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REVIEW ARTICLE

TWO AND THREE DIMENSIONAL FINITE ELEMENT SIMULATION AND EXPERIMENTAL CORRELATION OF FATIGUE CRACK CLOSURE

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A review is made of different experimental techniques and numerical methods used in the estimation of plasticity-induced fatigue crack closure under cyclic loading. Load interaction effects and temperature exposure on closure levels are highlighted. Some of the recent experimental works on estimation of closure levels with part through cracks are discussed. With the advent of large and fast computing power, it is possible to adopt numerical methods such as elastic-plastic finite element analysis to carry out parametric study from the viewpoint of design estimates. Researchers worldwide have carried out both two and three dimensional numerical simulation of crack closure phenomenon. The effect of constraint variation across the thickness on crack closure levels are better appreciated with three dimensional analysis. The authors have also contributed in this direction by carrying out both two and three dimensional numerical studies. The results of numerical analysis are compared with fractographic data obtained from a series of specially designed experiments. The load interaction effects on crack closure levels are numerically simulated and compared with the earlier experimental results reported in literature. For the first time, crack closure levels are estimated across the crack front under part through crack situations using three dimensional elastic-plastic finite element software. These configurations are very important in the present day, where damage tolerance concepts are used in the design of high performance aerospace and automobile components. This paper presents an overall picture of the work done in the area of fatigue crack closure and the different experimental techniques employed in the estimation of closure levels and the numerical procedures adopted in the estimation of crack closure levels.

Key Words: Fatigue Crack Growth; Crack Closure; Fractography; Elastic-Plastic Finite Element Analysis; Overload; Strain Hardening

Introduction

Fatigue crack growth has been a topic of intensive study by a number of researchers over the past two decades. The prediction of fatigue crack growth in structures under service is of great concern to the designer to ensure safety against catastrophic failure. It is known that, real life materials contain defects on a microscale. Engineering materials go through manufacturing process during which flaws may

be initiated as they are shaped into a structural component or a part of a machine. These structures could have in-built manufacturing defects, crack like flaws undetected by the non-destructive inspection. These will act as stress raisers and could act as sites for crack initiation under service loading.

Fatigue failures are found to occur in aerospace, marine, nuclear structures and in automobile components due to the initiation and propagation of cracks from holes, scratches or defects in the material. Such failures will lead to loss of life, destruction of equipment and severely reduce the life of these structures. It is reported in literature that, the origin of failures due to crack type of defects can be graded in order of their frequency of occurrence as surface cracks, corner cracks and cracks emanating from fastener holes. Such cracks propagate with elliptic or nearly elliptic crack fronts. So either the pre-existing flaws or cracks readily initiated in critical areas, grow under service loading. As the crack grows, there is a loss in the stiffness and strength of the structure. Finally fracture occurs, when the structure experiences a load more than or equal to the residual strength of the component. The total life span of the structure is the life time spent in service prior to failure.

The concepts of Safe-Life and Fail-Safe are used in aerospace structural design. In safe life, it is assumed that, the material is flaw free and is operating at a stress level that is low enough to result in low probability of failure during the safe life period. Whereas in the concept of fail safe design, it is presumed that, the structure is likely to have flaws or defects. The structure is designed to operate in the presence of these defects. Thus, a structure designed with this concept must maintain its load carrying capability until the defect is detected during periodic inspection and remedial action is taken. In fail safe design, use of multiple load paths, crack stoppers and use of materials with low growth rates are some of the characteristic features. Damage tolerant or fail safe design philosophy which is being applied to aircraft structures gives considerable importance to the methods which can accurately predict fatigue crack growth and residual strength.

Fatigue Crack Growth and Crack Closure

Paris *et al.*[1] examined a large amount of experimental data available and gave an empirical relation between stress intensity factor range (ΔK) and crack growth rate (da/dn). It is shown in the Fig. 1. He approximated that the crack growth rate by a straight line in log-log plot of (ΔK) *vs* (da/dn) in the major part of the crack growth life. Based on this observation, he proposed a relation,

$$\frac{da}{dn} = C(\Delta K)^m \qquad \ldots(1)$$

Where 'C' and 'm' are material constants. A more realistic representation is provided by the sigmoidal curve Fig. 1 that shows three distinct regimes, viz,

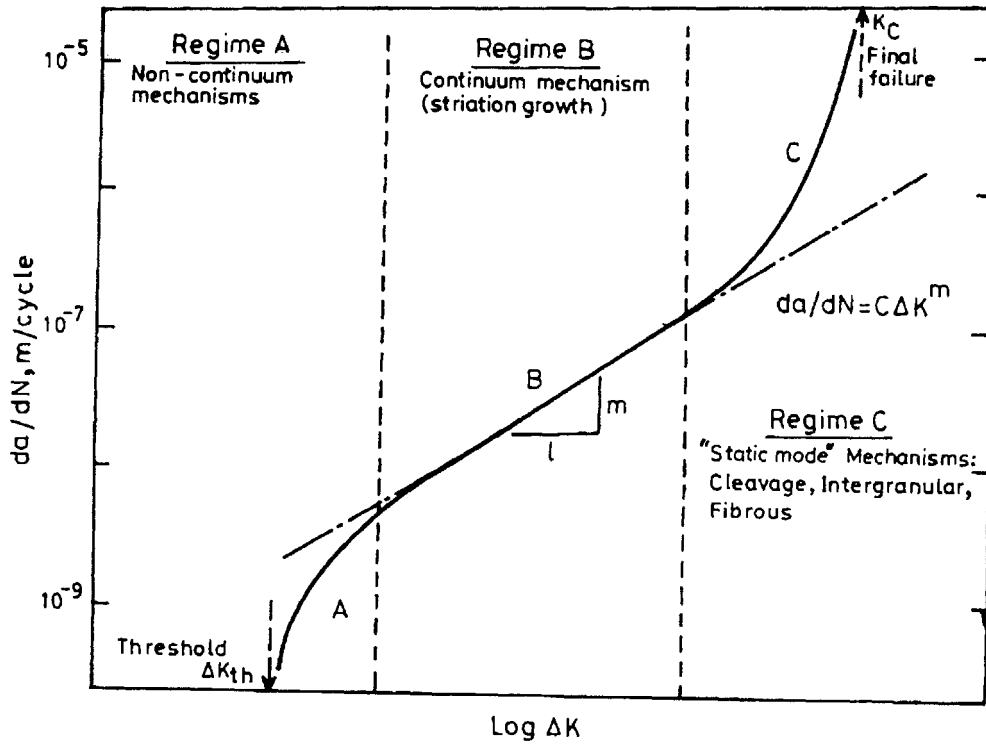

Fig 1 Sigmoidal variation of fatigue crack growth rate with ΔK

threshold regime, linear or Paris regime and fast fracture regime. When prediction of life based on eq(1) is made, it is normally assumed that only tensile part of the load cycle is contributing to the fatigue damage and crack tip surfaces close at zero load. This has been incorporated into many of empirical relationships describing fatigue crack growth response[2,3]. These empirical relationships required modifications in the later stages for many design applications. The Paris equation defined above does not account for the effect of stress ratio, R, (ratio of the minimum to maximum stress in a cycle), crack growth response near the threshold regime (ΔK_{th}) and crack growth response near the critical stress intensity factor (K_{IC} or K_c). Modifications to these relationships were proposed to consider the above effects[4,5].

The technical significance of crack closure as first observed experimentally by Elber[6,7], is generally considered as a predominant mechanism which accounts for stress ratio and crack growth retardation or acceleration under variable amplitude loading. The ultimate goal is to arrive at quantitative prediction of fatigue crack growth rate required for safety and economy reasons in aircraft structures, pressure vessels, marine and nuclear structural components wherein the application of damage tolerant philosophy has become mandatory.

Several types of closure mechanisms are now known. These include plasticity induced closure, roughness, asperity and oxide induced closure etc. The present study is restricted to the first one. Plasticity induced closure is caused by the residual plastic deformation remaining in the wake of an advancing crack tip. Crack closure phenomenon was able to give more satisfactory explanation for the influence of intermittent over load cycle on the crack propagation during constant amplitude loading[7,8], for general influence of stress ratio (R)[7,9]. It becomes essential in the quantitative knowledge of crack opening stress level (S_{op}) in the crack growth rate prediction under constant and variable amplitude loading. For variable amplitude loading, it requires prediction of S_{op} values cycle-by-cycle for entire load history applied. Prediction of S_{op} for cycle-by-cycle for entire load history is complex and so one finds it a difficult proposition to use crack closure phenomenon to estimate crack growth rates in a practical problem.

