PINSA **65**, **A**, No. 4, July 1999, pp. 481-509.

LOW-FREQUENCY FLUCTUATIONS AND ELECTROMAGNETIC WAVE SCATTERING AND CONVERSION IN MAGNETIZED PLASMAS

A G SITENKO

Bogolyubov Institute for Theoretical Physics, National Academy of Sciences of Ukraine-252143 Kyiv-143 (Ukraine)

(Received 22 September 1998; accepted 01 January 1999)

The problem of electromagnetic wave scattering in a plasma placed in a strong external magnetic field and its possible applications for plasma diagnostics is discussed. The scattering is associated both with longitudinal field fluctuations (charge density fluctuations) and with transverse electromagnetic fluctuations. Electromagnetic wave scattering by magnetic fluctuations is accompanied by wave conversion. Low-frequency fluctuation spectra of electric and magnetic fields are calculated for a plasma with a strong external magnetic field. Incoherent fluctuations and fluctuations associated with the excitation of collective modes (Alfven and magnetosonic waves) are separated out. The spectral distributions of the electric and magnetic field fluctuations in nonequilibrium plasmas are considered. Effective temperatures are introduced both for the collective Alfven and magnetosonic fluctuations and for the incoherent fluctuations in low-frequency range. The wave scattering cross-section is calculated and the conditions are revealed, under which conversion is the dominant process. Peculiar features of scattered waves spectral distribution depending on the parameters that characterize plasma state are treated. Experimental studies of wave conversion provide additional information concerning the plasma state which cannot be obtained from the data on the scattering by density fluctuations.

Key Words: Electromagnetic Wave Scattering; Magnetized Plasmas; Eigen Waves; Alfven and Magnetosonic Waves; Eigen Oscillation Frequencies; Non-equilibrium Plasmas

1 Introduction

The study of electromagnetic wave scattering spectra (both in laser and centimeter wave ranges) is an efficient method of plasma diagnostics in laboratory fusion research devices as well as in the near and outer space. Electromagnetic wave scattering is caused by fluctuations of charged particle density and other plasma parameters: current density, electric and magnetic fields. Spectra of scattered waves provide information on the density and temperature distributions in the plasma. A peculiarity of electromagnetic wave scattering in plasmas is coherent scattering by collective plasma excitations—combination scattering, that occurs along with Thompson incoherent scattering by individual plasma particles. Wave scattering by collective plasma fluctuations, in particular, makes it possible to find relative concentrations of charged particles and temperatures of individual plasma components. The phenomenon of electromagnetic wave combination scattering by collective plasma excitations was considered for the first time in Akhiezer *et al.*[1]. Subsequently a theory of electromagnetic wave scattering in plasmas was developed[2-6]. The detailed theory of scattering and transformation of waves in a magnetoactive plasma was worked out in[7]. The useful reviews of the electromagnetic wave scattering problem were presented in Refs. [8,9] Kinetic and fluid treatments of nonlinear wave interactions (wave mixing and electromagnetic wave scattering) in plasmas were compared and the difference between the two approaches was worked out[10,11]. The nonlinear kinetic equation for cold plasmas was shown to yield the same expression for the current that induses scattered waves as the one obtained in the fluid approximation. Increased activities in the controlled fusion research and employment of magnetic plasma confinement systems require improved methods of plasma state analysis. Experimental studies[12,13] of electromagnetic wave conversion under the scattering by magnetic fluctuations in plasmas were shown to be possible. As distinct from the wave scattering by fluctuations of charged particle density, interaction of the incident wave and the magnetic field fluctuations may be accompanied by the electromagnetic wave conversion, i.e., transformation of an ordinary electromagnetic wave into an extraordinary one or *vice versa.* Though the scattering cross-section is much greater than the conversion cross-section, under certain conditions one can distinguish conversion from scattering and find the spectral distribution of converted waves. Studies of converted wave spectral distributions provide information on plasma parameters other than those associated with wave scattering, in particular on the magnetic field fluctuation intensity distribution etc. It is obvious that information on the plasma state can be drawn from experimental data only, provided the theoretical spectra of magnetic and electric field fluctuations in the plasma are available. This paper is just devoted to the consideration of low-frequency electromagnetic fluctuations and scattering and conversion of electromagnetic waves in a plasma with a strong external magnetic field.

2 Plasma with Strong Magnetic Field

Electromagnetic plasma properties are completely described in terms of the plasma dielectric permittivity tensor $\varepsilon_{ij}(\omega,\vec{k})$ which depends on the frequency ω and the wave vector $\vec{k}$. The dielectric permittivity tensor of a plasma with external magnetic field was calculated in the kinetic approximation in[14].

For an equilibrium plasma or a nonisothermal plasma in which the particles are characterized by Maxwellian distributions with different temperatures, the dielectric permittivity tensor is determined by the expression[6]:

$$\varepsilon_{ij}(\omega,\vec{k})=\delta_{ij}-\sum_{\alpha}\frac{\omega_{p_\alpha}^2}{\omega^2}\left\{\begin{array}{l} e^{-\beta\alpha}\sum_{n=-\infty}^{\infty}\frac{z_0^\alpha}{z_n^\alpha}\pi_{ij}(z_n^\alpha) \\ \times\left[\varphi(z_n^\alpha)-i\sqrt{\pi}z_n^\alpha\ \exp(-z_n^{\alpha 2})\right]-2z_0^{\alpha 2}b_ib_j\end{array}\right\} \quad \ldots (1)$$

where the summation is over the kinds of particles α, $\vec{b}$ is the unit vector in the $\vec{B}_0$ directions,

$$\pi_{ij}(z_n)=\begin{pmatrix} \frac{n^2}{\beta}I_n & -in(I_n-I_n') & \frac{k_\perp}{|k_\parallel|}\sqrt{\frac{2}{\beta}}nz_nI_n \\ in(I_n-I_n') & \left(\frac{n^2}{\beta}+2\beta\right)I_n-2\beta I_n' & i\frac{k}{|k|}\sqrt{2\beta}z_n(I_n-I_n') \\ \frac{k_\perp}{|k_\parallel|}\sqrt{\frac{2}{\beta}}nz_nI_n & -i\frac{k}{|k|}\sqrt{2\beta}z_n(I_n-I_n') & 2z_n^2I_n \end{pmatrix} \quad \ldots (2)$$

$I_n(\beta)$ and $I_n'(\beta)=dI_n/d\beta$ are the modified Bessel function and its derivative,

$$z_n^\alpha=\sqrt{\frac{2}{3}}\frac{\omega-n\omega_{B_\alpha}}{|k_\parallel|s_\alpha},\quad \omega_{p_\alpha}^2=\frac{4\pi n_0e_\alpha^2}{m_\alpha},$$

$$s_\alpha^2=\frac{3T_\alpha}{m_\alpha},\ \beta_\alpha^2=\frac{k_\perp^2s_\alpha^2}{3\omega_{B_\alpha}^2},\ \omega_{B_\alpha}=\frac{|e_\alpha|B_0}{m_\alpha c}.$$

The tensor eq. (1) is written in a coordinate system in which the z axis is along the external magnetic field $\vec{B}_0$ and the x axis is in the plane of the vectors $\vec{k}$ and $\vec{B}_0$.

In the kinetic approach, the dielectric tensor of a magnetoplasma, unlike the tensor of the dielectric permittivity of a cold plasma, is non-Hermitian. The non-Hermitian part of the tensor eq. (1) is caused by the resonant interaction of charged particles and waves and arises if the resonance condition

$$k_\parallel v_\parallel=\omega-n\omega_{B_\alpha}$$

take place.

