Proc. Indian natn. Sci. Acad., B62, No. 2, pp. 159-168 (1996)

Comparison of Empirical Models to Simulate Crop Growth and Relative Growth Rate in Sunflower (*Helianthus annus* L.) Cultivars

T V RAMACHANDRA PRASAD*, S P PALANIAPPAN, C SUYAMBULINGOM and A RAJAGOPAL

Agronomy Department, Directorate of Soil and Crop Management Studies, Tamil Nadu Agricultural University, Coimbatore 641003

(Received on 6 February 1995; after revision 26 December 1995; Accepted on 16 April 1996)

A few mathematical models were tested to simulate crop growth (dry matter production -DMP) in sunflower cultivars cv. CO_2 — line and cv. MSFH-8-hybrid) in comparison with asymmetrical sigmoidal functions (Gompertz, Richards and logistic) and polynomials (third and fourth degree). The relative growth rate (RGR) from such regression model was compared with the classical method. The new mathematical models were linear + reciprocal, normal and Hoerl. These models were tested to fit the data on periodical DMP from a field trial conducted at Tamil Nadu Agricultural University, Coimbatore, India. The best new models meeting sensitivity test criteria to simulate DMP meaningfully were normal, linear + reciprocal (in both cultivars) and Hoerl in (cv. MSFH-8). These compared with the traditionally tested functions of Richards, polynomials of cubic or fourth degree (for both cultivars) and logistic (for cv. CO_2). The biological worthiness of these regression model was made by differentiating the functions. The RGR from regression models of Richards, normal, logistic, Hoerl (for all stages), polynomials of cubic or fourth degree (excluding early two stages) gave instantaneous RGR quite comparable to the classical method. The regression model showed better expression of RGR during post-flowering stage than the classical method. Sigmoidal function of Richards showed its versatility in simulating DMP and RGR better than other models. The early cultivar CO_2 showed faster growth rate than late cultivar, MSFH-8.

Key Words : Empirical models, Dry matter production, Relative growth rate, Sunflower

Introduction

Biomass accumulation in a crop is dependent on the rate of leaf production, leaf area expansion, period of leaf activity, quantum

Present Address: Agronomist, AICRP on Weed Control, Old Agricultural College, University of Agricultural Sciences, Hebbal Campus, Bangalore 560 024

of light intercepted by the canopy and radiation use efficiency with which the chemical energy is produced (Gallagher & Biscoe 1978, Kiniry et al. 1989). The classical growth analysis concept to elucidate growth difference in crop plants attempted since several decades (Hunt 1982) has certain limitations in explaining the crop growth. Of late, the functional approach based on regression developed in the 1960's (Richards 1969, Hunt 1982) has been reported to have overcome these problems and provides clear perception of time dependent phenomenon (Hunt 1982, Poorter 1989, Ramachandra Prasad et al. 1993).

Among the regression models tested, sigmoidal curves like Gompertz (Ramachandra Prasad & Shiva shankar 1992) and generalised logistic (Meek et al. 1991, Ramachandra Prasad et al. 1992) have explained the crop growth adequately. Similarly, Srinivasan et al. (1986) and Ramachandra Prasad and Shiva Shankar (1992) described versatility in using Richards function to describe crop growth more meaningfully. Nevertheless, various utility of polynomial functions in describing crop growth have been made by Poorter (1989). To simulate crop growth realistically, Hunt (1982) and Ramachandra Prasad et al. (1993) suggested good fitting of the data with third degree polynomial. However, Ramachandra Prasad et al. (1993) indicated that high degree polynomial may overfit the data leading to upward or downward estimates near the curve asymptote. In this paper, an effort has been made to test the suitability of mathematical models to simulate crop growth in sunflower cultivars in comparison with sigmoidal functions and polynomials. In addition, relative growth rate from the best models has been compared with the classical method.