Many of the studies reported in the literature considered load interaction effects adequately to predict fatigue crack growth under variable amplitude loading.

Prominent among these investigations were the ones which explain decrease in crack growth rate observed after a single overload cycle. Newman[10] developed a modified Dugdale model which considers the influence of plastically stretched material in the wake of the advancing fatigue crack which causes crack closure. This model estimates the contact stresses behind the crack tip which is used to estimate the crack opening stress level. Later on Brahma[11] in his work improved over this model by taking elastic strains also into considerations in the estimation of crack opening stress level. He modified this model suitably to account for temperature exposure effect on fatigue crack growth retardation.

There were a number of attempts to measure crack closure by experimental methods. To begin with, Elber observed the phenomenon as non-linear behavior of load vs. Crack opening displacement (COD) relationship in a fatigue crack growth test. Fig. 2 shows the relationship between the applied load and the displacement measured from a gauge. From this non-linearity, Elber reasoned out that the plastically stretched material on the crack lips cause the crack to close at positive applied stress level.

Consider a cracked panel made of ductile material subjected to external tensile load. As the stress in the vicinity of crack tip exceeds yield strength of the material and there is plastic deformation occurring around the crack tip as shown in Fig. 3. As the panel is gradually unloaded, compressive stresses develop in the vicinity of crack tip and the material yields in compression. Thus a zone of reversed plastic deformation is formed around the crack tip. As the crack grows with the application of load cycles, plastically stretched material is left in the wake of growing crack. This stretched material causes the crack faces to bear against each other and transmit compressive stresses during unloading part of the cycle. During unloading portion, the crack faces make contact with each other near the crack tip at a positive applied stress called the closing stress, S_{cl}. With further unloading

below this level, the contact area spreads and upon unloading to minimum stress, the crack faces establish contact over a length of the crack from the crack tip. During subsequent reloading phase, the length of contact gradually decreases as the load is increased. At certain applied load level called the opening stress level S_{op}, the crack faces become fully open, indicating, that the compressive stresses have been just overcome by the applied tensile load. Hence the effective range is the one over which the crack remains fully open which is less than the applied stress intensity range.

Therefore, the effective range becomes,

$$\Delta K_{eff} = K_{max} - K_{op} \text{ for } K_{op} \geq K_{min}$$
$$\Delta K_{eff} = K_{max} - K_{min} \text{ for } K_{op} \leq K_{min}$$

Where K_{op} is the stress intensity factor at crack opening stress level. Elber postulated that, only the applied load range over which the crack remains fully open is effective in propagating the crack. Over the years, large number of investigators have tried different techniques to measure crack opening stress levels. The major research effort was on the experimental methods. Considerable effort has been put forth to develop and improve upon the existing techniques for precise measurement of crack tip closure.

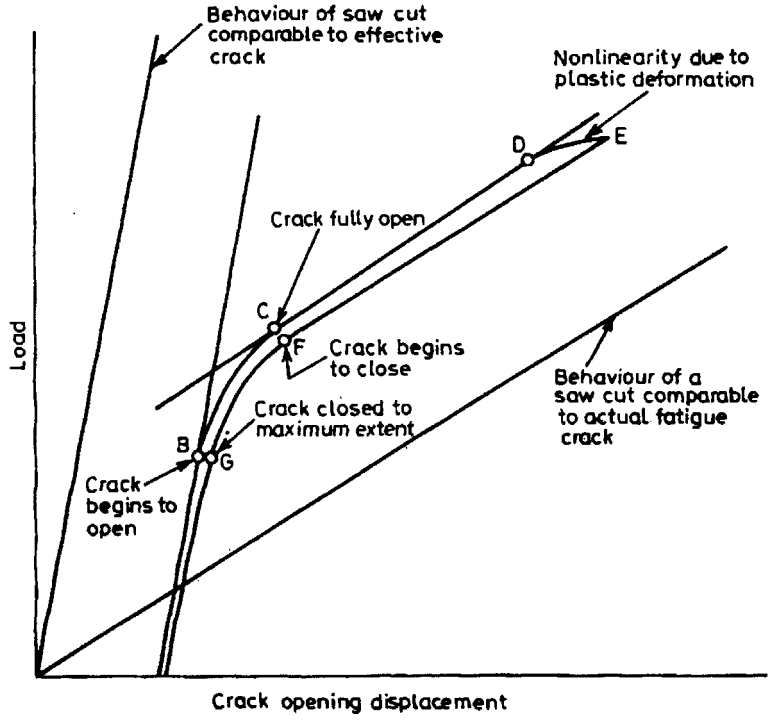

Fig 2 Load vs. crack opening displacement curves obtained during a cycle of loading and unloading

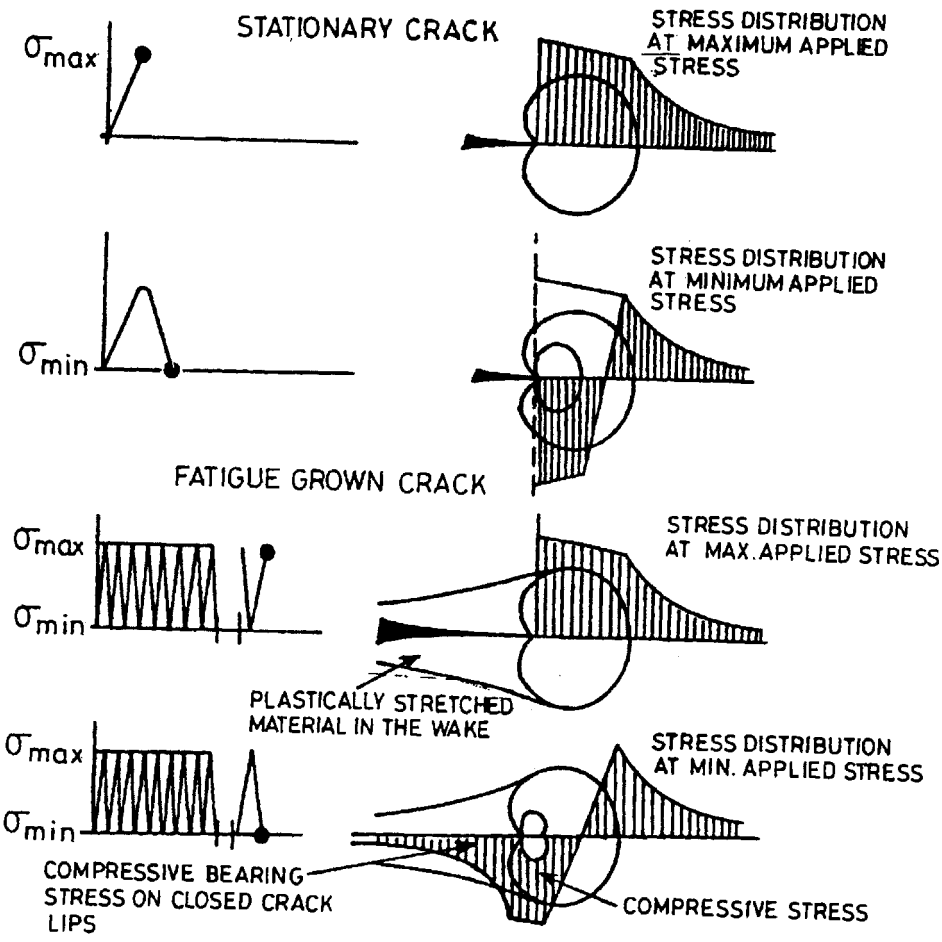

Fig 3 Wedge action of stretched material in the wake causing crack closure

With the advent of large and fast computing power, researchers have found it possible to adopt numerical methods such as elastic-plastic finite element analysis to carry out parametric study from the view point of design estimates.