In order to consider a plasma with strong external magnetic field $\vec{B}_0$ we introduce a dimensionless parameter

$$q^2 = \frac{k^2 s^2}{\omega_{B1}^2}$$

where $s = \sqrt{\frac{3T}{m}}$ is the electron thermal velocity (T is the plasma temperature, m is the electron mass) and $\omega_{B_i} = \frac{eB_0}{Mc}$ is the ion cyclotron frequency (e and M are the ion charge and mass, respectively). We assume that this parameter is small,

$$q^2 << 1.$$

We restrict the consideration to the spectral range of frequencies ω lower than the electron cyclotron frequency $\omega_{B_e} = \frac{eB_0}{mc}$ i.e.,

$$\omega << \omega_{B_e},$$

and employ approximate expressions for the plasma dielectric permittivity components, obtained under the assumption that $\frac{m}{M} << 1$ and expanded in power series of the small parameter q^2. In the coordinate system with the z-axis along the vector $\vec{B}_0$ and the x-axis in the plane of vectors $\vec{k}$ and $\vec{B}_0$, the components $\varepsilon_{ij}(\varpi, \vec{k})$ of the dielectric permittivity tensor take the form

$$\varepsilon_{11} = \varepsilon_1 + i\psi_1,$$

$$\varepsilon_{22} = \varepsilon_{11} - \frac{m}{M}\varepsilon_0\left\{1 + \varphi(z) - i\sqrt{\pi} z e^{-z^2}\right\}\frac{tg^2\vartheta}{z^2},$$

$$\varepsilon_{33} = 3\frac{M}{m}\varepsilon_0\left\{1 - \varphi(z) + i\sqrt{\pi} z e^{-z^2}\right\}\frac{1}{q^2 \cos^2\vartheta},$$

$$\varepsilon_{12} = i\{\varepsilon_2 + i\psi_1\}, \quad \ldots (3)$$

$$\varepsilon_{23} = -i\varepsilon\frac{\omega_{B_i}}{\omega}$$

$$\left\{1 - \varphi(z) + i\sqrt{\pi} z e^{-z^2} + \frac{1}{2}\left(1 - \varphi(z_1) + i\sqrt{\pi} z_1 e^{-z_1^2}\right)\right\} tg\vartheta,$$

$$\varepsilon_{13} = \frac{1}{2}\varepsilon_0\frac{\omega_{B_i}}{\omega}\left\{1 - \varphi(z_1) + i\sqrt{\pi} z_1 e^{-z_1^2}\right\} tg\vartheta.$$

Here ϑ is the angle formed by the vectors $\vec{k}$ and $\vec{B}_0$; $\varepsilon_0 = \frac{\omega_{p_i}^2}{\omega_{B_i}^2}$, where $\omega_{p_i}^2 = \frac{4\pi n_0 e^2}{M}$ is the square of the ion plasma frequency (we assume that $\varepsilon_0 >> 1$); $z = \sqrt{\frac{3}{2}}\frac{\omega}{ks\cos\vartheta}$ is the dimensionless frequency; $z_1 = \sqrt{\frac{3}{2}\frac{M}{m}}\frac{|\omega_{B_i} - \omega|}{ks\cos\vartheta}$; the function $\varphi(z)$ is defined by

$$\varphi(z) = 2ze^{-z^2}\int_0^z dx\, e^{-x^2}; \quad \ldots (4)$$

and the notation is introduced:

$$\varepsilon_1(\omega) = \frac{1}{2}\frac{\omega_{p_i}^2}{\omega}\left\{\frac{1}{\omega_{B_i} - \omega}\varphi(z_1) - \frac{1}{\omega_{B_i} + \omega}\right\},$$

$$\varepsilon_2(\omega) = \varepsilon_1(\omega) - \frac{\omega_{p_i}^2}{\omega_{B_i}(\omega_{B_i} + \omega)}, \quad \ldots (5)$$

$$\psi_1(\omega) = \frac{1}{2}\sqrt{\frac{3\pi}{2}\frac{M}{m}}\frac{\omega_{p_i}^2}{\omega ks\cos\vartheta}e^{-z_1^2}.$$

We note that

$$\varepsilon_{11}^2 + \varepsilon_{12}^2 = \varepsilon_1^2 - \varepsilon_2^2 + 2i\frac{\omega_{p_i}^2}{\omega_{B_i}(\omega_{B_i} + \omega)}\psi_1.$$

If $z_1^2 >> 1$, then we have

$$\varepsilon_1 \to \frac{\omega^2{}_{p_i}}{\omega_{B_i}^2 - \omega^2}, \quad \varepsilon_2 \to \frac{\omega}{\omega_{B_i}}\varepsilon_1.$$

In eq. (3), only the first terms of the q^2-expansions of general expressions for the permittivity tensor of a plasma with external magnetic field eq. (1) are retained. We note that individual components of the permittivity tensor eq. (3) differ in the order of magnitude with respect to the parameter q^2. The leading term of the expansion of the component ε_{33} is proportional to q^{-2}, and those of the components ε_{11} and ε_{22} are of the order of one.

The component ε_{33} of the dielectric permittivity tensor in a plasma taking account of ion motion is determined by

$$\varepsilon_{33} = 3\frac{M}{m}\varepsilon_0 \left\{ \begin{matrix} 2-\varphi(z)-\varphi(\mu z) \\ +i\sqrt{\pi}z\left(e^{-z^2}+\mu e^{-\mu^2 z^2}\right) \end{matrix} \right\} \frac{1}{q^2\cos^2\vartheta},$$

where $\mu^2 = \dfrac{M}{m}$.

We introduce the dispersion tensor $\Lambda_{ij}(\omega,\vec{k})$:

$$\Lambda_{ij}(\omega,\vec{k}) = \varepsilon_{ij}(\omega,\vec{k}) - \left(\delta_{ij} - \frac{k_i k_j}{k^2}\right)\eta^2 \qquad \text{... (6)}$$

where $\eta^2 = \dfrac{k^2c^2}{\omega^2}$ is the square of the wave refraction index and write its determinant in the form

$$\Lambda(\omega,\vec{k}) = A\eta^4 + B\eta^2 + C \qquad \text{...(7)}$$

where

$$A = \varepsilon_{11}\sin^2\vartheta + \varepsilon_{33}\cos^2\vartheta + 2\varepsilon_{13}\sin\vartheta\cos\vartheta,$$

$$\left.\begin{aligned} B &= 2(\varepsilon_{12}\varepsilon_{23} - \varepsilon_{22}\varepsilon_{13})\sin\vartheta\cos\vartheta \\ &-(\varepsilon_{11}\varepsilon_{22}+\varepsilon_{12}^2)\sin^2\vartheta \\ &-(\varepsilon_{22}\varepsilon_{33}+\varepsilon_{33}^2)\cos^2\vartheta - \varepsilon_{11}\varepsilon_{33}+\varepsilon_{13}^2 \end{aligned}\right\}, \qquad (8)$$

$$\begin{aligned} C &= \varepsilon_{11}\varepsilon_{22}\varepsilon_{33} + \varepsilon_{11}\varepsilon_{23}^2 \\ &- \varepsilon_{22}\varepsilon_{13}^2 + \varepsilon_{33}\varepsilon_{12}^2 + 2\varepsilon_{12}\varepsilon_{23}\varepsilon_{31} \end{aligned}$$

We see that the permittivity tensor components are contained in the coefficients A, B and C in a differing manner. If $\omega \ll \omega_{B_i}$, then $|\varepsilon_{13}| \ll |\varepsilon_{23}|$; if $\omega \lesssim \omega_{B_i}$, then $\varepsilon_{13} \approx \varepsilon_{23}$ but $|\varepsilon_{23}| \ll |\varepsilon_{12}| \approx |\varepsilon_{11}| \ll |\varepsilon_{33}|$.