Materials and Methods

A field experiment was conducted during October 1991 through January 1992 at the Tamil Nadu Agricultural University, Coimbatore (lat 11°N, 76.55′ E) under irrigated conditions to know performance of two cultivars viz., CO_2(line) and MSFH-8 (hybrid) under six plant densities (49,400 ro 148,100 plants ha^{-1}). An uniform fertilizer dose of 40 kg N, 8.74 kg P and 12.5 kg K ha^{-1} was applied. The total dry matter production (DMP) averaged over plant densities and replications was estimated at eight stages namely, 12, 24, 33, 41, 61 (coinciding flowering stage), 78, 91 (for cv. CO_2) and 106 (for cv. MSFH-8) days after sowing (DAS) from randmly selected five plants in each plot. Eight empirical models fitted to the DMP data were

Sigmoidal functions (as per Causton & Venus 1981)

a. Gompertz (non-symmetrical) : $DMP_t = DMP_m \exp. [-\exp(\beta-kt)]$ ... (1)

b. Logistic (autocatalytic) : $DMP_t = DMP_m[1 + \exp. (\beta-kt)]^{-1}$ ... (2)

c. Richards (extension of logistic) : $DMP_t = DMP_m [1+\exp. (\beta-kt)]^{-1/n}$ ... (3)

Polynomials

d. Third order (cubic) : $DMP_t = a + bt + ct^2 + dt^3$ ... (4)

e. Fourth order : $DMP_t = a + bt + ct^2 + dt^3 + et^4$ ... (5)

New Models

f. Linear + reciprocal : $DMP_t = f + gt + h/t$ ... (6)

g. Normal (Three Paramatric symmetrical) : $DMP_t = f \exp. [(t- g)^2/h]$ ... (7)

h. Hoerl [(multiple regression of DMP with t and log t)] : $DMP_t = f (g)^t (t)^h$ ... (8)

where DMP_t is the total dry matter production (g $plant^{-1}$) at time 't' in days, DMP_m = maximum final DMP assumed to be attainable under field conditions (71 and 82 g $plant^{-1}$ for cv. CO_2 and cv. MSFH-8 rspectively), β, k, a, b, c, d, e, f, g and h are constants to be estimated and indicates the shape of the curve, the value being -1 to 1 (Causton & Venus 1981). The models 4 to 8 were fitted using Curvefit programme (Thomas S. Cox, No. 102, Evergreen Street, Easley, SC 29640, 1986).

The above models performance in simulating DMP compared to the actual data was assessed based on sensitivity test criteria like, coefficient of determination (R^2), residuals sum of squares (RSS) {sum of [observed (0) - predicted responses $(P)]^2$}, chi-square test (χ^2) [Σ $(O-P)^2/P$] (as per Gunst & Mason 1980), and root mean square deviation (RMSD) [$\{\Sigma$ $(O-P)^2$ $/n\}^{0.5}$] (n = number of stages) (as per Keating & Wafula 1992, Ramachandra Prasad et al. 1993). The above models, if they satisfy all the sensitivity criteria i.e., higher R^2, non-significant, lower RSS and RMSD, are considered the best. RMSD is a measure of accuracy in models' prediction providing an average weighted difference between observed and predicted responses. RSS provides total variability in the models' overall predictions (Keating & Wafula 1992, Ramachandra Prasad et al. 1993).

By differentiating the best functions [$(1/DMP_t)$ (dMP_t/dt)] viz., polynomials of third and fourth degree, linear + reciprocal, Richards, logistic, normal and Hoerl, the computed relative growth rate (RGR) (mg g^{-1} day^{-1}) was compared with the RGR of classical method suggested by Watson (1952). RGR is rate of increase in dry matter produced per unit dry matter per unit time.

$$RGR = (\ln DMP_2 - \ln DMP_1)/(t_2 - t_1)$$

- classical method.

Where, DMP_1 and DMP_2 are dry matter production at time t_1 and t_2 respectively.

The significance of differences in RGR between classical and functional methods were assessed using paired-t test {t = Mean/ [standard deviation/(n-2)]$^{0.5}$, n = number of