Experimental Techniques for Measurement of Crack Closure

In one of the survey papers by Schijve[12] various methods to measure crack closure levels are reviewed. The different experimental techniques available for crack closure measurements are (i) direct observation of the crack tip by optical microscope[13,14], plastic replication[15], (ii) compliance measurement that indicates change of crack length during crack closure[16], (iii) indirect indications from crack growth observations[17], (iv) fractographic observations of fatigue failures[18,19].

From the most common technique of Load-COD data (Fig. 2), the point of change in slope will correspond to the transition from crack 'OPEN' to 'CLOSE' stage or vice versa. In reality crack closure does not occur abruptly but gradually over a portion of the load cycle. Because of the measuring location employed in the technique, one gets a global crack closure level and not the actual closure levels which exists at the crack tip. This makes the estimation of crack closure levels to be ambiguous. In order to overcome this, crack tip opening displacement (CTOD) gauges were developed to measure the displacement data near the crack tip. These gauges were mounted near the crack tip to get the local measurements near the crack tip[11]. Compared to COD data, the data obtained from CTOD measurements were able to explain the local effects near the crack tip. Here also, as the crack grows, the measurements made will be far away from the growing crack tip. In recent times techniques such as laser interferometry based measurements were employed (LI) in obtaining load-CTOD data. A typical laser interferometry load-displacement data[20] is shown in Fig. 4. Small indents were made at small distances Δa ahead of the crack tip. Negative values indicate indent locations ahead of the crack tip. Using laser interferometry (LI), measurements are made at these locations. As the crack grows with the applied cycle these indents initially ahead of the crack tip gets into the plastic wake. With the number of similar indents made, this technique becomes very efficient in obtaining the local closure levels by making plots of relative displacements from data recorded at different locations[20]. A widely used technique in post-processing the data is

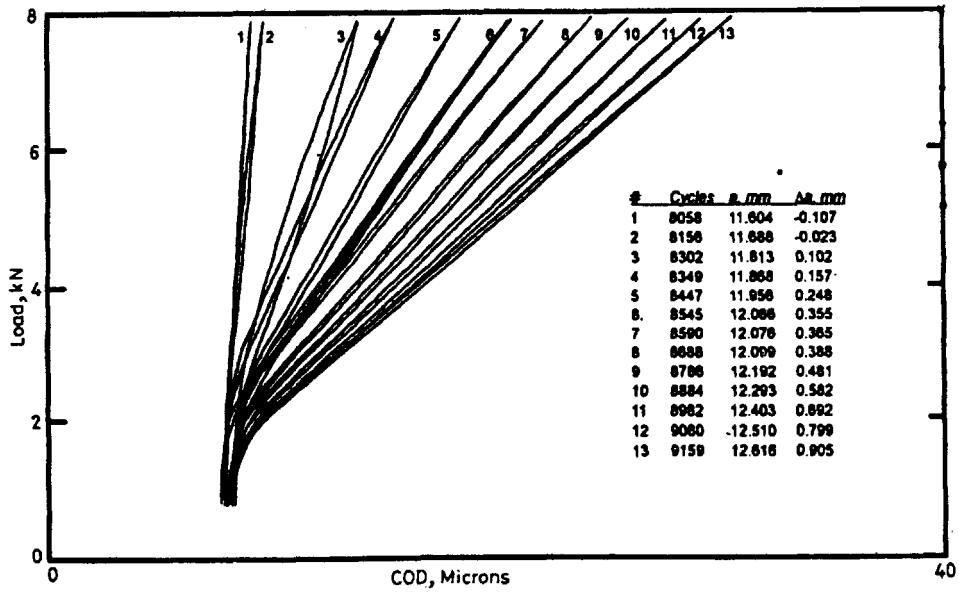

Fig 4 Typical laser interferometry measurement of Load-COD

to present the displacement data as an offset from the straight line approximation of the data segment used for unloading compliance. The load-displacement data converted to this format and non-dimensionalised are shown in Fig. 5. The change in slope indicates the onset of crack closure.

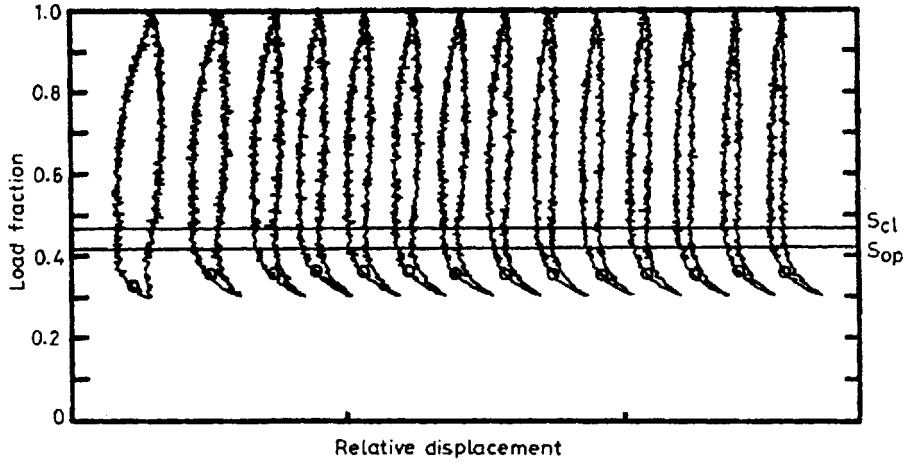

Fig 5 A typical plot of normalized Load vs. relative displacement

Fractographic Techniques

Pelloux[18] using high resolution electron fractography estimated crack opening stresses from the striation spacing induced by load cycles. Later, Sunder and Dash[19] considerably improved this technique by comparing the striations induced by load cycles with constant maximum stress and varying minimum stress. Further Sunder *et al.*[21] improved over this technique and modified the closure block, so that in a single closure block, one can estimate both closure and opening stress levels very precisely. This technique is qualitatively superior to others. It provides convenient, unambiguous and accurate determination of crack opening and closure stress levels especially under variable amplitude loading. With the technique giving very accurate measurements of closure levels, it can be used as a tool to validate results of numerical methods developed or analytical models proposed. A typical fractograph looks like the one shown in Fig. 6a-b. This particular fractography corresponds to an earlier study[47] conducted for a compact tension $C(T)$ specimen at a stress ratio of $R = 0.3$. By merely counting the number of equally spaced striations, the applied stress level at which the crack starts closing(S_{cl}) and the stress level at which the crack starts opening (S_{op}) can be precisely estimated. However fractographic techniques, though very accurate, are cumbersome and time consuming compared to conventional techniques. Also the material should give good and distinct striations patterns for the precise estimation of closure and opening levels. This is one of the draw backs of this technique.