Thus, we may disregard ε_{13} in eq. (8). We restrict the expansions of the coefficients A, B and C by the terms of the order of q^{-2} and one, then the determinant (7) reduces to

$$\Lambda(\omega,\vec{k}) = \varepsilon_{33} \left\{ (\eta^2\cos^2\vartheta - \varepsilon_{11})\left(\eta^2 - \varepsilon_{22} - \frac{\varepsilon_{23}^2}{\varepsilon_{33}}\right) + \varepsilon_{12}^2 + \frac{\eta^2}{\varepsilon_{33}}\left[\begin{matrix} (\eta^2-\varepsilon_{22})\varepsilon_{11}\sin^2\vartheta \\ +2\varepsilon_{12}\varepsilon_{23}\sin\vartheta\cos\vartheta - \varepsilon_{12}^2\sin^2\vartheta \end{matrix}\right] \right\} \qquad \text{... (9)}$$

The large quantity $\varepsilon_{33} = 0(q^{-2})$ is put before the curly brackets for convenience. The two first addends between the curly brackets are of the order of one, all the other terms are of the order of q^2.

The dispersion equation that determines the frequencies and damping rates of plasma eigenwaves reduces to the condition

$$\Lambda(\omega,\vec{k}) = 0. \qquad \text{... (10)}$$

The eigenfrequencies are determined by the real part of the dispersion equation in which thermal corrections contained in the expressions for the dielectric permittivity components may be neglected, while the wave damping rate is determined by the imaginary part of the dispersion determinant (9) that is given rise to by the thermal effects. In as much as imaginary corrections in the expressions for the permittivity tensor components are small and since for $\omega \ll \omega_{B_i}$ ($z_1^2 \gg 1$) we have $\varepsilon_{23} \approx -\dfrac{i}{3}\dfrac{m}{M}q^2\dfrac{\omega_{B_i}}{\omega}\varepsilon_{33}\sin\vartheta\cos\vartheta$, the dispersion determinant eq. (9) may be written in the approximate form

$$\Lambda(\omega,\vec{k}) = \varepsilon_{33}\left\{ \begin{matrix} (\eta^2\cos^2\vartheta - \varepsilon_1) \\ \times(\eta^2 - \varepsilon_1 - \varepsilon_0\Psi) - \varepsilon_2^2 - i(R_q + R_1) \end{matrix} \right\} \qquad \text{... (11)}$$

where

$$\begin{aligned} \varepsilon_0\Psi &= \varepsilon_{22} - \varepsilon_{11} + \frac{\varepsilon_{23}^2}{\varepsilon_{33}} = \frac{1}{2}\frac{m}{M} \\ &\times \varepsilon_0\left(3 + \varphi(z) - i\sqrt{\pi}ze^{-z^2}\right)\frac{tg^2\vartheta}{z^2}, \end{aligned} \qquad \text{... (12)}$$

$$\begin{aligned} R_q &= \eta^2\left[(\eta^2-\varepsilon_1)\varepsilon_1 + \varepsilon_2^2\right]\sin^2\vartheta\,\mathrm{Im}\frac{1}{\varepsilon_{33}} \\ &= \frac{\sqrt{\pi}}{3}\frac{m}{M}q^2\sin^2\vartheta \end{aligned}$$

$$\times\cos^2\vartheta\frac{\eta^2(\eta^2-\varepsilon_0)\varepsilon_1}{\varepsilon_0([1-\varphi(z)]^2+\pi z^2e^{-2z^2})}ze^{-z^2}, \quad \dots (13)$$

$$R_1=\left\{\eta^2(1+\cos^2\vartheta)-2\varepsilon_0\frac{\omega_{B_i}}{\omega_{B_i}+\omega}\right\}\psi_1 \quad \dots (14)$$

In the range of frequencies lower than the ion cyclotron frequency, $\omega << \omega_{B_i}$ (when $z_1^2 >> 1$), we have

$$\varepsilon_1^2-\varepsilon_2^2=\varepsilon_0\varepsilon_1 \ (\varepsilon_0=\frac{\omega_{p_i}^2}{\omega_{B_i}^2},\ \varepsilon_1=\frac{\omega_{p_i}^2}{\omega_{B_i}^2-\omega^2}). \quad \dots (15)$$

This relation was employed in deriving the expression for R_q.

In the range of very low frequencies, $\omega << \omega_{B_i}$ ($z^2 << z_1^2$), we can disregard the quantity R_1 in the dispersion eq. (10), whereas for frequencies close to the ion cyclotron frequency, $\omega \lesssim \omega_{B_i}$ ($z^2 >> z_1^2$) we can disregard the imaginary part of Ψ and the quantity R_q.

3 Eigenwaves of a Plasma with Strong External Magnetic Field

First of all we consider the range of frequencies much lower than the ion cyclotron frequency, $\omega << \omega_{B_i}$. We note that in this range

$$\varepsilon_2^2=\frac{\omega^2}{\omega_{B_i}^2}\varepsilon_1^2=\frac{2}{3}\varepsilon_1^2z^2q^2\cos^2\vartheta,\ \varepsilon_1=\varepsilon_0. \quad \dots (16)$$

Therefore, having disregarded the addends ε_2^2 and $i(R_q+R_1)$ of the order of q^2 between the curly brackets of the eq. (11) and put eq. (11) equal to zero we obtain two dispersion equations, i.e.,

$$\eta^2\cos^2\vartheta-\varepsilon_1=0, \quad \dots (17)$$

$$\eta^2-\varepsilon_1-\varepsilon_0\Psi=0. \quad \dots (18)$$

After neglecting the quantity Ψ in eq. (18), we obtain an expression for the frequency of the so-called fast magnetosonic wave, i.e.,

$$\omega_S=kv_A \quad \dots (19)$$

where $v_A=\frac{B_0}{\sqrt{4\pi n_0M}}$ is the Alfven speed. In the general case, the quantity Ψ is complex. When eq. (18) that contains Ψ is solved by means of successive approximations, the real part of this quantity determines the thermal correction with an accuracy to the square of the magnetosonic wave frequency which should be disregarded in terms of the approximation accepted. Taking into account the imaginary part of Ψ ($\Psi \to i\Psi''$), i.e.,

$$\Psi''=\frac{\sqrt{\pi}}{2}\frac{m}{M}tg^2\vartheta\frac{e^{-z^2}}{z}, \quad \dots (20)$$

we find the damping rate of the fast magnetosonic wave to be given by

$$\gamma_S=\frac{1}{2}\omega_S\Psi''(z_S,\vartheta) \equiv\frac{1}{2}\sqrt{\frac{\pi}{6}\frac{m}{M}}ks\sin\vartheta tg\vartheta e^{-z_s^2},$$

$$z_s^2=\frac{3}{2}\frac{v_A^2}{s^2}\frac{1}{\cos^2\vartheta}. \quad \dots (21)$$

The electric field vector of the fast magnetosonic wave is perpendicular to the plane of vectors $\vec{k}$ and $\vec{B}_0$, hence for the polarization vector we have

$$\vec{e}^{(S)}=(0,1,0). \quad \dots (22)$$

Since the component ε_{11} is real in this approximation, eq. (17) yields only the Alfven wave frequency given by

$$\omega_A=kv_A\cos\vartheta \quad \dots (23)$$

The electric field vector of the Alfven wave is perpendicular to $\vec{B}_0$ and lies in the plane of vectors $\vec{k}$ and $\vec{B}_0$, hence

$$\vec{e}^{(A)}=(1,0,0). \quad \dots (24)$$

In order to find the damping rate of the Alfven wave, we have to consider the term of the dispersion equation that is associated with the addends which have been disregarded in eq. (9) in course of deriving (17). Thus, we rewrite the determinant eq. (9) as

$$\Lambda(\omega,\vec{k}) = \varepsilon_{33}\left(\eta^2\cos^2\vartheta - \varepsilon_{11} - \varepsilon_0 P\right)\left(\eta^2 - \varepsilon_{22} - \frac{\varepsilon_{23}^2}{\varepsilon_{33}}\right) \quad \dots (25)$$

where

$$\varepsilon_0 P = -\left\{\varepsilon_{12}^2 + \frac{\eta^2}{\varepsilon_{33}}\left[\left(\eta^2 - \varepsilon_{22}\right)\times\varepsilon_{11}\sin^2\vartheta + 2\varepsilon_{12}\varepsilon_{23}\sin\vartheta\times\cos\vartheta\right]\right\}\Big/\left(\eta^2 - \varepsilon_{22} - \frac{\varepsilon_{23}^2}{\varepsilon_{33}}\right)$$

