h_n^2}\frac{d\zeta_n}{dt}\right]$$

$$G_2[\zeta_n(t) = H\left(t - \frac{\sqrt{x_n^2+\phi h_n^2}}{\beta_2}\right) \frac{(x_n^2 + h_n^2\phi^2)\, h_n\phi t}{\beta[t^2 - (x_n^2 + h_n^2\phi^2)/\beta_2^2]}\left[t^2 h_n^2\phi^2 + x_n^2\left(t^2 - \frac{x_n^2+\phi^2 h_n^2}{\beta_2^2}\right)\right] \qquad \ldots (38)$$

Case III

A line torque at $(0,0,h)$ with axis parallel to the x-axis.

$$\text{Let } Y = P_0 H(t)\frac{\partial}{\partial z}[\delta(x)\,\delta(z-h)]. \qquad \ldots (39)$$

$$\text{Therefore, } Y_3 = \frac{P_0\, q\, e^{-qh}}{p},$$

Eq. (17) then becomes

$$v_1(x,0,p) = \frac{P_0}{2\pi\beta_2^2 p}\int_{-\infty}^{\infty}\frac{e^{-(\eta_s h + i\xi x)}}{p}\, d\xi$$

$$= \frac{P_0}{\pi\beta_2^2 p} Re\int_0^{\infty} e^{-i\xi x - n_s h}\, d\xi \qquad \ldots (40)$$

In the usual manner, we get

$$v_1(x,0,p) = \frac{P_0}{\pi\beta_2^2} Re\int_{h/\beta_2}^{\infty} e^{-pt}\frac{d\zeta(t)}{dt}\, dt \qquad \ldots (41)$$

so that

$$\frac{\pi v(x,0,t)}{P_0}=\frac{1}{\beta_2^2}\frac{ht\,H\left(t-\dfrac{\sqrt{x^2+\phi^2h^2}}{\beta_2}\right)}{(x^2+h^2\phi^2)\left(t^2-\dfrac{x^2+\phi^2h^2}{\beta_2^2}\right)^{1/2}} \qquad \ldots (42)$$

Now if the time dependent part $H(t)$ of Y in (39) is replaced by $F(t)$, where

$$F(t)=t\,H(t)-t\,H(t-t_0)$$

i.e., $F(t)=t,\ t<t_0$

$$=0,\ t\geqslant t_0 \qquad \ldots (43)$$

Since the Laplace transform of $tH(t)$ is $1/p^2$, using the product theorem of Laplace transform, we get

$$\frac{\pi\, v(x,0,t)}{P_0}=\frac{1}{\beta_2^2}\left(\frac{h}{\phi^2h^2+x^2}\right)[A_1-A_2-A_3], \qquad \ldots (44)$$

where

$$A_1=\left(t^2-\frac{x^2+\phi^2\mathrm{h}^2}{\beta_2^2}\right)^{1/2} H\left(t-\frac{\sqrt{x^2+\phi^2h^2}}{\beta_2}\right),$$

$$A_2=\left[(t-t_0)^2-\frac{x^2+\phi^2h^2}{\beta_2^2}\right]^{1/2} H\left[(t-t_0)-\frac{\sqrt{x^2+\phi^2h^2}}{\beta_2}\right]$$

and

$$A_3=\frac{t_0(t-t_0)}{\left[(t-t_0)^2-\dfrac{x^2+\phi^2h^2}{\beta_2^2}\right]^{1/2}}-H\left[(t-t_0)-\frac{\sqrt{x^2+\phi^2h^2}}{\beta_2}\right] \qquad \ldots (45)$$

Numerical Results and Discussions

Numerical results are obtained for displacement component $v(x, 0, t)$ at different distances from the source for different values of the non-dimensional time only for

Case III.

For simplicity, we assume

$\tau=\beta_2 t/h_0$, $t_0=5h_0/\beta_2$, $h=lh_0$, $x=mh_0$.

and so eq. (44) reduces to

$$\frac{\pi\, v(x,0,t)\,\beta_2^3}{P_0}=\left(\frac{l}{\phi^2l^2+m^2}\right)\left[A_1'-A_2'-A_3'\right], \qquad \ldots (46)$$

where

$$A_1' = [\tau^2 - (\phi^2 l^2 + m^2)]^{1/2}\, H(\tau - \tau_0),$$

$$A_2' = [(\tau - 5)^2 - (\phi^2 l^2 + m^2)]^{1/2}\, H(\tau - \tau_0 - 5),$$

$$A_3' = \frac{5(\tau - 5)}{[(\tau - 5)^2 - (\phi^2 l^2 + m^2)]^{1/2}}\, H(\tau - \tau_0 - 5)$$

and

$$\tau_0 = \sqrt{(\phi^2 l^2 + m^2)}$$

The value of ϕ is taken for two different crystals as follows:-

ϕ for Magnesium = 1.92

and

ϕ for Thallium = 2.36.

These values of ϕ are taken from Payton[10].