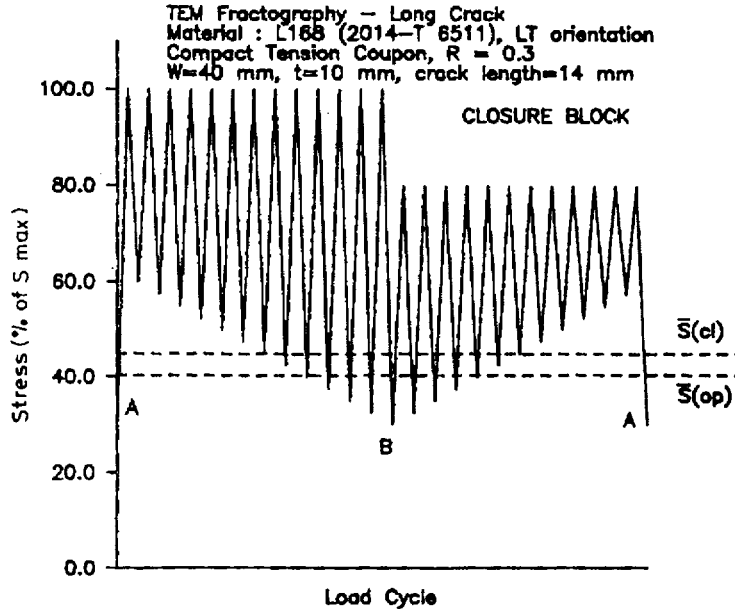

Fig 6a A typical closure block at R = 0.3 for compact tension (CT) specimen

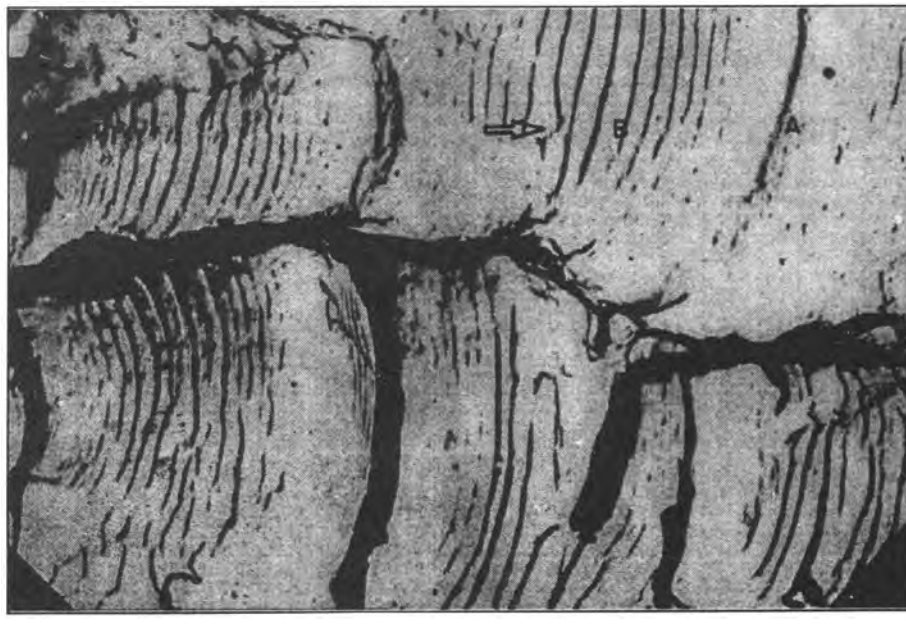

Fig 6b A typical fractograph at R = 0.3 for compact tension (CT) specimen

Variable Amplitude Loading

The situation under variable amplitude loading is quite complex because of load interaction effects when compared to constant amplitude loading. One of the simplest way employed in the estimation of crack growth life is to assume that the extent of crack growth under any load cycle is equal to the extent of crack growth under constant amplitude loading having the same load amplitude and mean load. This will be a more of conservative approach. Predictions thus obtained are considerably at variance with experimental observations. These load interaction effects can be classified broadly as 'acceleration' and 'retardation'. Under variable amplitude loading, the interaction effect is predominant.

One of the most dramatically observed interaction effects is 'retardation' in growth rate produced by a single tensile overload cycle applied in a constant amplitude load sequence. The crack growth behaviour after the application of tensile overload is shown in Fig. 7. For comparison, the expected crack growth rate without overload cycle is shown in the same figure. The applied overload cycle causes delay and the delay period is shown in the same figure. Earlier studies in this regard, on retardation in crack growth by single or multiple overload cycles were reported by Christenson[23], Hudson and Hardrath[24] and Hudson and Raju[25]. Elber[6,7] who discovered crack closure phenomenon explained the load interaction effects on the basis of crack closure model. Studies in the past have

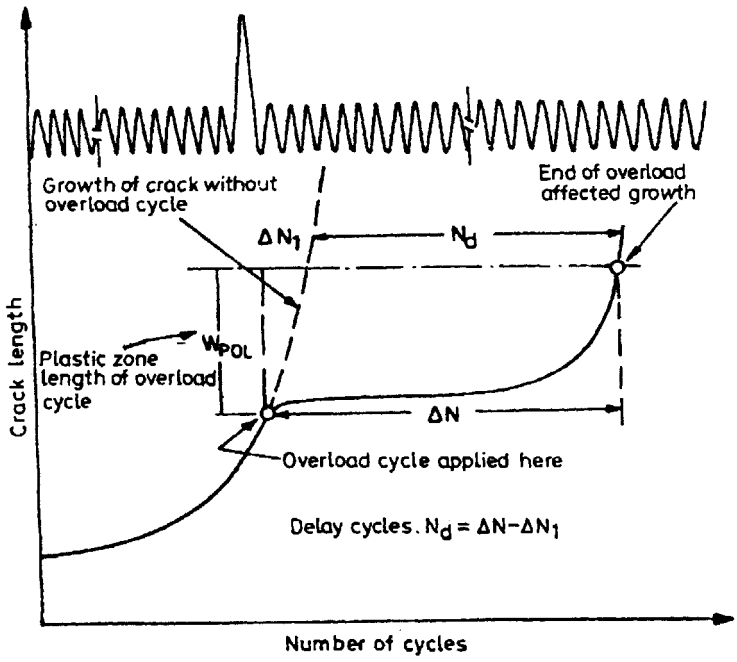

Fig 7 Crack growth behaviour after an overload cycle showing retardation effect

been carried out on the effect of compressive underloads following tensile overloads on retardation. Brahma[11] in his work has shown the effect of overload on the crack growth rate under different conditions of constant ΔK, increasing ΔK and decreasing ΔK with increasing crack length. A typical experimental result pertaining to SE(T) specimen explaining the effect of a single 100 % overload cycle interspersed between constant amplitude loading cycle for constant ΔK situation is shown in Fig. 8. The effect of overload is seen over a considerable portion of crack length. After certain amount of crack growth, the effect of overload cycle ceases to exists. From Fig. 8 the effect is seen upto a crack length of 21mm. For all the different load conditions, the experimentally measured Load-CTOD measurements and fractographs have shown the effects of overload cycle on crack growth rate.

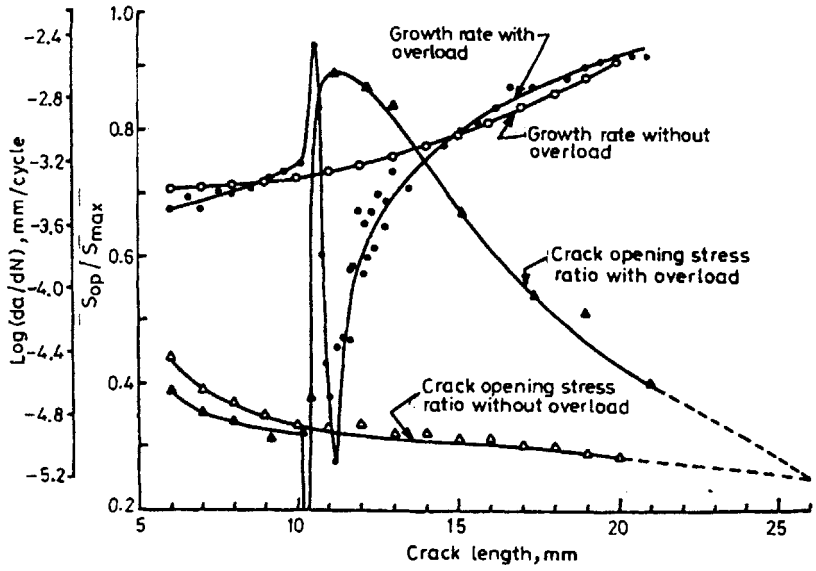

Fig 8 Effect of a single 100% overload cycle on growth rate and crack opening stress ratio : K increasing test

The thermal effects such as temperature gradient have considerable effect on fatigue crack growth rate. It was observed by earlier workers that, delay period due to an overload cycle reduced if the specimen is subjected to heat soak or temperature exposure[26,27]. Preliminary investigations were carried out on the effect of temperature exposure by Raju *et al.*[26] to see the effect of temperature exposure after the application of a tensile over load cycle on the retardation in fatigue crack growth. This phenomenon can be explained on the basis of crack closure phenomenon. Under various temperature gradients, it was reported that, the effect of temperature gradient is to accelerate the fatigue crack growth rate. It is equivalent to a situation of applying a negative compressive underloads. From the series of experiments conducted by Brahma[11] under controlled stress intensity, it was

found that, stresses continuously decrease with the duration of exposure. From Fig. 9, initially, stresses are relaxed at higher rate due to high value of residual compres-sive stresses. The rate of relaxation steadily decreases due to decreasing stress levels.