Then the dispersion equation for the Alfven wave takes the form

$$\eta^2\cos^2\vartheta - \varepsilon_{11} - \varepsilon_0 P = 0 . \quad \dots (26)$$

We retain only the imaginary part of P $(P \to iP'')$ in eq. (26),

$$\varepsilon_0 P'' = \frac{\varepsilon_1}{3}\frac{q^2\cos^2\vartheta}{(\eta^2-\varepsilon_1)^2 + (\varepsilon_0\Psi'')^2}\times\left\{2\varepsilon_0\varepsilon_1 z_1^2\Psi'' + \sqrt{\pi}\frac{m}{M}\frac{\eta^2(\eta^2-\varepsilon_1)(\eta^2-\varepsilon_0)\sin^2\vartheta}{\varepsilon_0[(1-\varphi(z))^2 + \pi z^2 e^{-2z^2}]}ze^{-z^2}\right\}, \quad \dots (27)$$

and find the damping rate of the Alfven wave to be given by

$$\gamma_A = \frac{1}{2}\omega_A P''(z_A,\vartheta) = \frac{1}{2}\sqrt{\frac{\pi}{6}}\frac{m}{M}ks\frac{\omega_A^2}{\omega_{B_i}^2}\times\cos\vartheta\left\{ctg^2\vartheta + \frac{tg^2\vartheta}{[1-\varphi(z_A)]^2 + \pi z_A^2 e^{-2z_A^2}}\right\}e^{-z_A^2},$$

$$z_A^2 = \frac{3}{2}\frac{v_A^2}{s^2} . \quad \dots (28)$$

The damping rates of both Alfven and fast magnetosonic waves were derived for the first time (see Ref. 15). We note that the ratio of Alfven to magnetosonic wave damping rates is of the order of q^2.

Now let us consider the wave dispersion in the frequency range $\omega \lesssim \omega_{B_i}$. We rewrite the dispersion determinant as

$$\Lambda(\omega,\vec{k}) = \varepsilon_{33}\left\{\eta^4\cos^2\vartheta - \eta^2\left(1+\cos^2\vartheta\right)\varepsilon_1 + \varepsilon_1^2 - \varepsilon_2^2 - iR\right\}, \quad \dots (29)$$

where

$$R = \left(\eta^2\cos^2\vartheta - \varepsilon_1\right)\varepsilon_0\Psi'' + R_q + R_1 . \quad \dots (30)$$

If $z_1^2 >> 1$, then, within the context of (15), the dispersion equation reduces to

$$\eta^4\cos^2\vartheta - \eta^2(1+\cos^2\vartheta)\varepsilon_1 + \varepsilon_1 - iR = 0 . \quad \dots (31)$$

We neglect the imaginary part and thus obtain an equation whose solutions determine the frequency-dependences of the refraction indices of plasma eigenwaves, i.e.,

$$\eta_\mp^2 = \frac{1}{2\cos^2\vartheta}\times\left\{(1+\cos^2\vartheta)\varepsilon_1 \mp \sqrt{(1+\cos^2\vartheta)^2\varepsilon_1^2 - 4\cos^2\vartheta\varepsilon_1}\right\} \quad \dots (32)$$

The eigenwave with the refraction index η_-^2 is referred to as ordinary wave (the coefficient before η^2 in eq. (31) is negative), the wave with the refraction index η_+^2 is called extraordinary wave.

Bearing in mind the relation of the refraction index to the frequency, i.e., $\eta^2 = \frac{k^2c^2}{\omega^2}$, we can reduce eq. (31) to the dispersion equation which determines the eigenwave frequency as a function of the wave vector, i.e.,

$$\frac{\omega_{B_i}^2}{\omega^4(\omega_{B_i}^2 - \omega^2)}\times\left[\omega^4 - \omega^2 k^2 v_A^2\times\left(1+\cos^2\vartheta + \xi^2\cos^2\vartheta\right) + k^4 v_A^4\cos^2\vartheta\right] - iR = 0, \quad \dots (33)$$

where

$$\xi^2 = \frac{k^2c^2}{\omega_{P_i}^2} = \frac{k^2 v_A^2}{\omega_{B_i}^2} .$$

We note that wave damping rates γ and absorption indices κ satisfy the relation

$$\omega_{\pm}^2 = \frac{1}{2}k^2 v_A^2 \left\{ \begin{array}{l} 1+\cos^2\vartheta+\xi^2\cos^2\vartheta \\ \pm\sqrt{\left(1+\cos^2\vartheta+\xi^2\cos^2\vartheta\right)^2-4\cos^2\vartheta} \end{array} \right\} \quad \dots (34)$$

The solution ω_+^2 corresponds to the fast magnetosonic wave,

$$\omega_+^2 = \omega_S^2,$$

whereas the one ω_-^2 is associated with the Alfven wave,

$$\omega_-^2 = \omega_A^2.$$

Indeed, for small values of the parameter ξ^2 ($\xi^2 << 1$), we have

$$\omega_+^2 \equiv \omega_S^2 = k^2 v_A^2 \left(1+\xi^2 ctg^2\vartheta\right),$$

$$\omega_-^2 \equiv \omega_A^2 = k^2 v_A^2 \left(1-\xi^2 ctg^2\vartheta\right)\cos^2\vartheta \quad \dots (35)$$

Having taken into account the imaginary part of eq. (33), we obtain the equation for the damping rates of the magnetosonic and Alfven waves, i.e.,

$$\gamma_S = \frac{1}{2\varepsilon_0}\frac{\omega_S^3}{\omega_A^2-\omega_S^2}\frac{R(\omega_S)}{\varepsilon_1(\omega_S)},$$
$$\gamma_A = \frac{1}{2\varepsilon_0}\frac{\omega_A^3}{\omega_S^2-\omega_A^2}\frac{R(\omega_A)}{\varepsilon_1(\omega_A)}, \quad \dots (36)$$

For $\xi << 1$, i.e., in the frequency range $z^2 << z_1^2$ but $z_1^2 >> 1$, these equations reduce to eqs. (21) and (28). In the frequency range $z^2 >> z_1^2 >> 1$, the quantities Ψ'' and R_q in eq. (30) may be disregarded and the damping is determined by the quantity R_1.

We note that the solution η_-^2 of the dispersion eq. (31) determines the refraction index of the fast magnetosonic wave and the one η_+^2 describes the refraction index of the Alfven wave, i.e.,

$$\eta_-^2 \equiv \eta_S^2, \; \eta_+^2 \equiv \eta_A^2. \quad \dots (37)$$

The imaginary term in eq. (31) yields approximate expressions for the absorption indices of the magnetosonic and Alfven waves, i.e.,

We neglect the imaginary part of eq. (33) and thus find the squared eigenfrequencies to be given by

$$\frac{\kappa_S}{\eta_S} = \frac{1}{2\cos^2\vartheta}\frac{R(\eta_S)}{\eta_S^2(\eta_S^2-\eta_A^2)},$$
$$\frac{\kappa_A}{\eta_A} = \frac{1}{2\cos^2\vartheta}\frac{R(\eta_A)}{\eta_A^2(\eta_A^2-\eta_S^2)} \quad \dots (38)$$

As the eigenwave frequency ω approaches the ion-cyclotron frequency ω_{B_i}, the refraction index of the Alfven wave tends to infinity[16],

$$\eta_A^2 \xrightarrow[\omega\to\omega_{B_i}]{} \frac{1+\cos^2\vartheta}{\cos^2\vartheta}\varepsilon_1(\omega), \text{ but } z_1^2 >> 1, \quad \dots (39)$$

whereas the refraction index of the magnetosonic wave remains finite,

$$\eta_S^2 \xrightarrow[\omega\to\omega_{B_i}]{} \frac{1}{1+\cos^2\vartheta}\varepsilon_0. \quad \dots (40)$$

