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A VECTOR ANALOG FOR THE MASS-MODEL AND PREDICTIONS

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A vector model for the masses of particles is suggested. Using this model a general mass equation is obtained. The mass parabola equation for nuclei is obtained, by a natural way, as a special case. Some predictions about nucleon-nucleon and quark-quark interactions are obtained. Interpretations for confined theory and negative mass concept are given.

Key Words : Vector Analog; Mass-Model; Mass Parabola Equation; Nucleon-Nucleon Interaction; Quark-Quark Interactions

Introduction

We know that, for any two vectors $\bar{a}$ and $\bar{b}$, if $\bar{c} = \bar{a} + \bar{b}$ we have

$$c \leq a + b, \qquad ...(1)$$

where a, b and c are the lengths of the vectors $\bar{a}$, $\bar{b}$ and $\bar{c}$ respectively, i.e., the length of the sum of two vectors is less than or equal to the sum of lengths of those two vectors. The equality case is satisfied if the angle between the two vectors equals zero.

Also, for a particle of mass m, which consists of two particles of masses m_1 and m_2, a similar relation -as (1)- is satisfied, where

$$m \leq m_1 + m_2, \qquad ...(2)$$

i.e., the mass of a complex particle is less than or equal to the sum of the masses of its constituents. The case of equality is satisfied for the system of total mass m, which consists of two individual particles of masses m_1 and m_2 which do not exist in a bound state.

Therefore, we try to construct a vector analog for masses of particles, such that the fusion of two particles corresponds to the sum of the two corresponding vectors of masses. Also we try to express some physical facts through this vector analog.

The Model

General Formalism

Any particle has its own field which cannot be separated apart from it. But, in the usual treatment we represent both of the particle and its field by different quantities. For example, the particle is represented by the mass, the charge, the spin and so on, and the field is represented by a vector. We try, introducing a vector analog

for the mass, to define a quantity for representing the particle and its field at the same time, i.e., we try to see the particle and its field in one bag.

This idea is considered as the physical basis to define a new vector whose magnitude is related to the mass of the particle and its direction is related to its field. We shall call this vector as a mass vector and denote it by $\overline{m}$ where $\| \overline{m} \| = \sqrt{m}$ and the unit vector $\hat{\overline{m}}$ in its direction is related to the field of the particle, such that the interaction between two particles m_1 and m_2 is represented by the scalar product $\hat{\overline{m}}_1 \cdot \hat{\overline{m}}_2$. Thus, we shall write the mass vector as follows

$$\overline{m} = \sqrt{m}\ \hat{\overline{m}} \ . \qquad \ldots(3)$$

This definition is not strange if we take into account the following facts - *(i)* The equivalence relation between the mass and energy, $E = m$ with $c = 1$, where c is the velocity of light. *(ii)* The binding energy of any system of particles due to any kind of force is a part of masses of the particles. *(iii)* The force between the particles - according to the quantum field theory - is translated by means of the particle of the field. All these facts lead to the strong relation between the mass of the particle and its field.

The physical meaning of $\hat{\overline{m}}_1 \cdot \hat{\overline{m}}_2$ which is given in the definition of the mass vector, can be, also, obtained considering the nonrelativistic limit of our mass equation which leads to the Hamiltonian of the system, see eqs. (34) and (35). Also, we found that, this combination of the particle and its field leads to a correct mass parabola equation of nuclei as a special case, see eq. (41). On the other hand, this concept of the mass vector does not contradict with any other stable physical concepts, laws and theory.

It seems to be that, the natural definition of the mass vector $\overline{m}$ is such that $\| \overline{m} \| = m$. But, we shall see that the given definition is more convenient to obtain the mass formulae.

The vector sum of two mass vectors $\overline{m_1}$ and $\overline{m_2}$ of two particles 1 and 2, respectively, is a mass vector $\overline{m}$ $(= \overline{m_1} + \overline{m_2})$ defined such that $\| \overline{m} \| = m$ where m is the mass of a new particle consisting of the two particles 1 and 2. In this definition, it is clear that, the mathematical process of addition of two mass vectors $\overline{m_1}$ and $\overline{m_2}$ represents the physical fusion of the two masses m_1 and m_2. Of course, there are no any mathematical or physical reason to suppose that $\overline{m_1} + \overline{m_2} \neq \overline{m_2} + \overline{m_1}$, therefore, we take

$$\overline{m_1} + \overline{m_2} = \overline{m_2} + \overline{m_1} \ , \qquad \ldots(4)$$

which means that the fusion of the two particles 1 and 2, in this order, means the same thing of the fusion of the two particles 2 and 1.

As in the case of usual vectors (of any dimension), we have

$$(\bar{m})^2 + (\bar{m}_1 + \bar{m}_2)^2 = \bar{m}_1^2 + \bar{m}_2^2 + 2\bar{m}_1 . \bar{m}_2 , \qquad ...(5)$$

where $\bar{m}_1 . \bar{m}_2$ is the scalar product of the two vectors $\bar{m}_1$ and $\bar{m}_2$,

i.e.,

$$||\bar{m}||^2 = ||\bar{m}_1||^2 + ||\bar{m}_2||^2 + 2\,||\bar{m}_1||\,||\bar{m}_2||\,\hat{m}_1 . \hat{m}_2 \qquad ...(6)$$

Then,

$$m = m_1 + m_2 - m = -2\sqrt{m_1}\sqrt{m_2}\,\hat{m}_1 . \hat{m}_2 , \qquad ...(7)$$

From this equation with $c = \hbar = 1$, we have, for the binding energy E_b of the new particle, that

$$E_b = m_1 + m_2 - m = -2\sqrt{m_1}\cdot\sqrt{m_2}\,\hat{m}_1 . \hat{m}_2 , \qquad ...(8)$$

It is clear, from Eq. (8), that the product $\bar{m}_1 . \bar{m}_2$ represents, in some way, the interaction between the two particles.