In recent times, experimental observations are reported in literature pertaining to part through cracks. Some of the interesting ones are by Troha *et al.*[28] and by Ravichandran *et al.*[29]. Troha *et al.*[28] in their study, adopted Newton interference method with a transparent polymethylmethacrylate (PMMA) to measure crack opening displacement and closure on surface flaw crack growth. The fatigue crack growth tests were conducted on rectangular plates loaded in bending. They have shown that, the crack fronts at the depth and at the surface positions close at different load levels during unloading portion of a fatigue cycle. In the study conducted by Ravichandran *et al.*[29], measurements of aspect ratios of the surface cracks of arbitrary shapes using a combined laser interferometry and photomicroscopic system is reported. These are some of the studies which try to explain the three dimensional aspects of part through cracks.

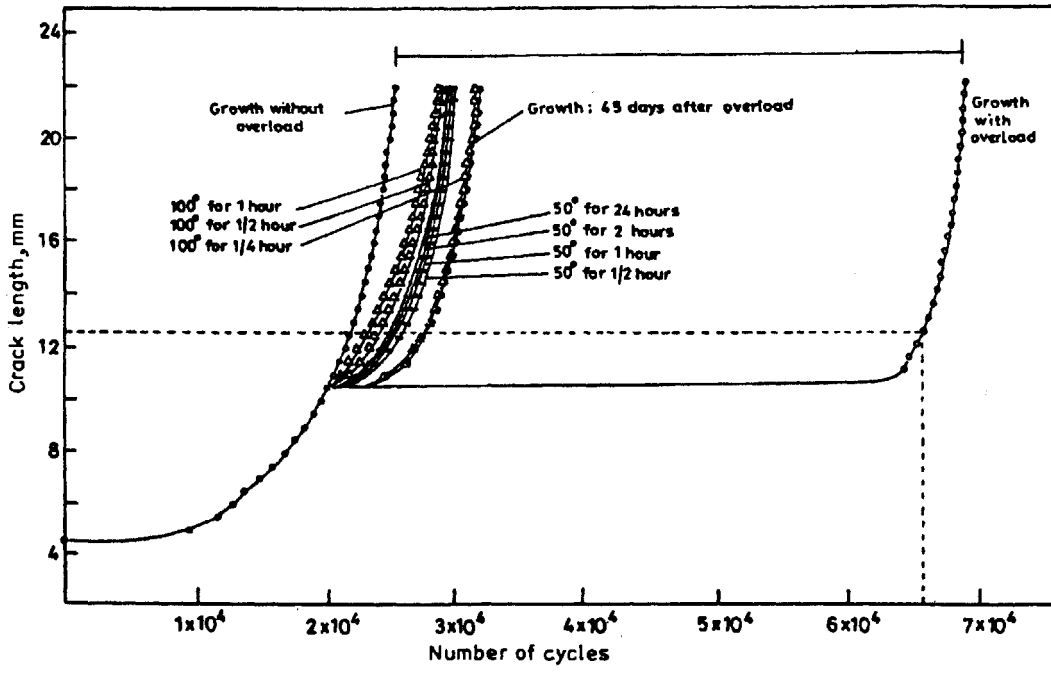

Fig 9 Crack length vs. Number of cycles : Effect of temperature exposure on retardation

Analytical and Numerical Methods for Estimation of Crack Closure levels

Certain analytical models[10,30,31] were developed based on a concept of Dugdale model[32] or strip yield model modified to consider the wedging action of the plastically stretched material in the wake of the advancing crack during unloading. Newman[10], Budiansky and Hutchinson[31] studied crack closure behavior from these models. The effect of three dimensional constraint on the crack closure behavior is difficult to address with these models. The transition from plane strain to plane stress is another complication of the development of the plastic wake behind the advancing crack. The wake field will depend on the K_{max} history and also on the state of stress and the yield limit of the material. It is a great concern to fatigue crack growth models for variable amplitude loading. To consider three dimensional effects, Newman[10], introduced a constraint factor to simulate plane stress and plane strain situation. Later on, this model was improved by Brahma[11] by considering elastic compression of the stretched material in his formulation. He used the modified model further to study the effect of temperature on fatigue crack closure. But these models, do not provide the ability to comprehensively study many aspects of the problem. With the advent of large and fast computing power recently, researchers found it possible to adopt numerical methods such as elastic-plastic finite element analysis to carry out parametric study from the view point of design estimates.

Two-Dimensional Finite Element Analysis

Two dimensional elastic-plastic finite element analysis of crack growth and closure under plane stress and plane strain conditions have been conducted by numerous investigators over the years. Newman[33,34,35,36] was one of the first to numerically study the crack closure phenomenon under plane stress condition. He investigated the crack closure in a center cracked panel under constant amplitude loading or two level block loading. The finite element model was composed of two-dimensional constant strain triangular elements and the material was assumed to be elastic-perfectly plastic. The calculated crack opening stresses under constant amplitude loading were qualitatively consistent with experimental measurements. At about the same time, study of crack closure with finite element method was published by Ohji and Ogura[37]. They applied their model to the study of crack growth from notches under variable amplitude loading. Nakagaki and Atluri[38] took a different approach to the problem, employing special crack tip elements and translation of the near crack tip mesh and studying mixed mode situation in addition to the standard mode 1 case. Blom and Holm[39] carried out numerical and experimental study of closure in the CT specimen at different stress ratios under plane stress and plane strain. Lalor and Sehitoglu[40] have extensively worked on the numerical study of crack closure phenomenon. They applied their elastic-

plastic finite element method to study the crack growth from notches under constant amplitude loading. Their analysis covered both plane stress and plane strain conditions. The results of their study are shown in Fig. 10. They have carried out the analysis at a stress ratio of R = —1.0. From their analysis, it is evident that, the extent of plasticity induced closure increases with crack length.

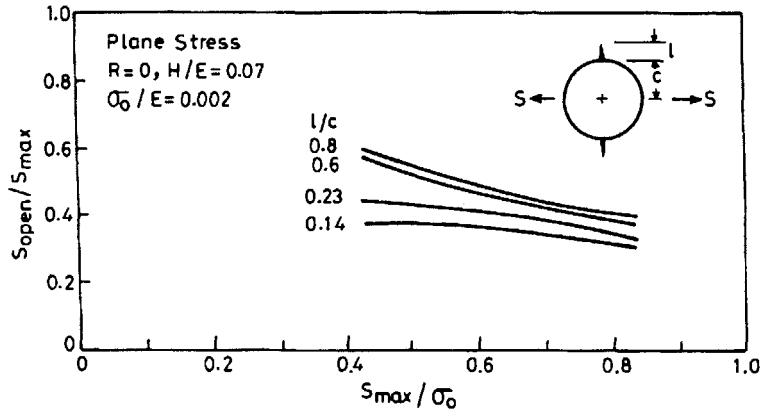

Fig 10 Change in normalized crack opening level as a function of crack length (R = —1)

Crucial issues in the Numerical Simulation

Later on it was McClung and Sehitoglu[41,42] who made an extensive review of the basic modelling issues and came out with certain criteria which had to be met in the finite element model. One of the important criteria was in the refinement of mesh along the crack line. The refinement is important for atleast two reasons. First, the mesh must be fine enough to capture the severe stress concentration at the crack tip. Second, the element spacing defines the crack growth increments, which should be as small as possible in order to simulate real crack growth. Extensive mesh refinement leads to large computational time and also requires many increments of crack growth (many cycles, many more load steps). A balance between these two considerations is required which does not seriously degrade the reliability of results.

Newman[35] was the first to identify and examine the problem of mesh refinement. Working with three meshes composed of constant strain triangles, he identified critical mesh spacing necessary for dependable solutions at different stress levels. He defined mesh spacing in terms of the ratio of crack growth increments to the plastic zone size at the maximum load. After a detailed analysis, taking all parameters into consideration, McClung and Sehitoglu[41,42] suggested a criterion for sufficient mesh refinement, so that results of numerical modelling are reliable. A limitation of FEM for fatigue crack growth model is that crack advance must

occur in relatively large discrete jumps corresponding to element sizes rather in a smooth continuous series of infinitesimally small values occurring in physical situations. Another limitation is that there is no unique criterion to explain, what actually causes the crack advance. Because of these limitations crack advance schemes are typically arbitrary.