Figs. 3 to 5 show the variation of $Kv(x, 0, t)$ where $K = (\pi\beta_2^3 P_0)$ for different values of τ and for different values of l and m, each of the figures contain three graphs, one for isotropic case and two for different crystals. For a comparison of the behaviour of the initial stages, the curve are drawn from the time of arrival of disturbance. All the curves show that for a given values of x the disturbance in the

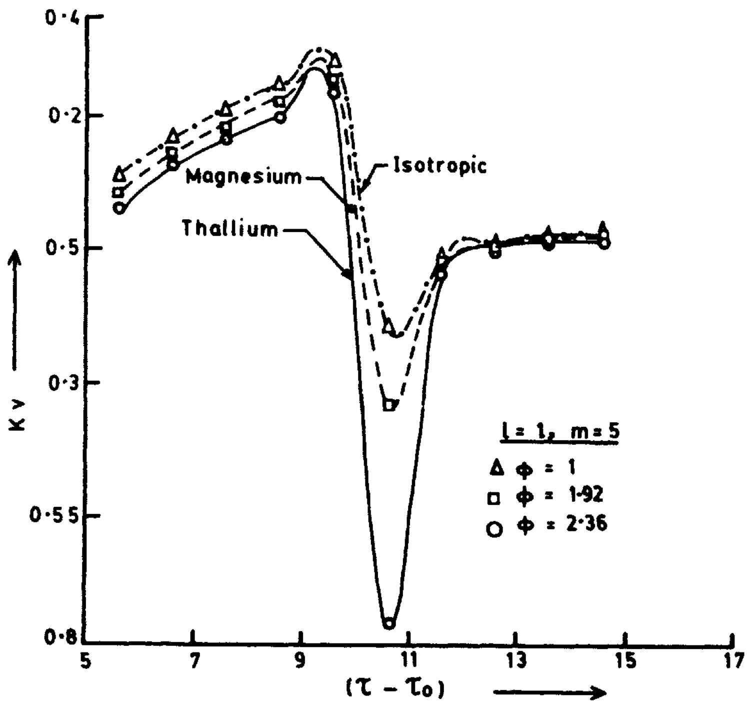

Fig 3 Variation of displacement with time

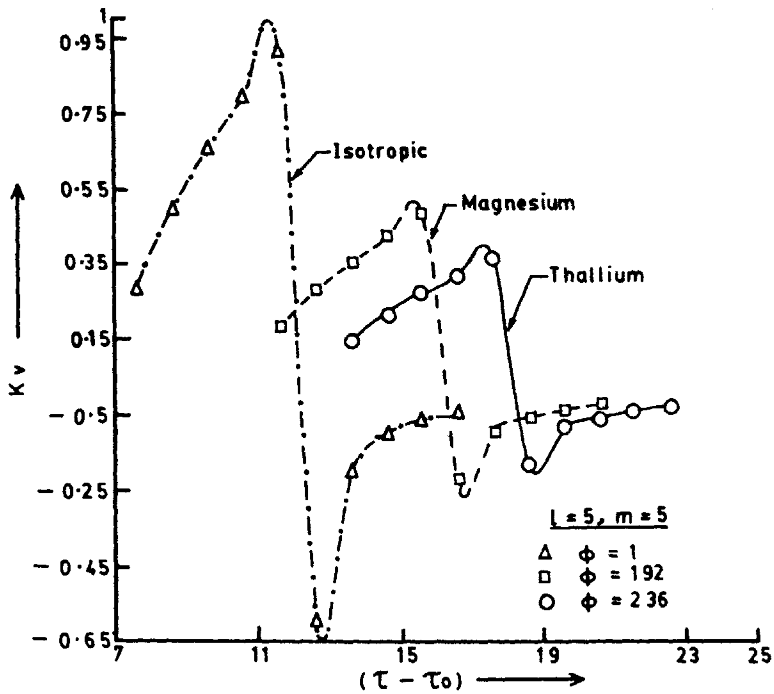

Fig 4 Variation of displacement with time

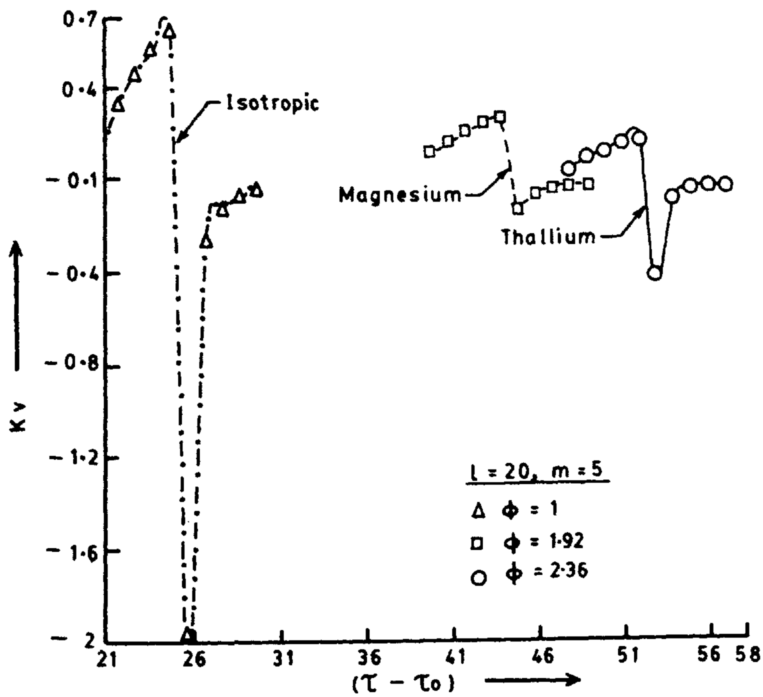

Fig 5 Variation of displacement with time

initial stages increases at first with increase of h but decreases when h increases beyond a certain value. It increases gradually with time $t = \sqrt{(x^2 + \phi^2 h^2)}/\beta_2 + t_0$, when there is a sudden jump followed by a slow recovery. It can be concluded that the discontinuous jump is due to the sudden disappearance of the acting force at $t = t_0$. Also it can be inferred that as we increase the ratio ϕ for different crystals the amplitude of the motion shortened but trends follow the same path as in the isotropic case.

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