Four-Mass Vector

In general, the field of a particle is presented by particle-particle interaction. Even if the law of the force is well known, a test particle is needed to describe the field or to obtain the form of its potential energy. For different test particles we obtain different pictures for the field. For example, proton-proton and electron-proton cases. Therefore, the (measured) picture of the field depends, not only on one particle, but on two particles, the source of the field and the test particle. Thus, practically, the relative direction between the two mass vectors, only, has explicit physical meaning.

However, we can represent the mass vector of a particle (in the sense of its definition) by a four-vector in a four-dimensional space, the so-called mass space, defined as follows

$$\bar{m} = (m^0, m^1, m^2, m^3), \qquad ...(9)$$

where m^0 represents the total mass m_i of the particle, by which we mean that "the measured mass of the particle m + a virtual mass part W used to introduce the field around the particle". Both m and W are not measured, in general, at rest. We shall take $m^0 = \sqrt{m_r}$. The three-dimensional vector (m^1, m^2, m^3) is related to the field of the particle such that $(m^1)^2 + (m^2)^2 + (m^3)^2 = W$ = (the virtual part of the mass). In

coordinate representation the three-vector (m^1, m^2, m^3) can be considered as a combination of three dimensional vectors $\bar{L}$, $\bar{\sigma}$ and $\bar{\tau}$ of angular momentum, spin and isospin of the particle such that, the particle-particle interaction is given by a correct form depending on scalar products ($\bar{\sigma}_1 . \bar{\sigma}_2$, $\bar{\tau}_1 . \bar{\tau}_2$, and so on) of these vectors for the two particles.

Thus,

$$\bar{m} . \bar{m} = (m^0)^2 - (m^1)^2 - (m^2)^2 - (m^3)^2 = m_r - W = \text{the measured mass } m. \quad \ldots(10)$$

For two particles with the mass vectors $\bar{m}_1$ and $\bar{m}_2$, if $\bar{m} = \bar{m}_1 + \bar{m}_2$, we have

$$m = (\bar{m}_1 + \bar{m}_2)^2 = \bar{m}_1 . \bar{m}_1 + \bar{m}_2 . \bar{m}_2 + 2\bar{m}_1 . \bar{m}_2 = m_{1i} - W_1 + m_{2i} - W_2 + 2(\sqrt{m_{1i}}\sqrt{m_{2i}} - W_{12}), \quad \ldots(11)$$

where m_{ii} and W_i, $i = 1,2$, have the same sense of m_i and W in eq. (10) and W_{12} is the scalar product of the two three-vectors (m_1^1, m_1^2, m_1^3) and (m_2^1, m_2^2, m_2^3). From Equation (10) we can write eq. (11) in the form

$$m = m_1 + m_2 + 2\sqrt{m_1}\sqrt{m_2}\left(\sqrt{1 + \frac{W_1}{m_1}}\sqrt{1 + \frac{W_2}{m_2}} - \frac{W_{12}}{\sqrt{m_1}\sqrt{m_2}}\right), \quad \ldots(12)$$

where m_1, m_2 are the measured masses of the two particles and W_1, W_2 and W_{12} describe the fields of m_1, m_2 and the interaction between them, respectively. The quantity in the bracket can be considered as $\hat{m}_1 . \hat{m}_2$ i.e.,

$$\hat{m}_1 . \hat{m}_2 = \sqrt{1 + \frac{W_1}{m_1}}\sqrt{1 + \frac{W_2}{m_2}} - \frac{W_{12}}{\sqrt{m_1}\sqrt{m_2}} \quad \ldots(13)$$

Thus $\hat{m}_i$ is related to the field of m_i and eq. (12) takes the form

$$m = m_1 + m_2 + 2\sqrt{m_1}\cdot\sqrt{m_2}\ \hat{m}_1 . \hat{m}_2$$

which is the same form of eq. (7)

In our formalism, the four-rest mass vector is

$$\bar{m}_0 = (m_{0_r}, \bar{W}_0), \quad \ldots(14)$$

where m_{0_r} is the total mass at rest, $\bar{W}_0$ is three-virtual mass vector $(\|\bar{W}_0\| = \sqrt{W_0})$ which gives the field of the particle at rest. If

$$\bar{m} = (m_r, \bar{W}), \quad \ldots(15)$$

for the particle in a moving state, then

$$\bar{m} \neq \bar{m}_0, \quad \ldots(16)$$

because, at least $m \neq m_0$ = the measured mass at rest. Therefore, in general

$$m_{0i} \neq m_i \text{ and } \overline{W}_0 \neq \overline{W} \qquad \ldots(17)$$

i.e., in the suggested formalism the field of the particle depends on the state of motion of the particle.

The Mass Vector under Lorentz Transformation

The norm of any vector in a Minkovski space must be invariant under the Lorentz transformation. Under this transformation the mass vector components in two spaces are related as follows :-

$$m'^0 = \frac{m^0 + \alpha m^1}{\sqrt{1-\alpha^2}} \quad , \quad m'^1 = \frac{m^1 + \alpha m^0}{\sqrt{1-\alpha^2}}, \qquad \ldots(18)$$

$$m'^2 = m^2 \quad \text{and} \quad m'^3 = m^3$$

where α is a transformation parameter. Here, this parameter cannot take the same meaning of v/c, where v is the velocity of one system with respect to the other. We require that the values of $\|\overline{m}\|$ in all spaces which are related by Lorentz transformation (18) to be the same. This because all mass spaces must be equivalent. The mass vector is a state vector characterized by $\|\overline{m}\|$ and the transformation from one space of states to the other must give an equivalent space which have the same states $\|\overline{m}\|$ with different forms $\overline{m}'$. Taking $\alpha = v/c$, we have $m = m_0$ while the correct relation is $m = m_0/\sqrt{1-\alpha^2}$. Therefore, we understand the mass space as an associated space to coordinate reference, where $\|\overline{m}\|$ gives the mass of the particle as it is measured in this usual four-dimensional coordinate reference.

Using the definition of $\overline{m}$ where $m^0 = \sqrt{m_i}$, from (18), we have

$$m_i \cdot W = m'_i \cdot W' \ , \qquad \ldots(19)$$

i.e.,

$$m = m' \ , \qquad \ldots(20)$$

i.e., the measured mass in any two mass spaces is the same.