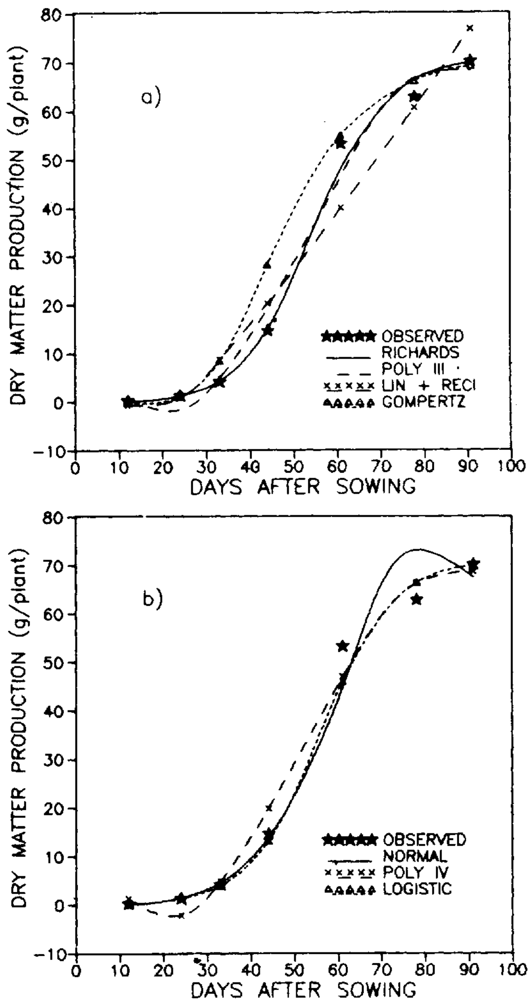

Figure 1 Empirical models describing dry matter production in cv. CO_2

stages}, as suggested by Gunst and Mason (1980).

Results and Discussion

Dry Matter Production

Dry matter production (DMP) increased from 0.27 g to 70.2 g $plant^{-1}$ in cv. CO_2 (at 91 DAS) (figure 1) and 0.25 g at 12 days after sowing (DAS) to 81.6 g $plant^{-1}$ in cv. MSFH-8 (at 106 DAS) (figure 2).

In cv. CO_2, of the nine models tested, only six models showed goodness of fit in simulating DMP closer to the actual data (table 1, figure 1a and b). The new models with higher R^2 were normal and linear + reciprocal as against already known functions like Richards, logistic and polynomials of third and fourth degree. All these models possessed lower values of residuals sum of squares (RSS), root mean square deviation (RMSD) and non- significant chi-square (χ^2), as compared to other models like Gompertz and Hoerl which showed less goodness-of-fit.

Whereas in cv. MSFH-8, three best models were Richards, third and fourth degree polynomials which simulated DMP by 96.8 to 98.4% of the actual data (table 2) and showed goodness of fit. The next best models were normal, Hoerl and linear + reciprocal (R^2 of 95 to 99%). However, these models showed significant χ^2 values due to over or lower predictions either in the beginning or later stage of the crop growth (see figure 2a and b). Other models with relatively more heteroscedasticity were Gompertz and logistic.

In both the cultivars, Richards function simulated DMP most nearer to the observed data at all stages and showed the least sensitivity test criteria (figures 1a and 2a).

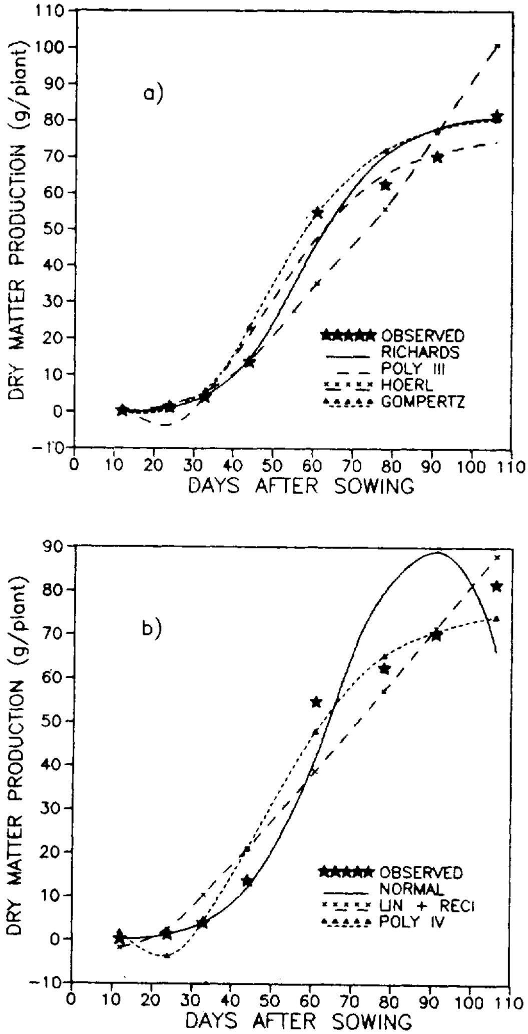

Figure 2 Empirical models describing dry matter production in cv. MSFH-8

Polynomials of third and fourth degree estimated DMP meaningfully from second and third stages onwards respectively in cv. MSFH-8 and from 33- 91 DAS in cv. CO_2. Polynomials predictions were not realistic in the earlier stages (figures 1a, b and 2a, b). Further, linear + reciprocal and normal functions predicted DMP more realistically at all stages except first stage in the former and