For $\omega \to \omega_{B_i}$, the absorption indices of the Alfven and magnetosonic waves are given by the expressions

$$\frac{\kappa_A}{\eta_A} = \frac{\varepsilon_0}{2\varepsilon_1(\omega_A)}\psi_1(\eta_A) = \frac{\sqrt{\pi}}{2}|z_1|e^{-z_1^2}, \quad \dots (41)$$

$$\frac{\kappa_S}{\eta_S} = \frac{1}{8}\frac{\varepsilon_0^2}{\varepsilon_1^2(\omega_S)}\psi_1(\eta_S) = \frac{1}{4}\sqrt{\frac{2\pi}{3}\frac{m}{M}}\frac{s}{v_A}\frac{\cos\vartheta}{\sqrt{1+\cos^2\vartheta}}z_1^2 e^{-z_1^2} \quad \dots (42)$$

where $z_1^2 = \frac{3}{2}\frac{M}{m}\frac{(\omega_{B_i}-\omega)^2}{k^2 s^2\cos^2\vartheta}$. In the vicinity of the ion cyclotron frequency ω_{B_i}, the wave-vector-dependences of the Alfven and magnetosonic wave eigenfrequencies, ω_A and ω_S, are given by

$$\omega_A^2 = \frac{\omega_{B_i}^2}{1+\frac{\omega_{B_i}^2}{k^2c^2}\frac{1+\cos^2\vartheta}{\cos^2\vartheta}}, \quad (k^2c^2 >> \omega_{B_i}^2) \quad \dots (43)$$

$$\omega_S^2 = k^2 v_A^2\left(1+\cos^2\vartheta\right), \quad \omega_S \to \omega_{B_i}. \quad \dots (44)$$

$$\frac{\gamma_A}{\omega_A}=\frac{\kappa_A}{\eta_A},\frac{\gamma_S}{\omega_S}=\frac{\kappa_S}{\eta_S}. \qquad \dots (45)$$

At last, if the eigenfrequency is equal to the ion cyclotron frequency, $\omega=\omega_{B_i}$ (i.e., when $z_1^2=0$), the expressions for the dielectric permittivity components in eq. (3) yield the dispersion equation

$$\eta^4\cos^2\vartheta+\frac{1}{4}\eta^2\left(1+\cos^2\vartheta\right)\varepsilon_0$$
$$-\frac{\varepsilon_0^2}{2}-i\left[\eta^2\left(1+\cos^2\vartheta\right)-\varepsilon_0\right]\psi_1=0$$

In the limiting case $\eta^2\to\infty$, this equation reduces to

$$\eta^3\cos^2\vartheta-\frac{i}{2}\sqrt{\frac{3\pi}{2}\frac{M}{m}}\varepsilon_0\frac{c}{s}\frac{1+\cos^2\vartheta}{\cos^3\vartheta}=0. \qquad \dots (46)$$

The solution of the latter equation is given by

$$\eta=\frac{\sqrt{3}+i}{2}\left\{\frac{1}{2}\sqrt{\frac{3\pi}{2}\frac{M}{m}}\varepsilon_0\frac{c}{s}\frac{1+\cos^2\vartheta}{\cos^3\vartheta}\right\}^{1/3}. \qquad \dots (47)$$

Thus, the Alfven wave is strongly damping ($\mathrm{Im}\,\eta\sim\mathrm{Re}\,\eta$) for the resonance frequency $\omega=\omega_{B_i}$.

4 General Description of Fluctuations in Equilibrium Plasmas

Both electric and magnetic field fluctuations in equilibrium plasmas are completely determined by the plasma dielectric permittivity tensor $\varepsilon_{ij}(\omega,\vec{k})$. According to the fluctuation-dissipation theorem, the spectral distribution of electric field fluctuations is given by[6]:

$$\left\langle E_iE_j\right\rangle_{\vec{k}\omega}=4\pi i\frac{T}{\omega}\left\{\Lambda_{ji}^{-1}-(\Lambda_{ij}^{-1})^*\right\}, \qquad \dots (48)$$

where T is the plasma temperature and Λ_{ij}^{-1} is the inverse of the dispersion tensor eq. (6). We introduce the algebraic complement λ_{ij} of the tensor eq. (6)

$$\lambda_{ij}\Lambda_{jk}=\Lambda\delta_{jk},\Lambda\equiv\left\|\Lambda_{ij}\right\|,$$

then the inverse tensor may be represented in the form

$$\Lambda_{ij}^{-1}=\frac{\lambda_{ij}}{\Lambda}. \qquad \dots (49)$$

In the coordinate system in which the z-axis is directed along the magnetic field $\vec{B}_0$ and the x-axis lies in the plane of vectors $\vec{k}$ and $\vec{B}_0$, the components of the algebraic complement λ_{ij} are described by the equations

$$\lambda_{11}=\left(\eta^2-\varepsilon_{22}\right)\left(\eta^2\sin^2\vartheta-\varepsilon_{33}\right)+\varepsilon_{23}^2,$$

$$\lambda_{22}=\left(\eta^2\cos^2\vartheta-\varepsilon_{11}\right)\left(\eta^2\sin^2\vartheta-\varepsilon_{33}\right)-\left(\eta^2\sin\vartheta\cos\vartheta+\varepsilon_{13}\right)^2,$$

$$\lambda_{33}=\left(\eta^2\cos^2\vartheta-\varepsilon_{11}\right)\left(\eta^2-\varepsilon_{22}\right)+\varepsilon_{12}^2, \qquad \dots (50)$$

$$\lambda_{12}=-\lambda_{21}=\left(\eta^2\sin^2\vartheta-\varepsilon_{33}\right)\varepsilon_{12}-\left(\eta^2\sin\vartheta\cos\vartheta+\varepsilon_{13}\right)\varepsilon_{23},$$

$$\lambda_{13}=\lambda_{31}=\left(\eta^2-\varepsilon_{22}\right)\left(\eta^2\sin\vartheta\cos\vartheta+\varepsilon_{13}\right)+\varepsilon_{12}\varepsilon_{23},$$

$$\lambda_{23}=-\lambda_{32}=\left(\eta^2\cos^2\vartheta-\varepsilon_{11}\right)\varepsilon_{23}-\left(\eta^2\sin\vartheta\cos\vartheta+\varepsilon_{13}\right)\varepsilon_{12}.$$

The spectral distribution of magnetic field fluctuations is related to the electric field fluctuation distribution as given by

$$\left\langle B_iB_j\right\rangle_{\vec{k}\omega}=\eta^2\in_{ikm}\in_{jln}\frac{k_kk_l}{k^2}\left\langle E_mE_n\right\rangle_{\vec{k}\omega}, \qquad \dots (51)$$

where $\in_{ikm}$ is a completely antisymmetric third-rank tensor. We make use of the relation

$$\in_{ikm}\in_{jln}=\begin{vmatrix}\delta_{ij} & \delta_{il} & \delta_{in}\\ \delta_{kj} & \delta_{kl} & \delta_{kn}\\ \delta_{mj} & \delta_{ml} & \delta_{mn}\end{vmatrix},$$

and thus obtain a relation between the spectral distributions of magnetic and electric field fluctuations in the plasma, i.e.,

$$\langle B_i B_j \rangle_{\vec{k}\omega} = \eta^2 \left\{ \begin{aligned} &\left(\delta_{ij} - \frac{k_i k_j}{k^2}\right)\left(\langle \vec{E}^2 \rangle_{\vec{k}\omega} - \frac{1}{k^2}\left\langle (\vec{k}\vec{E})^2 \right\rangle_{\vec{k}\omega}\right) \\ &-\left(\delta_{im} - \frac{k_i k_m}{k^2}\right)\left(\delta_{jn} - \frac{k_j k_n}{k^2}\right)\langle E_n E_m \rangle_{\vec{k}\omega} \end{aligned} \right\}. \quad \ldots (52)$$