Four Momentum Vector and the Rest Mass Vector

In the usual relativistic theory, the four momentum vector for any particle is defined as

$$p = (E/c, \overline{P}), \qquad \ldots(21)$$

where E is the total energy and $\overline{p}$ is the three-momentum vector. Since

$$m_0^2 = E^2/c^4 - \overline{P}^2/c^2 = \text{ invariant quantity.} \qquad \ldots(22)$$

Thus, we can define a four-rest mass vector as follows

$$\overline{m}_0 = \left(E/c^2, \overline{P}/c \right) \qquad \ldots(23)$$

This definition can not be extended to include the mass of the particle in general moving state. Since, in this case $\|\overline{m}\|$ $(= m = E/c^2)$ is not invariant under the same Lorentz transformation.

Also, this definition is not related to the field of the particle. It is only related to dynamical quantities of the particle E and $\overline{P}$. For example, for two particles,

$$\overline{m}_{01} = \left(E_1/c^2, \overline{P}_1/c \right), \qquad \ldots(24)$$

and $$\overline{m}_{02} = \left(E_2/c^2, \overline{P}_2/c \right), \qquad \ldots(25)$$

then $$\overline{m}_{01} + \overline{m}_{02} = \left((E_1 + E_2)/c^2, (\overline{P}_1 + \overline{P}_2)/c \right), \qquad \ldots(26)$$

which represents the four-momentum vector of the system of two free independent particles divided by c. $\|\overline{m}_{01} + \overline{m}_{02}\|$ is not the rest mass of new free particle consisting of two interacting particles.

Finally, the only common thing between the vectors

$$\overline{m}_0 = \left(m_{0i}, \overline{W}_0 \right),$$

and

$$\overline{m}_0 = \left(E/c^2, \overline{P}/c \right),$$

is that, in both cases, the magnitude of three-vector $\overline{W}_0$ in the first and of $\overline{P}/c$ in the second is a part of the zero component.

Generalization of Many-Body System

Now, we can generalize eq. (7) to the complex particle which contains more than two constituents as follows. Let $\overline{m}_1, \overline{m}_2, \ldots \overline{m}_n$ are the mass vectors of n particles which fuse together to consist a new particle of mass vector $\overline{m}$, then

$$\overline{m} = \sum_{i=1}^{n} \overline{m}_i \qquad \ldots(27)$$

Therefore,

$$\|\overline{m}\|^2 = \|\sum_{i=1}^{n} \overline{m}_i\|^2 = \sum_{i=1}^{n} \|\overline{m}_i\|^2 + \sum_{i \neq j} \overline{m}_i . \overline{m}_j, \qquad \ldots(28)$$

i.e.,

$$m = \sum_{i=1}^{n} m_i + \sum_{i \neq j} \lambda_{ij} \sqrt{m_i} \sqrt{m_j}, \qquad \ldots(29)$$

where $\lambda_{ij} = \hat{\overline{m}}_i \cdot \hat{\overline{m}}_j$. This is the principle result of this work and we can call it the general mass formula where the mass parabola equation of nuclei can be considered as a special case of it eq. (41).

Nuclear Mass Formula

In the nuclear case, we-at first-note that, the different protons (neutrons) in the nucleus exist in different states of energy and spin. Therefore, we can expect that, the mass vector of each proton (neutron) is different from the mass vectors of the other protons (neutrons). The different states of energy mean different velocities, and then different values of masses, according to special theory of relativity. The different spin, isospin and other quantities characterizing the state of the particle may lead to different orientation of the mass vectors. Even with very small difference in masses, due to the non-relativistic velocity of nucleons inside the nucleus and to equality of different proton (neutron) masses at rest, the mass vectors of the different nucleons are different and must be related to the energy levels (or, in general, to the states) of the nucleons. Only, as an approximation, we can consider that, the norms of the mass vectors of all protons (neutrons) are the same, i.e.,

$$\|\overline{m}_i\| = \sqrt{m_{p(n)}}, \text{for all protons (neutrons)}$$

and $\overline{m}_i \neq \overline{m}_j$ for any two different protons (neutrons). Thus, from Equation (29) for a nucleus A_{X_Z}, with the mass number A and proton number Z, we have the following mass formula

$$m = Zm_p + Nm_n + 2\sum_{i>j} \lambda_{ij}\sqrt{m_i}\sqrt{m_j}, \qquad \ldots(30)$$

where $N = A - Z$ is the number of neutrons in the nucleus. From this equation, with $c = \hbar = 1$, the binding energy of the nucleus is given by

$$E_b = (Zm_p + Nm_n) - m$$

$$= -2\sum_{i>j} \lambda_{ij}\sqrt{m_i}\sqrt{m_j} \qquad \ldots(31)$$

The parameters λ_{ij}, $i \neq j$, $i, j = 1, \ldots, A$ are related to the orbits occupied by i^{th} and j^{th} nucleons and, in general, are different from each other.

The Physical Meaning of λ_{ij}

As we know that, the mass of a particle is a relativistic quantity and obeys to the special relativity laws. Thus, using the mass centre frame of reference, the left hand side of eq. (29) represents the rest mass of the system and the mass m_i of i^{th} particle

is dependent on the velocity v_i of the particle with respect to the centre of mass of the system, i.e.,

$$m_0 = \sum_i \frac{m_{0i}}{\sqrt{1-v_i^2}} + 2\sum_{i>j} \frac{\sqrt{m_{0i}}}{\sqrt[4]{1-v_i^2}} \frac{\sqrt{m_{0j}}}{\sqrt[4]{1-v_j^2}} \lambda_{ij}, \qquad \ldots(32)$$

where the index 0 denoted the rest mass, and lij can be, also, dependent on v_i and v_j. If we consider that the velocity v_i is small enough with respect to the velocity of light and is approximately the same for all particles, $v_i = v$ for all i, we obtain, in the non-relativistic limit, that

$$m_0 = \sum_i m_{0i}\left(1+\frac{1}{2}v^2\right) + 2\sum_{i>j} \sqrt{m_{0i}}\sqrt{m_{0j}}\,\lambda_{ij}\left(1+\frac{1}{2}v^2\right) \qquad \ldots(33)$$