Table 1 *The best functional models predicting dry matter production (DMP, g plant^{-1}) in sunflower cv. CO2*

Equation No.	Functional form	R^2	RSS+	RMSD	χ^2
1	$DMP_t = 71 \exp. [-\exp (3.254–0.0758t)]$	0.974**	219.4	5.6	44.9**
2	$DMP_t = 71 [1 + \exp. (6.877–0.123t)]^{-1}$	0.989**	68.3	3.2	1.5
3	$DMP_t = 71 [1 + \exp. (5.817 –112t)]^{-1.25}$	0.990**	39.8	2.4	0.8
4	$DMP_t = 21.5–2.59 t + 0.08t^{-2} - 0.503 \times 10^{-3} t^3$	0.984**	87.0	3.5	3.1
5	$DMP_t = 26.5–3.23 t + 0.104t^2–0.859 \times 10^{-3} t^3 +1.739 \times 10^{-6} t^4$	0.984**	82.5	3.4	3.7
6	$DMP_t = 43.96 + 1.287 t + 334.9/t$	0.953**	276.5	6.3	9.1
7	$DMP_t = 74.44 \exp. [(t-81.85)^2 /– 854.4]$	0.997**	186.1	5.2	3.3
8	$DMP_t = 0.579 \times 10^{-4} (0.9906)^t (t^{3.355})$	0.977**	928.9	11.5	20.4**

R^2 = Coefficient of determination; RSS = Residuals sum of squares; RMSD = Root mean square deviation; χ^2 = Chi-square test; ** = Significant at P = 0.01; DMP_t = DMP at time 't'; t = days after sowing.

Table 2 *The best empirical models to predict crop growth in sunflower cv. MSFH-8*

Equation No.	Functional form	R^2	RSS+	RMSD	χ^2
1	$DMP_t = 82 \exp. [-\exp 3.169 - 0.067t)]$	0.983**	244.2	5.9	37.9**
2	$DMP_t = 82 [1+ xp. (6.133 - 0.0921 t)^{-1}$	0.937**	622.4	8.8	21.7**
3	$DMP_t = 82 [1+ \exp. (4.035 - 0.0846 t)]^{-2}$	0.980**	191.2	4.9	3.2
4	$DMP_t = 3.28 - 0.84 t + 0.036t^2 - 0.201 \times 10^{-3} t^3$	0.968**	266.7	5.8	8.4
5	$DMP_t = 38.43 - 4.78t+ 0.164t^2 - 1.736 \times 10^{-3} t^3 + 0.605 \times 10^{-5} t^4$	0.984**	171.0	4.6	7.3
6	$DMP_t = 33.79 + 1.132 t + 221.7 / t$	0.949**	424.0	7.3	14.6*
7	$DMP_t = 89.73 \exp. [(t-88.68)^2 / - 1002]$	0.992**	1066.9	11.6	15.5*
8	$DMP_t = 0.216 \times 10^{-4} (0.9803)^t (t^{3.745})$	0.978**	843.2	10.3	16.4*

+Explanations for abbreviations are provided as foot note in table 1.

last stage in the latter functions (see figures 1a, b and 2b). Normal functions estimated maximum DMP a stage prior to harvest and lower estimates at harvest. While linear + reciprocal predicted negative value in the first stage. This model gave progressive decreasing rate with advancement in age of the crop, as observed with sigmoidal functions. This showed suitability in predicting crop growth in cv. CO_2. Further in cv. MSFH-8, Hoerl function predicted DMP nearer to the actual data at all stages except at maturity, wherein predicted DMP was relatively higher (figure 2a).