Interest in Finite element modelling of fatigue crack closure over the years has made investigators to look into the aspect of crack closure behavior for finite thickness plates. Blom *et al.*[39] Lalor and Sehitoglu[40], W Sun *et al.*[43] and Fleck[44,45] have analyzed the closure behavior under plane strain conditions. It was found that the crack opening loads under plane strain condition to be much less than those under plane stress condition.

Comparison With Experimental Results

Recently, the authors have carried out numerical simulation of their fractographic observations[20,22,47,51,52]. For this purpose, sophisticated fractographic experiments were designed and executed at National Aerospace Laboratories, Bangalore. For numerical simulation, two dimensional elastic plastic non linear finite element software was developed to carry out crack closure analysis under both constant and variable amplitude loading. Under plane strain condition, the phenomenon of plane strain locking was overcome by field consistency[46] in the finite element formulation. In this formulation, total energy is split into volumetric and distortional components and reduced integration was carried out for evaluating volumetric energy. In one of the study conducted earlier[47] for compact tension $CT(T)$ specimen at an applied stress ratio of $R = 0.1$, the numerical estimates of crack closure/opening stress level are shown in Fig. 11. The corresponding fractographic estimates

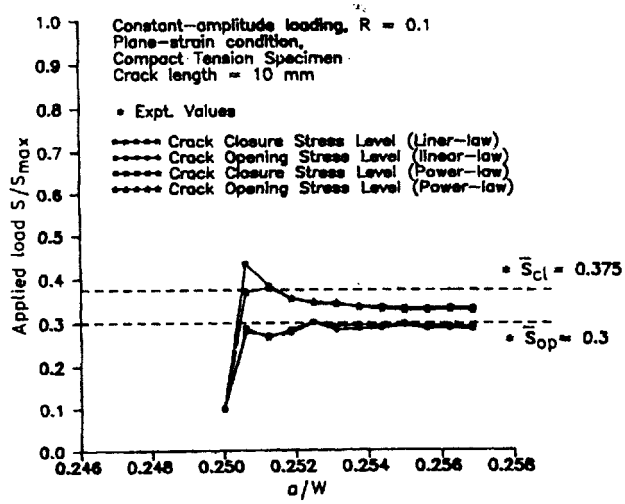

Fig 11 Normalized crack closure and opening stress as a function of crack extension for $R = 0.1$

are also shown in the same figure for the purpose of comparison. The numerical analysis showed good correlation with the fractography measurements. Similarly from finite element analysis, the Load-COD data is recorded[47]. For this purpose, material points were chosen ahead of initial crack tip 'a'$_i$. Corresponding to each of these material points, the Load versus COD data were recorded. One of the typical results corresponding to compact tension (CT) specimen at a stress ratio of R= 0.1 is shown in Fig. 12. The corresponding laser interferometry measurements[20] are shown in Fig. 13. Also the numerical results of Load-COD data compared with the laser interferometry measurements showed excellent correlation.

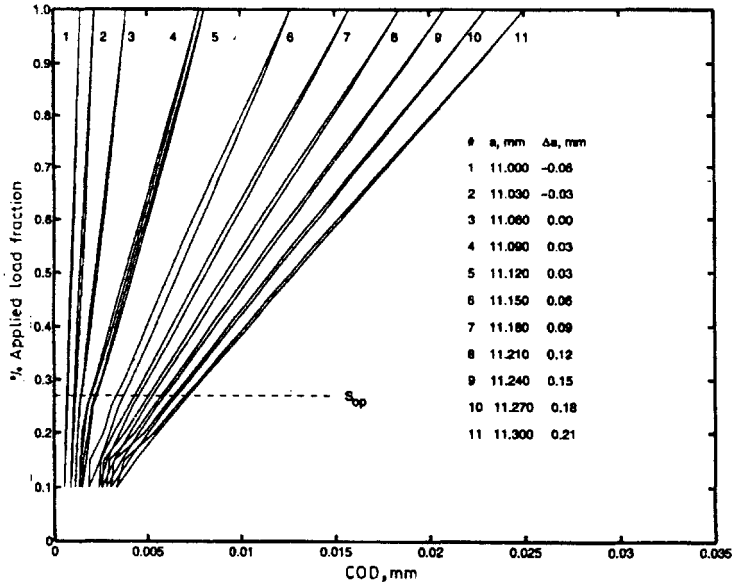

Fig 12 Load-COD plot for R = 0.1, Crack length = 11 mm

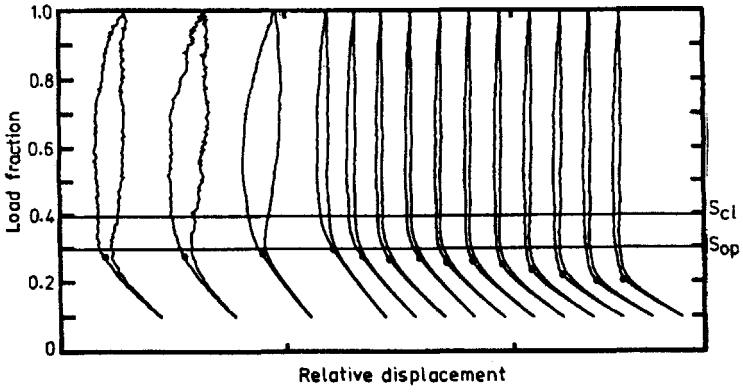

Fig 13 A plot of Load vs. relative displacement at R = 0.1 Laser interferometry measurement

Not much of numerical simulation has been reported in literature to explain the load interaction effects. In this direction the work has been carried out by the authors to explain the load interaction effect numerically[52]. For this purpose, a novel approach has been proposed to numerically simulate the retardation effect observed under a constant amplitude loading situation interspersed with a single overload cycle. For the sake of comparison, configuration and loading conditions for which experimental results[11] available in the literature are considered for numerical simulation. The results of numerical study as shown in Fig. 14 was able to explain the load interaction effects indicating retardation effects after the application of over load cycle. The numerical trend compares very well with the experimental observation Fig. 8.

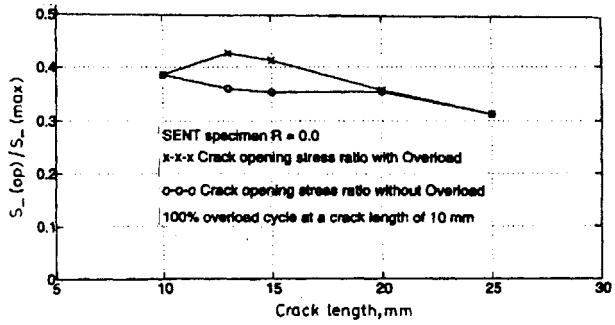

Fig 14 Effect of a single 100% overload cycle on crack opening stress ratio

Three Dimensional Effects

Most failures occur from the initiation and propagation of crack from holes, scratches or defects in the material. Such failures will lead to loss of life, destruction of equipment and severely reduces the life of these structures. It is reported in literature, the origin of failures due to crack in order of frequency of occurrence to be surface cracks, corner cracks and cracks emanating from fastener holes. Such crack propagate with elliptic or nearly elliptic in shapes. These cracks are truly three dimensional. Fatigue crack growth and fracture tests of metallic materials have shown shear lip formation during the transformation from flat mode at mid-thickness to a slant type of failure near to the free boundaries. This transformation is attributed to the loss in the constraint level. Because of these constraint effects, the inelastic deformation *i.e.* shape and size of plastic zone does vary across the thickness. Thus the crack closure phenomenon which creeps in because of inelastic deformation under cyclic loading varies across the thickness. It makes the crack growth rate to vary across the thickness. Two dimensional elastic-plastic finite element analysis of these crack configurations which considers only in-plane constraints are usually unsatisfactory. Accurate predictions can be made only by solving three dimensional boundary value problem where out of plane constraint is also taken into considerations.