Since $|\lambda_{ij}| < 1$ (sometimes $|\lambda_{ij}| \ll 1$) for $i \neq j$ and $(v/c) \ll 1$, the term $\sum_{i>j} \sqrt{m_{0i}}\sqrt{m_{0j}}\,\lambda_{ij} v^2$ can be neglected to obtain

$$m_0 = \sum_i m_{0i} + \sum_i \frac{1}{2} m_{0i} v^2 + 2\sum_{i>j} \sqrt{m_{0i}}\sqrt{m_{0j}}\,\lambda_{ij},$$

i.e.

$$m_0 = \sum_i m_{0i} + T + 2\sum_{i>j} \sqrt{m_{0i}}\sqrt{m_{0j}}\,\lambda_{ij}, \qquad \ldots(34)$$

where T is the total kinetic energy of the particles in the system in the mass centre frame of reference and $\lambda_{0ij} = \lim_{v/c \to 0} \lambda_{ij}$.

We can obtain relation between the quantities λ_{ij} and the potential energy of the system introducing. for example, the definition of non-relativistic mass operator in the form[1]

$$\hat{M} = \sum_i m_{i0} + \hat{T} + \sum_{i>j} \hat{V}_{ij}, \qquad \ldots(35)$$

where $\hat{V}_{ij}$ is the two-body potential energy operator, $\hat{T}$ is the kinetic energy operator and the many-body forces are neglected. If the system is in the ground state $|\psi>$, considering the mass centre frame of reference, we get

$$m_0 = \langle\psi|\hat{M}|\psi\rangle = \sum_i m_{0i} + \sum_{i>j} \langle\psi|\hat{T} + \hat{V}_{ij}|\psi\rangle. \qquad \ldots(36)$$

Comparing this equation by our eq. (34), we have

$$2\sum_{i>j}\sqrt{m_{oi}}\sqrt{m_{oj}}\,\lambda_{oij}=\sum_{i>j}\bar{V}_{ij}, \qquad \ldots(37)$$

where $\bar{V}_{ij}=\langle\psi|\hat{V}_{ij}|\psi\rangle$. Since, we consider only two-body interaction and every ij-term in both sides corresponds only to i^{th} and j^{th} particles we can take

$$\bar{V}_{ij}=2\sqrt{m_{oi}}\sqrt{m_{oj}}\,\lambda_{oij},$$

i.e.

$$\lambda_{oij}=\frac{1}{2\sqrt{m_{oi}}\sqrt{m_{oj}}}\bar{V}_{ij}, \qquad \ldots(38)$$

which gives the physical meaning of the parameter λ_{ij} as a representation of ij-interaction.

Special Case of Mass Parabola Equation

As an approximation for $i \neq j$, we shall assume that $\lambda_{ij} = \lambda_{pp}$ for $i, j \leq Z$, $\lambda_{ij} = \lambda_{nn}$ for $Z+1 \leq i, j \leq A$ and $\lambda_{ij} = \lambda_{np}$ $(= \lambda_{pn})$ for $Z+1 \leq i \leq A$ and $j \leq Z$ (or $i \leq Z$ and $Z+1 \leq j \leq A$). Thus eq. (30) takes the form,

$$m = Zm_p + Nm_n + 2\lambda_{pp}\left(\frac{Z^2-Z}{2}\right)m_p + 2\lambda_{nn}\left(\frac{(A-Z)^2-(A-Z)}{2}\right)m_n + 2\lambda_{np}(A-Z)\,Z\sqrt{m_n}\sqrt{m_p}, \qquad \ldots(39)$$

i.e.,

$$m=\left(\lambda_{pp}\,m_p+\lambda_{nn}\,m_n-2\lambda_{np}\sqrt{m_n}\sqrt{m_p}\right)Z^2 + \left(m_p-m_n-\lambda_{pp}\,m_p+\lambda_{nn}(1-2A)\,m_n+2\lambda_{np}\,A\sqrt{m_p}\sqrt{m_n}\right)Z + \left(\lambda_{nn}\,(A-1)+1\right)m_n\,A. \qquad \ldots(40)$$

Then,

$$m=\gamma Z^2+\beta Z+\alpha A, \qquad \ldots(41)$$

where

$$\left.\begin{aligned}\gamma&=\lambda_{pp}\,m_p+\lambda_{nn}\,m_n-2\lambda_{np}\sqrt{m_n}\sqrt{m_p},\\ \beta&=m_p-m_n-\lambda_{pp}\,m_p+\lambda_{nn}(1-2A)\,m_n+2\lambda_{np}\sqrt{m_p}\,m_n,\\ \alpha&=\left(\lambda_{nn}(A-1)+1\right)m_n,\end{aligned}\right\} \qquad \ldots(42)$$

This is a mass parabola equation which is similar to well-known Weizsäcker mass formula[2]

$$m = \gamma_1 Z^2 + \beta_1 Z + \alpha_1 A \cdot \qquad \ldots(43)$$

where γ_1, β_1 and α_1 are defined in terms of the coefficients C_{vol}, C_{sur}, C_{asy}, C_{coul} and C_{pair} which are related to volume, surface-, asymmetry-, Coulomb- and pairing energy as follows

$$\gamma_1 = \frac{4C_{asy}}{A} + \frac{C_{coul}}{A^{1/3}},$$

$$\beta_1 = -4C_{asy} - (m_n - m_p),$$

and

$$\alpha_1 = m_n - \left(C_{vol} - C_{asy} - \frac{C_{sur}}{A^{1/3}}\right) \pm \frac{C_{pair}}{A^{3/2}} \text{ (or zero)}, \qquad \ldots(44)$$

where the last term in α_1 equals zero for odd-A and equals $\mp\frac{C_{pair}}{A^{3/2}}$ for even-A depending upon whether Z is even or odd. The difference between the two eqs. (41) and (43) is, only, the method of parameterization which are further discussed in the following.