Comparing logistic function of both the cultivars, β/k signifying the time for the point of inflection (i.e. point of maximum absolute growth rate as per Causton and Venus 1981) showed that cv. CO_2 was earlier by over 10 days (55.9 days) than cv. MSFH-8 (66.6 days) simulating 10 days difference in maturity under field conditions. The 'n' which indicates the shape of the curve was 0.8 for cv. CO_2 and 0.5 for cv. MSFH-8. Whenever logistic function fits in well, n of Richards model approaching one will also fits the growth data more meaningfully, as seen in cv. CO_2 here. Similarly, logistic curve simulated crop growth with less heteroscedasticity in tomato, sorghum and winter wheat (Meek et al. 1991) and maize cv. Deccan hybrid (Ramachandra Prasad et al. 1992). While for sunflower and maize, Causton and Venus (1981), and Ramachandra Prasad and Shiva Shankar (1992) noticed better description of crop growth with Richards model, as observed here. Similarly, in describing ear or pod growth, Richards function gave better fit to the data in greengram, blackgram (Chandra Babu & Kailasam 1988) and rice (Li & Senadhira 1988, Kailasam et al. 1989). Further in this study, cubic polynomials and splined cubic polynomials with two knots gave better description of growth in crops including maize (Mithorpe & Moorby 1979, Hunt & Evans 1980, Hunt 1982, Ramachandra Prasad et al. 1993). Increasing the degree showed less improvement in the predictability or lowering the degree underfitted the data in the present study. Although Meek *et al.* (1991) observed Gompertz as the best growth curve for cotton and sugarbeet, in the present study, it simulated very low DMP upto 24 DAS, followed by over estimations upto 44 DAS (figure 1a and 2a). As a result χ^2 was significant and showd lack of fit. In the present study, three parametric functions Richards, cubic polynomials, Hoerl, normal and linear + reciprocal in both cultivars appeared to be better growth curves, while two parametric function - logistic was better for cv. CO_2. However, Meek et al. (1991) ruled out use of four parametric functions - Richards and cubic polynomial in describing crop growth in cotton, sugar beet, sorghum, tomato and wheat, due to heteroscedasticity in models estimations. While Trapani et al. (1992) obtained better description of aerial biomass of sunflower cultivars with cubic polynomial exponential for stages prior to anthesis and linear for stages after anthesis.

Relative Growth Rate (RGR)

In the classical method, mean RGR consistently decreased from 131.6 mg at 12-24 DAS to 8.5 mg g^{-1} day^{-1} 79-91 DAS in CO_2 and from 137.4 mg at 12-24 DAS to 7.9 mg at 62.78 DAS with slight increase further to 10.0 mg g^{-1} day^{-1} at 92-106 DAS in MSFH-8 (figure 3 and 4).

In both the cultivars, RGR from regression model of Richards was quite comparable to RGR of classical method at all stages.

Similarly, normal function in both cultivars at all stages except at harvest and Hoerl function in cv. MSFH-8 at all stages gave RGR similar to classical method (figures 3a and 4a). The mean RGR (R) in these models were 78.1 mg in classical, 78.8 mg in Richards, 74.3 mg in logistic, 77.9 mg in normal functions in cv. CO_2; 68.9 mg in classical, 73.9 mg in Richards, 75.1 mg in Hoerl and 65.8 mg g^{-1} day^{-1} in normal function in cv. MSFH-8.

Further, these regression models gave relatively higher RGR at 62-78 DAS (immediately after flowering) than the classical method, showing faster growth rates after flowering simulating biological phenomenon. Shivakumar and Shaw (1978) in soybean and Ramachandra Prasad et al. (1993) in maize explained change in crop growth during post-flowering period in regression models than the classical method. In the classical method, drop in RGR after 61 DAS was quite considerable perhaps due to increased respiration at flowering with concomitant increase subsequently in seed dry mass, as observed by Shivakumar and Shaw (1978) in soybean and Ramachandra Prasad and Shiva Shankar (1992) in corn. However, immediately after anthesis, heads start bending as seed-filling progresses. This is better explained by higher RGR observed during 62-78 DAS in the regression models in both the cultivars. Nevertheless, this steady rate decreased further at harvest in these regression models, yet lower than the classical