5 Spectral Distributions of Electric Field Fluctuations

Making use of the eq. (3) for components of the dielectric permittivity tensor in the case $q^2 << 1$ we give the algebraic component tensor.eq. (50) in the form

$$\lambda_{ij} = \lambda_{ij}^{long} - \varepsilon_{33} \begin{pmatrix} \eta^2 - \varepsilon_{22} - \frac{\varepsilon_{23}^2}{\varepsilon_{33}} & 0 & 0 \\ 0 & \eta^2 \cos^2\vartheta - \varepsilon_{11} & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \ldots (53)$$

where $\lambda_{ij}^{long} = \eta^2(\eta^2 - \varepsilon_{22})\frac{k_i k_j}{k^2}$ is longitudinal part of tensor λ_{ij} which we can disregard in the range of frequencies $\omega > \sqrt{\frac{m}{3M}} qkv_A \cos\vartheta$. Thus, only the components λ_{11} and λ_{22} are of the same order of magnitude in the limiting case $q^2 \to 0$, i.e.,

$$\lambda_{11} \to \varepsilon_{33}\varepsilon_0(\omega^2 - k^2 v_A^2 + i\omega^2\Psi'')/\omega^2,$$

$$\lambda_{22} \to \varepsilon_{33}\varepsilon_0(\omega^2 - k^2 v_A^2 \cos^2\vartheta)/\omega^2$$

(all the other components λ_{33}, λ_{12}, λ_{13} and λ_{23} are of higher orders of smallness).

In the range of frequencies ω much lower than the ion cyclotron frequency ω_{B_i}, the dispersion determinant reduces in the limiting case $q^2 \to 0$ to the form

$$\Lambda(\omega,\vec{k}) \to \varepsilon_{33}\varepsilon_0^2\left(\omega^2 - k^2 v_A^2 \cos^2\vartheta + i\omega^2 P''\right) \times \left(\omega^2 - k^2 v_A^2 + i\omega^2\Psi''\right)/\omega^4. \quad \ldots (54)$$

We remind the reader that $|\Psi''| << 1$ and the quantity $P'' \approx 0(q^2)$ must be taken into account only under the condition $\omega^2 - k^2 v_A^2 \cos^2\vartheta \to 0$.

We make use of eqs. (53) and (54) and, according to the definition eq. (48), obtain a formula for the spectral distribution of electric field fluctuations in a plasma with strong external magnetic field in the range of frequencies much lower than the ion cyclotron frequency ($\omega << \omega_{B_i}$). It is given by

$$\langle E_i E_j \rangle_{\vec{k}\omega} = 8\pi T \frac{v_A^2}{c^2}\omega \left\{ e_i^{(A)} e_j^{(A)} \frac{\omega^2 P''}{\left(\omega^2 - k^2 v_A^2 \cos^2\vartheta\right)^2 + \left(\omega^2 P''\right)^2} + e_i^{(S)} e_j^{(S)} \frac{\omega^2 \Psi''}{\left(\omega^2 - k^2 v_A^2\right)^2 + \left(\omega^2 \Psi''\right)^2} \right\}, \quad \ldots (55)$$

where $\vec{e}^{(A)}$ and $\vec{e}^{(S)}$ are the polarization vectors of the Alfven and magnetosonic waves eqs. (24) and (22). We see that the spectral distribution of fluctuations has two maxima associated with Alfven and magnetosonic fluctuation oscillations. These maxima are manifested in different components of the fluctuation spectral distribution tensor because of differing polarizations of the fluctuation electric field vectors. The maximum associated with the Alfven fluctuation oscillations occurs in the spectral distribution of the component $\langle E_1^2 \rangle_{\vec{k}\omega}$ whereas the one corresponding to the magnetosonic fluctuation oscillations is manifested, respectively, in the spectral distribution of the component $\langle E_2^2 \rangle_{\vec{k}\omega}$. These distributions were derived in [7] for the limiting case $\gamma_A << \omega_A$ and $\gamma_S << \omega_S$.

The components $\langle E_1^2 \rangle_{\vec{k}\omega}$ and $\langle E_2^2 \rangle_{\vec{k}\omega}$ of the tensor spectral distribution of electric field fluctuations are shown in Fig. 1 (a,b) as functions of the dimensionless frequency $\tilde{z} \equiv \frac{\omega}{ks}$ and the angle ϑ. Numerical calculations were performed[17] for the values of the parameters $\Theta^2 \equiv \frac{3}{2}\frac{v_A^2}{s^2}$ and q^2 which correspond to the actual conditions of real plasma with the concentration $n_0 = 1.2\cdot 10^{14}\,cm^{-3}$, temperature $T = 10^4$ eV, and external magnetic field $B_0 = 3.4\cdot 10^4$ Gs. These values are $\Theta^2 = 0.01$ and $q^2 = 0.1$ (the spectral distributions are normalized to the quantity $W = 8\pi \dfrac{T}{\omega_{B_i}}$).

The spectral distribution $\left\langle E_1^2 \right\rangle_{\vec{k}\omega}$, (Fig. 1a) shows a broad maximum produced by the low-frequency incoherent fluctuations along with the narrow