Comparing the two eqs. (41) and (43) we have

$$\frac{4C_{asy}}{A} + \frac{C_{coul}}{A^{1/3}} = \lambda_{pp} m_p + \lambda_{nn} m_n - 2\lambda_{np}\sqrt{m_n}\sqrt{m_p}, \qquad \ldots(45a)$$

$$-4C_{asy} - (m_n - m_p) = (1 - \lambda_{pp}) m_p + (\lambda_{nn}(1-2A) - 1) m_n + 2\lambda_{np} A \sqrt{m_n}\sqrt{m_p} \qquad \ldots(45b)$$

and

$$m_n - \left(C_{vol} - C_{asy} - \frac{C_{sur}}{A^{1/3}}\right) \mp \frac{C_{pair}}{A^{3/2}} \text{ (or zero)} = (\lambda_{nn}(A-1) + 1) m_n. \qquad \ldots(45c)$$

From these three eqs. (45) we get

$$\lambda_{nn} = \frac{1}{(A-1) m_n} \left[\mp \frac{C_{pair}}{A^{3/2}} \text{ (or zero)} - \left(C_{vol} - C_{asy} - \frac{C_{sur}}{A^{1/3}}\right)\right], \qquad \ldots(46a)$$

$$\lambda_{pp} = \frac{1}{(A-1) m_p} \left[\mp \frac{C_{pair}}{A^{3/2}} \text{ (or zero)} - \left(C_{vol} - C_{asy} - \frac{C_{sur}}{A^{1/3}}\right) + C_{coul} A^{2/3}\right], \qquad \ldots(46b)$$

and $$\lambda_{np} = \frac{1}{(A-1)\sqrt{m_n}\sqrt{m_p}} \left\{\mp \frac{C_{pair}}{A^{3/2}} \text{ (or zero)} - \left(C_{vol} - \frac{C_{sur}}{A^{1/3}}\right) - \frac{(A-2)}{A} C_{asy} + \frac{C_{coul}}{2A^{1/3}}\right\}. \qquad \ldots(46c)$$

The similarity among (46*a*), (46*b*) and (46*c*) is clear and represents the isospin

invariant of nuclear force in Weizsäcker formula. The little difference between them can be understood as the result of electric force and asymmetry effect.

Discussion

The Weizsäcker mass formula plays an important role in nuclear physics to determine the nuclear mass, binding energy of nuclei, the energy of nucleon-separation and so on. This formula depends on the coefficients C_{vol}, C_{sur}, C_{asy}, C_{coul} and C_{pair}, each of which is related to some concrete property of nuclear force. All these properties can be called "group properties". Since, it is related to the group of nucleons as a whole. Also, all these properties of nuclear force must appear in *p-p*, *n-n* and *p-n* interaction. Therefore, the reformulation of this formula to create a new form depending upon some coefficients which are directly related to these interactions, may be useful. Our eq. (41), which is the special case of general nuclear mass formula (30), satisfies this requirement where λ_{nn}, λ_{pp} and λ_{np} play the required role to represent *n-n*, *p-p* and *n-p* interaction.

Also, the coefficients C_{vol}, C_{sur}, C_{asy}, C_{coul} and C_{pair} are constants not related to the energy- shells where the nucleons take place. These shells have a deeper relation with the nucleon-nucleon interaction and then with the binding energy. Therefore, it is interesting to have this formula where the coefficients are related to the states of nucleons in the nucleus. The general nuclear mass formula (30) satisfies this requirement, where the coefficient λ_{ij} is related—as explained before—to the states of the two interacting particles. Of course, an equation with this required condition is more strongly related to nuclear structure than the usual mass parabola equation of Weizsäcker.

The formula (30) contains large number of nucleon-nucleon parameters λ_{ij} even with light nuclei. But, as a first step, we can consider—as an approximation—that, every energy shell characterized by a constants λ_{nn}, λ_{pp} and λ_{np}. This approximation is more accurate than the approximation which is corresponding to the mass parabola equation with λ_{nn} is constant for any *n-n* pair, λ_{pp} is constant for any *p-p* pair and λ_{np} is constant for any *n-p* pair in the nucleus. Thus, with the suggested approximation for the first-shell nuclei we have only three parameters $\lambda_{p_1p_1}$, $\lambda_{n_1n_1}$, and $\lambda_{n_1p_1}$ where the index 1 denote the first-shell. For nuclei with the first-and second-shell we have, only, nine parameters $\lambda_{p_1p_1}, \lambda_{n_1n_1}, \lambda_{n_1p_1}, \lambda_{p_2p_2}, \lambda_{n_2n_2}, \lambda_{n_2p_2}, \lambda_{p_1p_2}, \lambda_{n_1n_2}, \lambda_{n_1p_2}$ (or $\lambda_{p_2n_1}$) and so on. In the general case the number of λ_{ij} is $A(A\text{-}1)$.

In the mass parabola approximation, using eq. (46), it is easy to obtain the values of λ_{nn}, λ_{pp} and λ_{np} where the values of C_{vol}, C_{sur}, C_{asy}, C_{coul} and C_{pair} are well known in the liquid drop model. But, for more accurate approximation we shall need more complicated calculations.

Some Notes and Predictions

The source of the mass vector idea—in our model—is the similarity between the triangle inequality (1) of the usual mathematical vectors and the inequality (2) of masses which is correct in all branches of physics except in some cases of particle physics where the confined theories take place and the mass of a particle is heavier than the sum of the masses of its constituent particles. However, we must note that, for defined mass vector (in our model) the relations (1) [or (2)] may be unsatisfied. Also, its norm may be negative as in the case of space-time vector of special relativity. These two possibilities have no contradiction with mathematical construction of the model and have, at the same time, physical interpretations, as we see below.