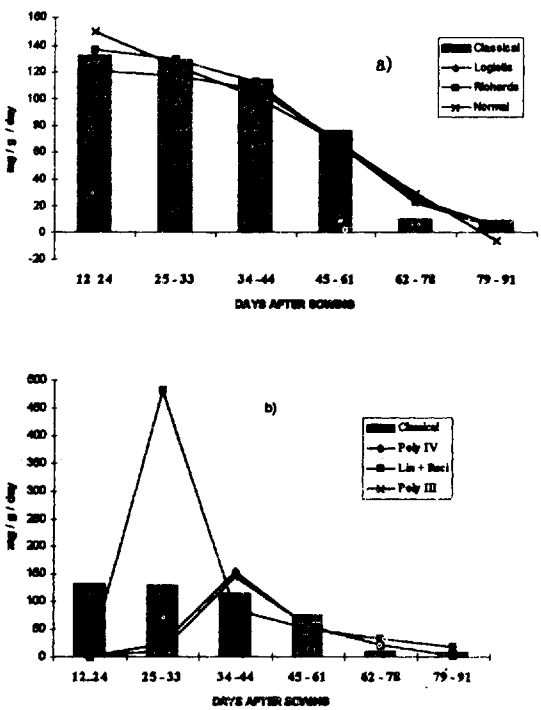

Figure 3 Relative growth rate in cv. CO_2-comparison of functional models and classical method

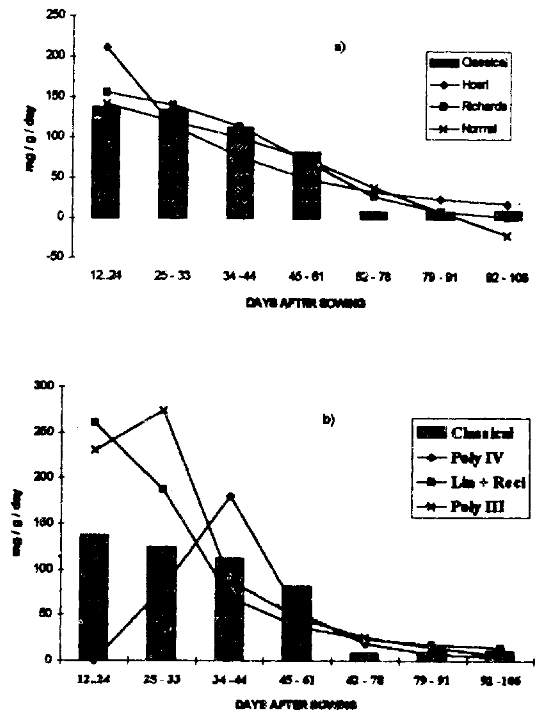

Figure 4 Relative growth rate obtained from functional model and classical method in cv. MSFH-8

method, which seems to be realistic biologically as also stated by Jolliffe et al. (1990). The regression models take into account of integrated crop growth development stages, as also shown by Sivakumar and Shaw (1978) in soybean and Ramachandra Prasad et al (1993) in corn by comparing classical and regression methods.

In cv. CO_2, both the polynomials predicted negative DMP at 12 and 24 DAS and consequently had very low RGR values (–541 mg and –495 mg g^{-1} day^{-1} in cubic and fourth degree respectively) than the classical method. Similarly in cv. MSFH-8, RGR's during 12-24 DAS and 25-33 DAS were lower than the classical method, as the predicted DMP was negative. Excluding earlier two stages, RGR's from polynomials in both the cultivars were comparable to the classical method (see figures 3b and 4b). As observed in this study with cubic and fourth degree polynomials at very early stages, Hunt and Evans (1980), and Ramachandra Prasad et al. (1993) in maize recorded negative value of RGR in the initial stage with splined cubic polynomials of two knots and cubic polynomials respectively.

Linear + reciprocal model although showed decreasing RGR with advance in crop growth in both the cultivars, to gave higher RGR than the classical method at most of the stages (see figures 3b and 4b). Whereas logistic model gave good fitting of DMP in cv. CO_2 and predicted RGR comparable to classical method at all stages (figure 3a).

Thus comparing all stages with less standard error (SE) to simulate instantaneous RGR comparable to the classical method, Richards model was the best (3.1 in CO_2 to 3.0 mg in MSFH-8), followed by normal (3.1 in CO_2 to 2.9 mg in MSFH-8), logistic (2.9 mg in CO_2) and Hoerl (3.1 mg in MSFH-8). The paired 't' test showed non-significant difference in all the models between regression model and the classical method.