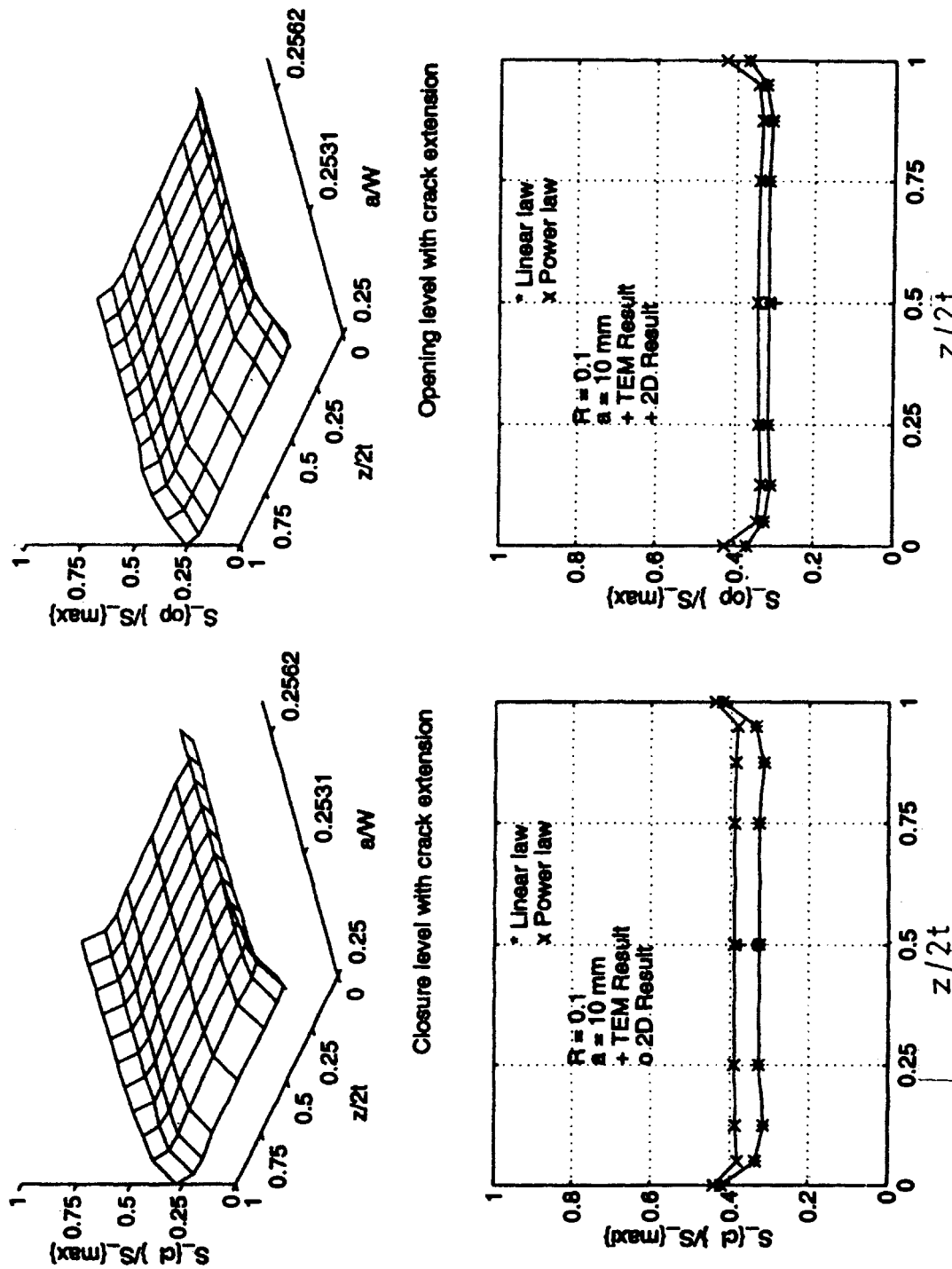

Fig 15 Normalized crack closure and opening stress as a function of crack extension at $R = 0.1$ for (CT) specimen

Chermahini *et al.*[48,49] carried out three dimensional elastic-plastic analysis of center cracked specimen and showed that the variation of crack closure levels across the thickness of a specimen. Later on Newman *et al.*[50] in their work showed the constraint variation in cracked bodies through their finite element analysis. In the contributions by Chermahini *et al.*[48,49] and Newman *et al.*[50], the behavior of the material was treated as perfectly plastic after the material has yielded. Once the hardening takes place, the Bauschinger effect is also felt when the material near the crack tip undergoes reverse yielding.

Authors to have carried out three dimensional finite element analysis[51] and compared with their earlier experimental measurements of fractography and laser interferometry measurements[20]. Here again for the same compact tension C(T) specimen at different applied stress ratios of $R = 0.1$, 0.3, 0.5 and 0.7 corresponding to initial crack lengths of 10, 14, 16 and 18 mm, the variation in closure/opening stress levels across the crack front were analyzed. A typical finite element analysis result corresponding to a stress ratio of $R = 0.1$ is shown in Fig. 15. The corresponding fractographs and two dimensional results are also shown in the same figure. The finite element analysis shows that, the crack closure/opening stress levels does vary across the thickness.

In the literature not much of work is reported in the numerical study on crack closure with part through cracks. As already mentioned only a few experimental studies have been reported. The significant ones are by Troha *et al.*[28] and Ravichandran *et al.*[29]. In order to understand the crack closure behavior under the situation of part through cracks, authors initiated work in this direction. The different typical configurations considered were surface crack, corner crack and embedded crack. For this purpose, a configuration for which experimental observations were carried out by Ravichandran *et al.*[29] was considered. It was an elliptic surface crack specimen with remote stress. Numerical analysis was carried out for the same type of loading and geometric configurations. The corresponding results are shown in Fig. 16 which also shows the stabilized values of closure/opening stress levels. For the purpose of comparison with experimental observations, the fatigue crack growth rate da/dn vs. ΔK and da/dn vs. ΔK_{eff} are shown in Fig. 17 a-b. The stabilized values of opening stress levels were made use of in the calculation of ΔK_{eff} range. The corresponding experimental results are shown in Fig. 18 a-b. When the numerical results were compared with the experimental observations it showed good correlation. From the analysis, it was clear, that, fatigue crack rate was higher at mid plane. It means that, the elliptic surface crack tries to grow first across the thickness and later develops into a part through crack. On similar lines analysis was carried out for corner and embedded crack situations which are equally important in the damage tolerant design of high performance aerospace structures.