The other important note about the model is clear from eq. (10) and the transformations (18), where we can consider our model as a trial to construct a unified formula—in some sense—for different kinds of fields. The elements of a mass space are the mass vectors of different particles and for each vector the components (m^1, m^2, m^3), see eq. (9), are related to the field of the particle, where $(m^1)^2 + (m^2)^2 + (m^3)^2$ = the virtual mass of the particle field W. Therefore, considering only electromagnetic interaction, for example, we have a specific mass space in which the virtual mass W for each vector is related to the electromagnetic field of the particle. Thus, we can consider different mass spaces for different kinds of interaction, i.e. each mass space represents some kind of force (or combination of forces). We can see that, there are four principle mass spaces corresponding to the four principle kinds of forces (strong, electromagnetic, gravitational and weak). The other different combinations of these forces give 11 mass spaces, one of them corresponds to the total field of the particle. In all these 15 mass spaces the particle has the same definite measured mass which is related to the space-time reference where it is moving. For another space-time reference we have another set of 15 mass spaces characterized by another value of the particle measured mass relating to the used space-time reference. Finally, for the transformation from a mass space to another i.e. the transformation from one kind of force to another kind ! – the value of the parameter a in the relation (18) is needed. But, we do not have until now-a clear way to determine it.

Now, we present, using our model, some predictions which may be useful to motivate the model.

(i) Nucleon-Nucleon Interaction in Nuclei

In Table I, the values of the neutron-neutron parameter λ_{nn}, the proton-proton parameter λ_{pp}, and neutron-proton parameter λ_{np}, are presented. These values are calculated using the relations (46) for some nuclei obeying the liquid drop model where $A \geq 4$. These parameters are related to nucleon-nucleon interaction see eq. (38) and determine the nuclear binding energy, (see eq. (31)). It is clear that, for

all nuclei $\lambda_{nn} > 0$, $\lambda_{pp} > 0$ and $\lambda_{np} < 0$, which means that the contribution of *n-n* and *p-p* interaction in the binding energy of a nucleus leads to its disconnection. At the same time, the holding of the nucleus is due to *n-p* interaction. These results interpret the absence of *n-n* and *p-p* bound system. This conclusion can not be obtained directly from the liquid drop model and in agreement with Pauli exclusive principle.

Table I

Values of the coefficients λ_{ij} for some nuclei

Nucleus	λ_{nn}	λ_{pp}	λ_{np}
$^{4}He_{2}$	5.09×10^{-3}	5.62×10^{-3}	-5.84×10^{-3}
$^{6}Li_{3}$	3.64×10^{-3}	4.05×10^{-3}	-3.66×10^{-3}
$^{12}C_{6}$	1.25×10^{-3}	1.55×10^{-3}	-2.40×10^{-3}
$^{14}N_{7}$	1.08×10^{-3}	1.36×10^{-3}	-2.05×10^{-3}
$^{40}Ca_{20}$	2.84×10^{-4}	4.70×10^{-4}	-8.13×10^{-4}
$^{50}V_{23}$	2.19×10^{-4}	3.91×10^{-4}	-6.59×10^{-4}
$^{202}Hg_{80}$	4.23×10^{-5}	1.49×10^{-4}	-1.75×10^{-4}

If a nucleus has Z protons and N neutrons, then, we have $(Z-1)Z/2$ proton pairs, $(N-1)N/2$ neutron pairs and NZ neutron-proton pairs. Thus, the number of neutron-proton pairs is more than the number of neutron pairs and protons pairs together by $[A-(Z-N)^2]/2$, where $A = Z + N$. Since $\lambda_{nn} < \lambda_{pp} < |\lambda_{np}|$, then, on the basis of our model, the binding energy of the nucleus increases with decreasing of $|Z-N|$. Therefore, the more stable nuclei—as it is well known—when $N = Z$.

(ii) Quark-quark Interaction in Baryons and Mesons

If we consider the down quark and up quark rest masses are equal, i.e., $m_d = m_u$ (=363 MeV), the nucleon mass m_N and Δ—particles mass m_Δ are given by[3]

$$m_{N(\Delta)} = 3m_u \mp \frac{3}{4}\frac{\hbar^2}{m_q^2}A' , \qquad ...(47)$$

where the positive sign for the Δ-particle and $A' = 50\ (\frac{2m_u}{h})^2$ MeV/c^2. At the same time, this mass equation, in our model, considering the quarks at rest (as an approximation) and have the same mass vectors (as another approximation) is, from eq. (29), given by

$$m_{N(\Delta)} = 3m_u + 6\lambda_{qq}^{N(\Delta)} m_u, \qquad ...(48)$$

where $\lambda_{qq}^{N(\Delta)}$ is the quark-quark interaction parameter in the nucleon (Δ-particle) Thus, comparing these two equations we have

$$\lambda_{qq}^{N(\Delta)} = (\mp)\,0.069, \qquad ..(49)$$

Also, under the same approximations, the corresponding two mass equations in the $\pi(\rho)$- meson case are[3]

$$m_{\pi(\rho)} = 2m_u (\mp)\frac{1}{4}\frac{\hbar^2}{m_u^2}A, \qquad ...(50)$$

and (in our model)

$$m_{\pi(\rho)} = 2m_u + 2\lambda\frac{\pi(\rho)}{q\bar{q}}\,m_u, \qquad ...(51)$$

respectively, where, in eq. (50), $A = 160\left(\frac{2m_u}{\hbar}\right)^2$ MeV/c^2, and the positive sign for ρ-particle. In this case m_u = 310 MeV. Thus, we get the quark-antiquark parameter $\lambda_{q\bar{q}}^{\pi(\rho)}$ has the value

$$\lambda\frac{\pi(\rho)}{q\bar{q}} = (\mp)\ 0.268, \qquad ...(52)$$

From the values (49), (52) and Table I, taking into account the physical meaning of the parameter λ_{ij}, eq. (38), we can give the relative strength of the nucleon-nucleon, quark-quark and quark- antiquark interaction, Table II. We must note that, the value of quark-quark interaction parameter, eq. (49), may not be accurate. This because, the accurate form of the last term of our mass eq. (48) is

$$2.2\sqrt{m_u}\sqrt{m_d}\,\lambda_{ud} + 2\sqrt{m_u}\sqrt{m_u}\,\lambda_{uu},$$

and the values of λ_{ud} and l_{ud} may be positive or negative and, therefore, the total value of this term doesn't give the accurate value of λ_{qq} which may be the same as the value of $\lambda_{q\bar{q}}$ for quark-antiquark interaction. In any case, the quark-quark and quark-antiquark interactions enough larger than the nucleon-nucleon interaction, such that the existence of the quark out side the nucleon is not expectable.