In conclusion, dry matter production in sunflower cultivars can be best simulated using empirical models of Richards, normal, cubic or fourth degree polynomials, linear + reciprocal, logistic (for cv. CO_2) and Hoerl (for cv. MSFH-8). Considering the biological utility, instantaneous relative growth rate from Richards, normal logistic, Hoerl (for all stages), polynomials of cubic or fourth degree (excluding early stages) models was comparable to the classical method.

Acknowledgements

This study formed part of the Topical Research of the Ph.D. programme undertaken at Tamil Nadu Agricultural University, Coimbatore. Authors are thankful to Dr R Devendra, University of Agricultural Scinces, Bangalore, for this kind help in fitting the computer programme.

References

Causton D R and Venus J C 1981 *The Biometry of Plant Growth*, pp. 16-85 (London : E. Arnold Publishers).

Chandra Babu R Kailasam C 1988 Functional growth model for pod development in greengram and blackgram; *J. Agron. Crop Sci.* **160** 356-358

Gallagher H N and Biscoe P V 1978 Radiation, absorption, growth and yield of cereals; *J. Agric. Sci.* (Camb.) **91** 47-60

Gunst R F and Mason R L 1980 *Regression Analysis and its Application : A data - Oriented pproach* p. 220-235 (*New York: Marcel Dekker Inc.)

Hunt R 1982 *Plant Growth Curves. The Functional Approach to plant Growth Analysis* London: E Arnold Publishers) 284 pp

—— and Evans G C 1980 Classical data on the growth of maize: Curve fitting with statistical analysis; *New Phytol.* **86** 155-180

Jolliffe P A, Tarimo A J P and Eaton G W 1990 Plant growth analysis: Growth and yield component responses to population density in forage maize; *Ann. Bot.* **65** 139-147

Kailasam C, Chandra Babu R, Vijayalakshmi C and Sree Rangaswamy S R 1989 A functional growth analysis model for grain development in *Oryza sativa* L; *J. Agron Crop Sci.* **162** 354-356

Keating B A and Wafula B M 1992 Modeling the fully expanded area of maize leaves; *Field Crops Res.* **29** 163-176

Kiniry J R, Jones C A, O'Toole J C, Blanchet R, Cableguenne M and Spaner D A 1989 Radiation use efficiency in biomass accumulation prior to grain-filling for grain-crop species; *Field crops Res.* **20** 51-64

Li G F and Senadhira D 1988 Models for panicle growth simulation; *Inrternational Rice Res. Newsletter* **13** 21

Meek D W Hutmacher R B, Mackey B E and Davis K R 1991 Heteroscedasticity in whole plant growth curves developed from non-replicated data; *Agron J.* **83** 417-424

Milthorpe F L and Moorby J 1979 An Introduction to Crop Physiology; Second Edition, pp. 181-217 (London: Cambridge Univ. Press)

Poorter H 1989 Plant growth analysis: towards a synthesis of the classical and the functional approach; *Physiol. Plant.*v **75** 237-244

Ramachandra Prasad T V Krishnamurthy K and Kailasam C 1992 Functional crop and cob growth models of maize (*Zea mays* L.) cultivars; *J. Agron. Crop Sci.* **168** 208-212

—— Devendra R and Kailasam C 1993 Polynomial functional models to simulate crop growth in maize (*Zea mays* L.) cultivars; *J. Agron. Crop Sci.* **171** 55-64

—— and Shiva Shankar K 1992 Mathematical models to describe dry matter and cob growth in maize (*Zea mays* L.) cultivars; *Maydica* **37** 185-190

Richards F J 1969 The Quantitative Analysis of growth; in *Plant Physiology A Treatise. V A Analysis of growth* pp. 1-76 ed F.C. Steward (London:Academic Press)

Sivakumar M V K and Shaw R H 1978 Methods of growth analysis in field grown soybeans (*Glycine max* (L.) Merill.]; *Ann. Bot.* **42** 213-222

Srinivasan P S, Mohan Naidu K and Ranganathan C R 1986 Richards function — a functional growth analysis moel for rice cultivars (*Oryza sativa* L.) *Curr. Sci.* **5** 201-203

Trapani N, Hall A J, Sadras V O and Villella F 1992 Ontogenetic changes in radiation use efficiency of sunflower (*Helianthus annuus* L.) crops; *Field Crops Res.* **29** 301-316

Waston D J 1952 The physiological basis of variation in yield; *Adv. Agron.* **4** 101-145