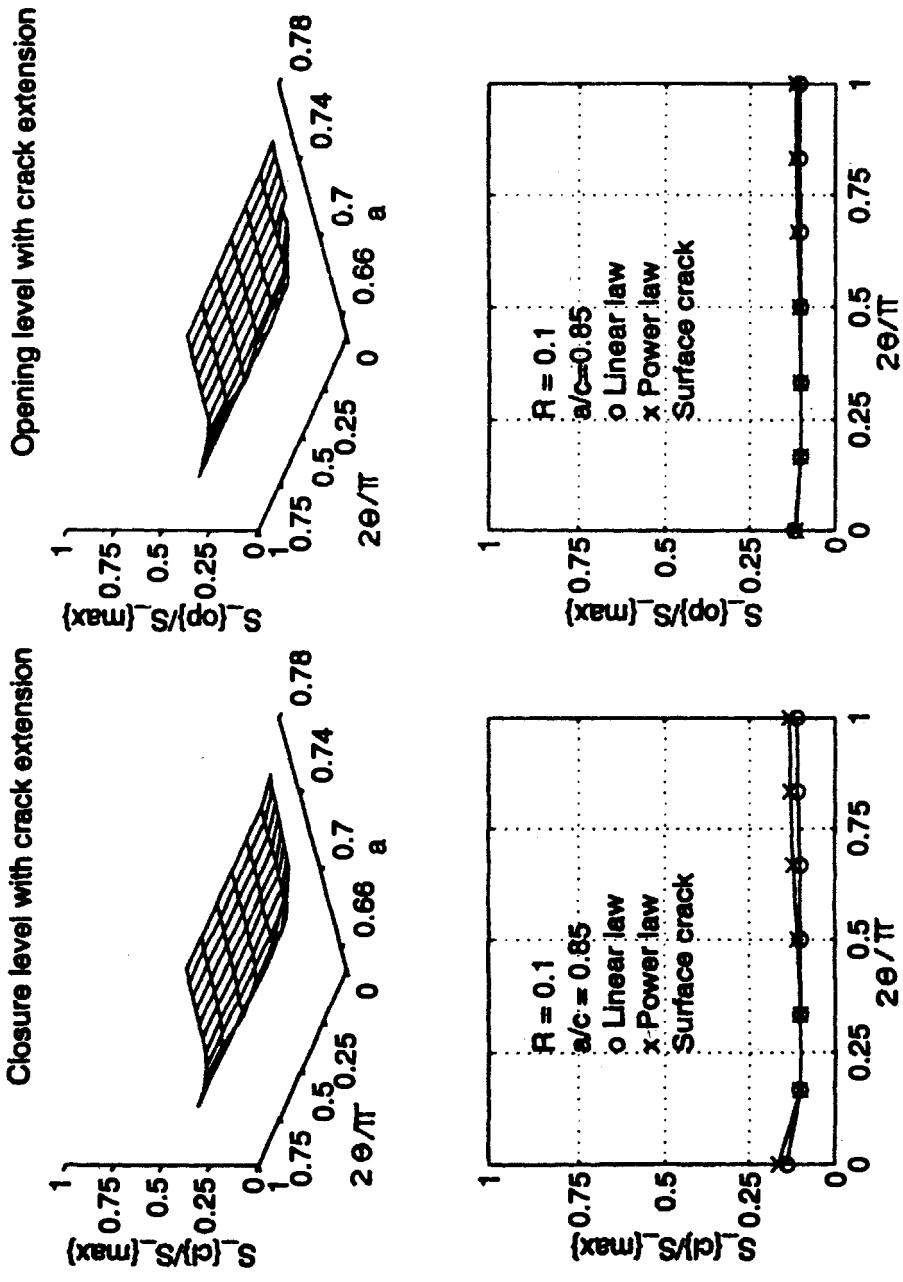

Fig 16 Normalized crack closure and opening stress as a function of crack extension at $R = 0.1$ for elliptic surface crack situation

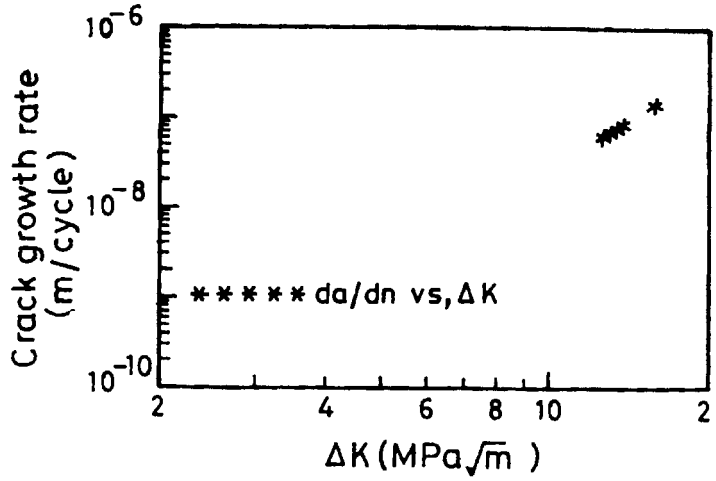

Fig 17(a) Variation in crack growth rates as a function of ΔK across elliptic crack front for surface crack situation

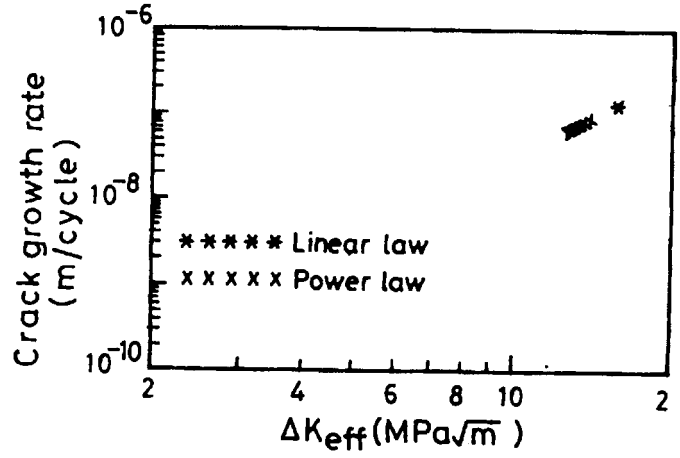

Fig 17(b) Variation in crack growth rates as a function of ΔK_{eff} across elliptic crack front for surface crack situation

Conclusions

An overall review of the different experimental techniques and numerical procedures employed in the estimation of crack closure levels was made. Some of the limitations of the different experimental techniques are discussed. No doubt, technique like fractograph measurements give very accurate results but it is quite cumbersome and time consuming compared to conventional techniques. One can say that, the finite element method is a powerful and reliable technique for the simulation of fatigue crack growth and the analysis of fatigue crack closure.

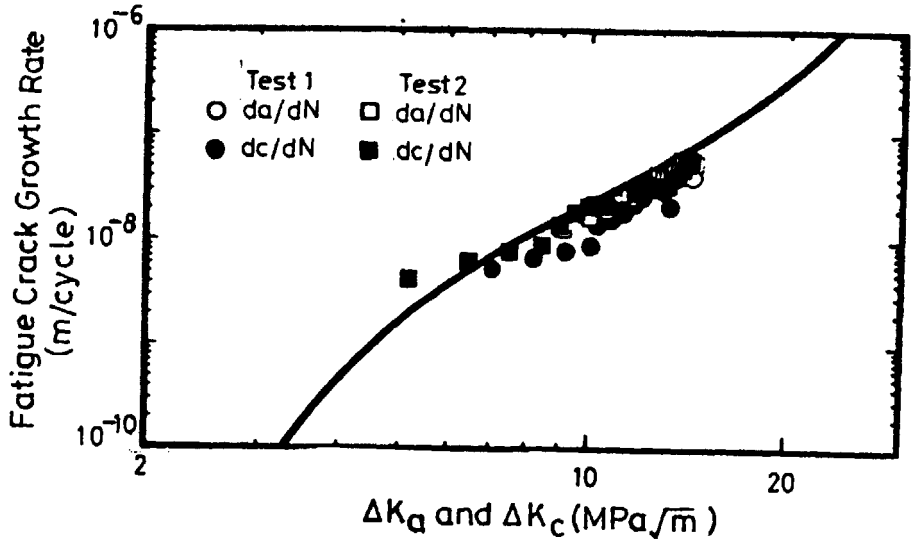

Fig 18(a) Experimental variation in crack growth rates as a function of ΔK across elliptic crack front for surface crack situation

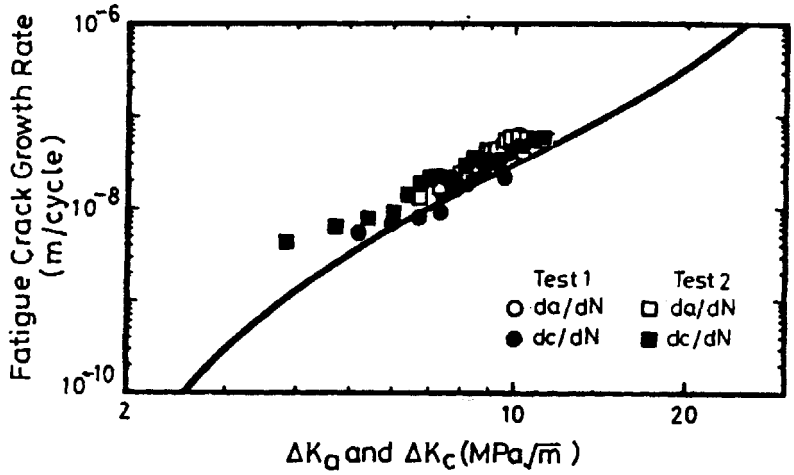

Fig 18(b) Experimental variation in crack growth rates as a function of ΔK_{eff} across elliptic crack front for surface crack situation

Although the failure criterion used in growing the crack was arbitrary, the numerical results have shown that, the calculated crack opening/closure levels are consistent with the results from fractograph measurements. The effect of constraint variation on crack closure levels across the thickness was better understood from the three dimensional studies with through and part through cracks. The load interaction effects can also be simulated using finite element techniques. Finally one can conclude that, the numerical analysis will help in parametric study from design considerations.

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