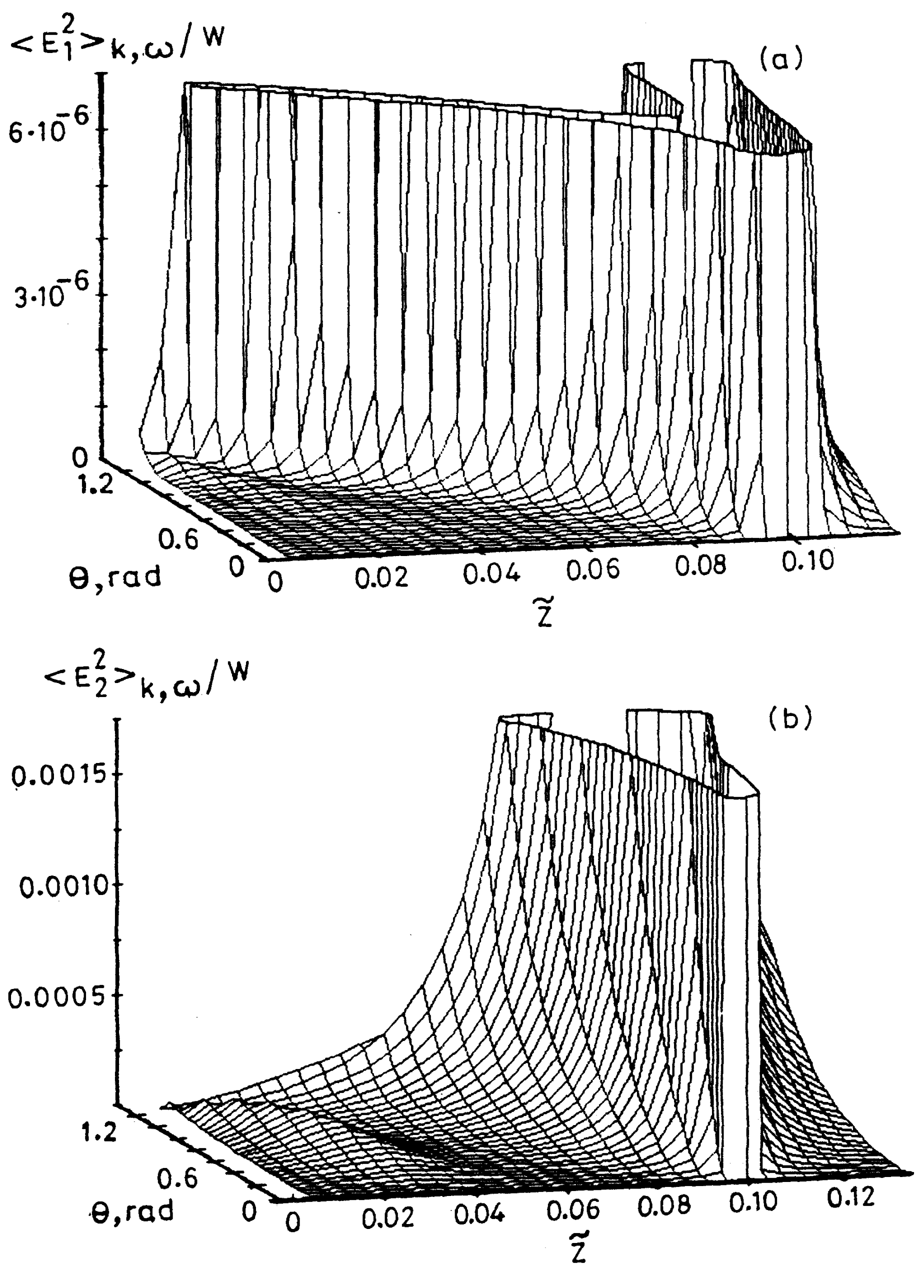


Fig. 1 The spectral distributions of the electric field fluctuations $\left\langle E_1^2 \right\rangle_{\vec{k}\omega}$ (a) and $\left\langle E_2^2 \right\rangle_{\vec{k}\omega}$ (b) in the frequency range $\omega << \omega_{B_i}$ as functions of the dimensionless frequency $\tilde{z} = \sqrt{\dfrac{3}{2}}\dfrac{\omega}{ks}$ and the angle ϑ ($w = 8\pi \dfrac{T}{\omega_{B_i}}$).

maximum at the frequency $\tilde{z} = \Theta \cos\vartheta$ ($\omega = kv_A \cos\vartheta$) that is given rise to by the Alfven fluctuation oscillations. In $\langle E_1^2 \rangle_{\vec{k}\omega}$, a maximum at the frequency $\tilde{z} = \Theta$ ($\omega = kv_A$) is also observed; it is given rise to by the magnetosonic fluctuation oscillations. The value of this maximum is by several orders of magnitude lower than the Alfven fluctuation maximum in the spectrum of $\langle E_1^2 \rangle_{\vec{k}\omega}$. The value of the Alfven fluctuation maximum in the spectral distribution $\langle E_1^2 \rangle_{\vec{k}\omega}$, is given in Fig. 2(a) as function of angle ϑ. The minimal value of this maximum lies in

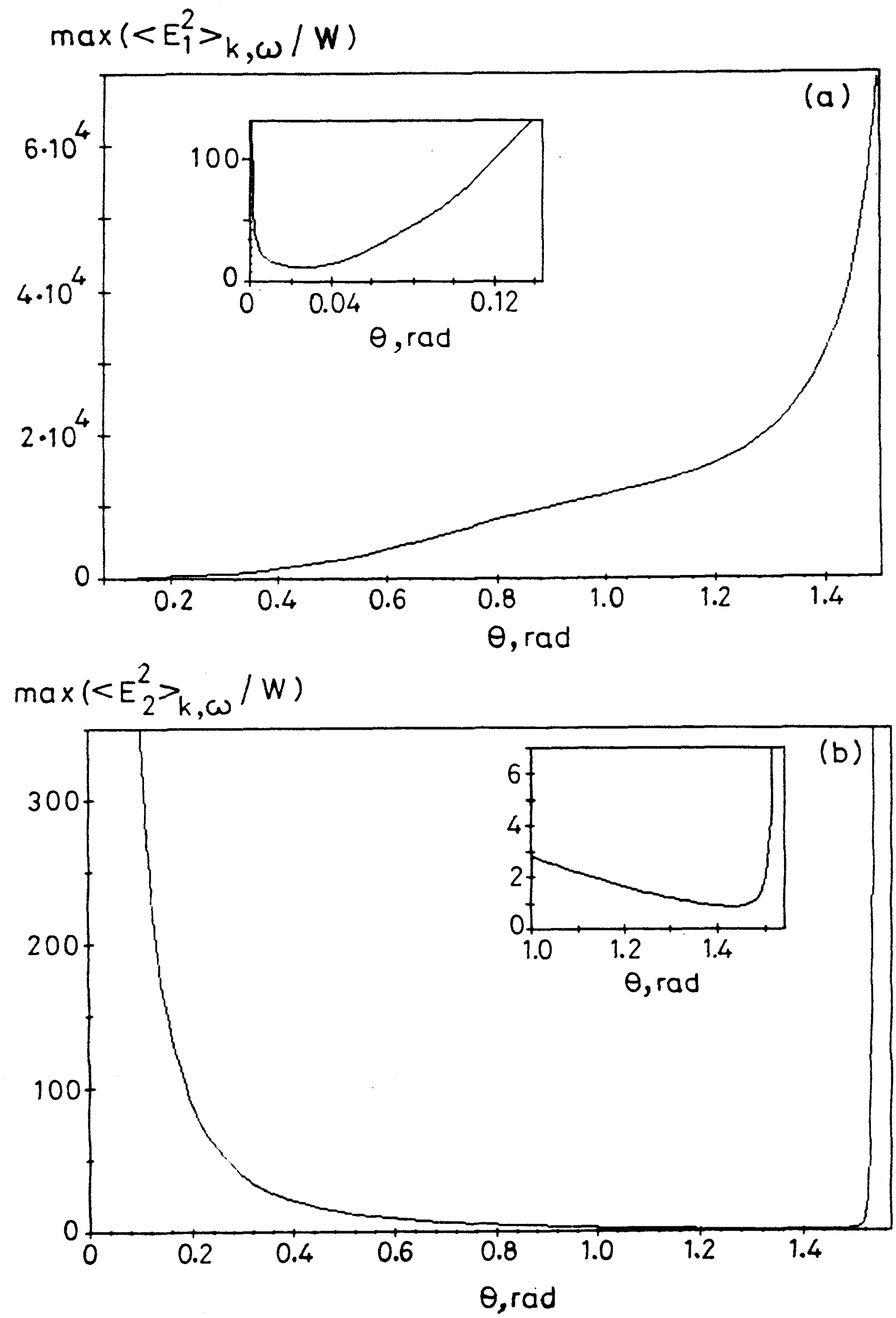


Fig. 2 The values of the maxima in the spectral distributions of the electric field fluctuations which are associated with Alfven fluctuation oscillations (a) and with magnetosonic fluctuation oscillations (b) as function of the angle ϑ.

range of angle $\vartheta \approx 0.04$ rad; the width of the maximum attains its greatest value in this range. The spectral distribution $\langle E_2^2 \rangle_{\vec{k}\omega}$ has two maxima (Fig.1 b). The broad maximum in the low-frequency range is associated with incoherent fluctuations. The narrow maximum at the frequency $\tilde{z} = \Theta^{\cdot}$ $(\omega = kv_A)$ corresponds to the magnetosonic fluctuation oscillations. This maximum is by several orders of magnitude higher that the broad low-frequency maximum. In the spectral distribution $\langle E_2^2 \rangle_{\vec{k}\omega}$, the minimal value of the maximum associated with magnetosonic oscillations and the maximal value of its width are attained for $\vartheta \approx 1{,}45$ rad, Fig.2 (b).