Table II

Relative strength of the nucleon-nucleon, quark-quark and quark-antiquark interactions taking the strength of nucleon-nucleon interaction as a unit

Interaction	Relative strength
N-N	1
q-q	12
$q\text{-}\bar{q}$	44

(iii) Confined Theory - An Interpretation

The inequality (2) of masses is not satisfied for some particles as a composite system of quarks, for example, as Δ-particle and ρ-meson, where the confined theories take place. Also, it is satisfied for other particles as nucleons and π-mesons. If we interpret the decreasing in the masses as the binding energy in the later cases of nucleons and π–mesons, what we can say about the increasing in the masses in the first case ? Therefore, we try to give a general interpretation for both cases in the framework of our model.

Consider a particle of measured mass m, which consists of two particles of measured masses m_1 and m_2. From eq. (11) we have

$$m = m_1 + m_2 + 2(\sqrt{m_{1t}}\sqrt{m_{2t}} - W_{12}), \qquad ...(53)$$

where m_{it} are the total mass of i^{th} particle and W_{12} are the virtual mass of interaction. For a moment, let us consider that the two particles are identical, such that $m_{1t} = m_{2t} = m_t$. Thus

$$\sqrt{m_{1t}}\sqrt{m_{2t}} - W_{12} = m_t - W_{12}. \qquad ...(54)$$

But,

$$\text{the measured mass of a particle} = m_t - W, \qquad ...(55)$$

where m_t is the total mass and W is the virtual mass of particle field. Therefore, we can consider the quantity $m_t - W_{12}$—eq. (54)—as a measured mass produced through the interaction of the two identical particle. Thus the same sense can be used for the quantity $\sqrt{m_{1t}}\sqrt{m_{2t}} - W_{12}$, where we can consider it as a measured mass produced through the interaction of the two particles 1 and 2. Therefore, eq. (53) can be written as

The measured mass of the composite particle

= the measured mass m_1 + the measured mass m_2

+ 2 the measured mass of interaction. ...(56)

For the general case of n constituent particles we have

the measured mass of the composite particle

$$= \sum_{i=1}^{n} m_i + 2\sum_{i<j} (\sqrt{m_{it}}\sqrt{m_{jt}} - W_{ij}), \qquad ...(57)$$

where

$$\sum_{i<j} (\sqrt{m_{it}}\sqrt{m_{jt}} - W_{ij}), \qquad ...(57')$$

is the measured mass of interaction.

Therefore, if

$$\sum_{i<j} (\sqrt{m_{it}}\sqrt{m_{jt}} - W_{ij}) < 0, \qquad ...(58)$$

i.e., the measured mass of interaction is negative, the measured mass of the composite particle is less than the sum $\sum_{i=1} m_i$ as in the cases of nucleons, π-mesons and nuclei. Also, if

$$\sum_{i<j} (\sqrt{m_{it}}\sqrt{m_{jt}} - W_{ij}) > 0, \qquad ...(59)$$

i.e., the measured mass of interaction is positive, the measured mass of the composite particle is larger than the sum $\sum_{i=1} m_i$ as in the Δ particle and ρ-meson cases, and the confined theories are satisfied.

In fact, considering—for simplicity—the two-particle bounded system, we believe that, in the case where $\sqrt{m_{1t}}\sqrt{m_{2t}} - W_{12} < 0$, the virtual mass of interaction W_{12} is large enough such that the virtual masses of particle fields W_1 and W_2 are not enough to complete the interaction. Therefore, parts of measured masses of the particles m_1 and m_2 are used to complete the process. On the other hand, the case $\sqrt{m_{1t}}\sqrt{m_{2t}} - W_{12} > 0$, where the measured mass of interaction is positive, means that the virtual mass of interaction W_{12} is small enough such that the interaction is complete and the residue of the virtual masses of the fields introduced a positive measured mass of interaction which leads to the increase of the measured mass of composite particle.

In the general case of n constituents, the terms of the sum (57') may be positive or negative (due to the nature of the interaction between the particles) and sign of the final resultant leads to decreases or increases of the mass of the system.

Finally, in the case

$$\sqrt{m_{1t}}\sqrt{m_{2t}} - W_{12} = 0, \qquad ...(60)$$

we get

$$m = m_1 + m_2 ,$$

i.e., no part of measured masses of the interacting particles is used in the interaction and no part of virtual masses is added to $m_1 + m_2$ due to the interaction. This means that, the virtual masses of the interacting particles are only used and completely in the interaction and there is no new composite particle.

(iv) The Hole Theory

For a particle of measured mass m, total mass m_t and virtual mass W, we know that $m = m_t - W$. Thus, if $W = m_t$ we have zero measured mass particle, i.e., $m = 0$. This is the case of the particle of the field as photon and gluon where the total mass of the particle is used to construct the field. Also, if $W < m_t$, then $m = m_t - W > 0$ which is the usual case of the particle with positive mass.

Now, we consider the case where $W > m_t$, which means that the measured mass of the particle $m < 0$. Since $W > m_t$, then, the field which is represented by W used all available mass m_t and still in need to another mass (or energy) to construct themself. Thus, this field can swallow a lot of masses before coming to the stable situation where $m \geq 0$. This give us a chance to consider that the field of a negative measured mass as a hole, which is strong or weak relative to $m<<0$ or not. As a result

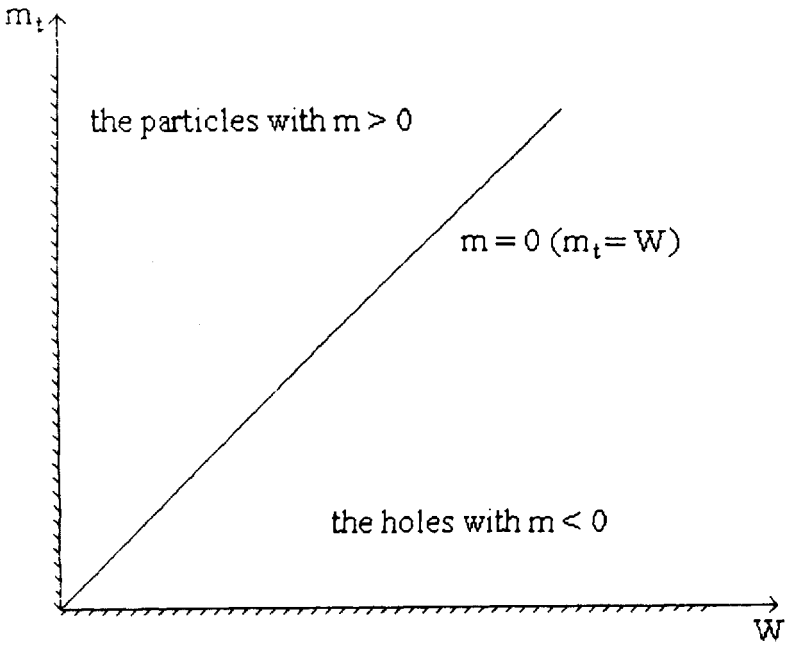

Fig 1 A representation of different cases of the measured mass m of the particles where $m>0$, $m > 0$ and $m < 0$

of this conclusion we can consider the black hole in the cosmos as a special case of the fields with very large negative masses. We can represent the three cases of positive, zero and negative mass by Fig. 1.

The Model Parameters

In our model we have two kinds of parameters. The first kind of the parameters are λ_{ij}, which are related to the mass eq. (29), and the second is the transformation parameter α which is related to eq. (18).

The parameters λ_{ij} have been obtained from eqs. (7), (11) and (29) as

$$\lambda_{ij}=\frac{1}{\sqrt{m_i}\sqrt{m_j}}(\sqrt{m_{it}}\sqrt{m_{jt}}-W_{ij}), \qquad ...(61)$$

which can be written as

$$\lambda_{ij}=\sqrt{\frac{\sqrt{m_i}\sqrt{m_j}-W_{ij}}{m_i}}\sqrt{\frac{\sqrt{m_i}\sqrt{m_j}-W_{ij}}{m_j}} \qquad ...(62)$$

Thus, using a given physical meaning of the quantity $(\sqrt{m_{it}}\sqrt{m_{jt}}-W_{ij})$ in eq. (56) as the measured mass produced through the interaction of the two particles i and j, we can consider the quantity $(\sqrt{m_{it}}\sqrt{m_{jt}}-W_{ij})/m_i$ as the measured mass produced through the interaction for each measured mass unit of the interacting particle i. Therefore, if the two interacting particles have, approximately, the same measured mass ($m_i \approx m_j$) as in λ_{nn}, λ_{pp} and λ_{np} cases (Eqs. (39)) we get

$$\frac{\lambda_{ij}}{2}=\frac{\sqrt{m_{it}}\sqrt{m_{jt}}-W_{ij}}{m_i+m_j}, \qquad ...(63)$$

where $\lambda_{ij/2}$ can be considered as the measured mass produced through the interaction for each measured mass unit of the two interacting particles. This is a concrete dynamical statement concerning the parameter λ_{ij} which is based on our ansatz.

However, this statement, practically, cannot be used, because eq. (63) may be written as

$$\frac{\lambda_{ij}}{2}=\frac{\sqrt{m_i+W_i}\sqrt{m_j+W_i}-W_{ij}}{m_i+m_j},$$

therefore, to determine λ_{ij}, some information about the virtual mass of interaction W_{ij} and the virtual masses of particle fields W_i and W_j must be known, which are not available in this stage of the model and of the particles knowledge.

For a realistic representation of this situation we consider the following example. The quark-antiquark (gluon) virtual cloud around the proton, from the present point of view of quark-parton models, is considered as a part of the hadronic matter and plays an important role in the dynamics of strong interaction. The mass of this virtual part of hadronic matter of proton is not contained in its measured mass and it is not measured—at least—until now. If we assume that, the mass of virtual cloud

around the proton represent the virtual mass W of the strong field of proton in our model, we have a realistic representation for W and we can consider the total mass m_t of proton equals the sum of its measured mass and the mass of the virtual cloud around it, which we hope to determine it to solve the problem of the parameters λ_{ij} using only the possibilities of the presented model.

Now, for the parameter α of eq. (18), we consider the following simple case. Consider a particle of measured mass m with two mass vectors $\bar{m} = (m^0, m^1, m^2, m^3)$ and $\bar{m} = (m'^0, m'^1, m'^2, m'^3)$ in two different mass spaces which are related to two different kinds of forces, where the virtual masses of the particle fields in the two spaces are W and W', respectively. Let $m^2 = m^3 = 0$, then, from (18), $m'^2 = m'^3 = 0$. But, in general

$$W = (m^1)^2 + (m^2)^2 + (m^3)^2,$$

thus, in the suggested case, we get $W = (m^1)^2$ and $W' = (m'^1)^2$. Therefore, noting that, $m^0 = \sqrt{m_t}$, we have

$$\sqrt{W'} = \frac{\sqrt{W} + \alpha\sqrt{m_t}}{\sqrt{1-\alpha^2}}. \qquad \ldots(64)$$

Solving this equation with respect to α we get

$$\alpha = \frac{-\sqrt{W}\sqrt{m+W} \pm \sqrt{W'}\sqrt{m+W'}}{m+W+W'} \qquad \ldots(65)$$

where $m_t = m + W$. For the transformation from a mass space to itself where $W = W'$, the parameter α must be zero. Therefore, we will take the positive sign in this equation and eq. (65) becomes

$$\alpha = \frac{\sqrt{W'}\sqrt{m+W'} - \sqrt{W}\sqrt{m+W}}{m+W+W'}. \qquad \ldots(66)$$

From this equation we can see that $|\alpha| < 1$ and the parameter of a transformation and its inverse between two mass spaces have the same value but with different sign.

Thus, taking into account the above realistic proton example, and considering, for example, the transformation performed between the mass spaces of the electromagnetic and strong forces, to determine α we need to know the virtual mass W of photon cloud around the particle and also the virtual mass W' of gluon cloud around the same particle. Therefore, the dynamical significance of parameter α is coming from the dynamical roles of the photon and the glucon clouds around the particle.

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