Fig.1 gives the spectral distributions of $\langle E_1^2 \rangle_{\vec{k}\omega}$ and $\langle E_2^2 \rangle_{\vec{k}\omega}$ for angles ϑ from 0 to $\pi/2$. Spectral distributions in the angle ranges from 0 to $\pi/2$ and from $\pi/2$ to π are related as given by

$$\langle E_i^2 \rangle_{\vec{k}\omega}\Big|_{\pi-\vartheta} = \langle E_i^2 \rangle_{\vec{k}\omega}\Big|_{\vartheta}, \; 0 \le \vartheta \le \frac{\pi}{2}.$$

The magnetosonic perturbation is transverse (the polarization vector of the fast magnetosonic wave is perpendicular to the wave vector, $\vec{e}^{(S)}\vec{k} = 0$; the electric field of Alfven perturbations has a longitudinal component as well. To find the spectral distribution of the longitudinal electric field, we multiply the total distribution eq. (55) by $\frac{k_i k_j}{k^2}$ and thus separate out the longitudinal component

$$\langle E_l^2 \rangle_{\vec{k}\omega}^{(A)} = 8\pi T \frac{v_A^2}{c^2} \omega \sin^2\vartheta \frac{\omega^2 P''}{(\omega - k^2 v_A^2 \cos^2\vartheta)^2 + (\omega^2 P'')^2} \quad \dots (56)$$

The spectral distribution of charge density fluctuations associated with Alfven perturbations in the plasma is determined by the relation

$$\langle \rho^2 \rangle_{\vec{k}\omega}^{(A)} = \frac{k^2}{16\pi^2} \langle E_l^2 \rangle_{\vec{k}\omega}^{(A)} \quad \dots (57)$$

The spectral distribution of the charge density fluctuations eq. (57) has a maximum for frequencies corresponding to Alfven fluctuation waves and a broad maximum in the range of low frequencies associated with incoherent plasma fluctuations.

For very low frequencies, one has to take into account the ion motion which determines the frequency-dependence of the quantities Ψ'' and P'' [18]:

$$\Psi'' \to \sqrt{\pi \frac{m}{M}} tg^2\vartheta \frac{e^{-\frac{M}{m}z^2}}{z}, z^2 \ll \frac{m}{M}; \quad \dots (58)$$

$$P'' \to \sqrt{\pi \frac{m}{M}} tg^2\vartheta \frac{e^{-\frac{M}{m}z^2}}{z}, z^2 \ll \frac{m}{M}. \quad \dots (59)$$

In the low-frequency range $z^2 \ll \frac{m}{M}$, the spectral distribution eq. (57) takes the form

$$\langle \rho^2 \rangle_{\vec{k}\omega}^{(A)} = \frac{1}{6}\sqrt{\frac{\pi}{2}} \frac{e^2 n_0}{\omega_{p_i}} \frac{\omega^2}{\omega_{p_e}^2} \frac{s^2}{v_A^2} ka \sin\vartheta t g^3\vartheta e^{-\frac{M}{m}z^2},$$

$$a^2 = \frac{T}{4\pi n_0 e^2} \quad \dots (60)$$

We note that intensities of both charge density and longitudinal electric field fluctuations $\langle \rho^2 \rangle_{\vec{k}\omega}^{(A)}$ and $\langle E_l^2 \rangle_{\vec{k}\omega}^{(A)}$ tend to zero in the low-frequency range.

In the low-frequency domain dominant contribution to the incoherent charge density fluctuations is associated with the longitudinal field λ_{ij}^{long}. The spectral distribution of such incoherent fluctuations is described by

$$\langle \rho^2 \rangle_{\vec{k}\omega} = \frac{1}{2}\sqrt{\frac{\pi}{2}} \frac{e^2 n_0}{\omega_{p_i}} k^3 a^3 \frac{e^{-\frac{M}{m}z^2}}{\cos\vartheta}, \quad z^2 \ll \frac{m}{M} \quad \dots (61)$$

The spectrum of electron density incoherent fluctuations in the low-frequency domain is as follows

$$\langle \delta n_e^2 \rangle_{\vec{k}\omega} = \frac{1}{2}\sqrt{\frac{3\pi}{2}}\sqrt{\frac{M}{m}} \frac{n_0}{ks\cos\vartheta} e^{-\frac{M}{m}z^2}, \; z^2 \ll \frac{m}{M} \quad \dots (62)$$

The low-frequency longitudinal fluctuations in the case of an anisotropic plasma with different temperatures $T_\perp$ and $T_\parallel$ with respect to the strong external magnetic field $\vec{B}_0$ were considered in detail in Refs. [19, 20].

We pass to the consideration of short-wavelength electric field fluctuations $kv_A \lesssim \omega_{B_i}$, i.e., fluctuation excitations in the frequency range near the ion cyclotron frequency $\omega \lesssim \omega_{B_i}$. We take the dispersion determinant in the form (29) which can be rewritten as

$$\Lambda(\omega,\vec{k}) = \varepsilon_{33}\frac{\varepsilon_0\varepsilon_1(\omega)}{\omega_4}\left(\omega^2-\omega_A^2+2i\omega_A\gamma_A\right) \times\left(\omega^2-\omega_S^2+2i\omega_S\gamma_S\right), \quad \dots (63)$$

where ω_A^2 and ω_S^2 are the squares of the Alfven and magnetosonic wave frequencies given by eq. (34), and γ_A and γ_S are the relevant damping rates eq. (36).

We employ eqs. (53), (63), and (48) of the spectral distribution of electric field fluctuations to obtain the general expressions which hold for a wide frequency range $\omega \lesssim \omega_{B_i}$. These expressions are given by

$$\langle E_1^2\rangle_{\vec{k}\omega} = 8\pi T\frac{\omega}{\varepsilon_0}\times \frac{\left\{ (\omega^2-\omega_S^2)\left(\omega^2-\frac{k^2c^2}{\varepsilon_1(\omega)}\right)2\omega_A\gamma_A+(\omega^2-\omega_A^2) \times\left[\left(\omega^2-\frac{k^2c^2}{\varepsilon_1(\omega)}\right)2\omega_S\gamma_S-(\omega^2-\omega_S^2)\frac{\omega^2}{\varepsilon_1(\omega)}(\varepsilon_0\Psi''+\psi_1)\right]\right\}}{\left[(\omega^2-\omega_A^2)^2+4\omega_A^2\gamma_A^2\right]\left[(\omega^2-\omega_S^2)^2+4\omega_S^2\gamma_S^2\right]}, \quad \dots (64)$$

$$\langle E_2^2\rangle_{\vec{k}\omega} = 8\pi T\frac{\omega}{\omega_0}\times \frac{\left\{ (\omega^2-\omega_A^2)\left(\omega^2-\frac{k^2c^2}{\varepsilon_1(\omega)}\cos^2\vartheta\right)2\omega_S\gamma_S+(\omega^2-\omega_S^2) \times\left[\left(\omega^2-\frac{k^2c^2}{\varepsilon_1(\omega)}\right)\cos^2\vartheta\right]2\omega_A\gamma_A -(\omega^2-\omega_A^2)\frac{\omega^2}{\varepsilon_1(\omega)}(\varepsilon_0P''+\psi_1)\right\}}{\left[(\omega^2-\omega_A^2)^2+4\omega_A^2\gamma_A^2\right]\left[(\omega^2-\omega_S^2)^2+4\omega_S^2\gamma_S^2\right]}. \quad \dots (65)$$

As distinct from the long-wavelength fluctuations, the short-wave Alfven and magnetosonic fluctuation excitations are not separated in individual components of the spectral distribution. The distributions of both components eqs. (64) and (65) have maxima associated with both Alfven and magnetosonic fluctuation excitations. We remind the reader that the expression for the dispersion determinant (63) and hence the spectral distributions eqs. (64) and (65) are valid provided the condition $z_1^2 >> 1$ is satisfied.

If $z^2 << z_1$ (i.e., $\omega << \omega_{B_i}$), then the spectral distributions eqs. (64) and (65) reduce to

$$\langle E_1^2\rangle_{\vec{k}\omega} = 8\pi T\frac{v_A^2}{c^2}\omega_A\frac{2\omega_A\gamma_A}{(\omega^2-\omega_A^2)^2+4\omega_A^2\gamma_A^2}, \quad \dots (66)$$

$$\langle E_2^2\rangle = 8\pi T\frac{v_A^2}{c^2}\omega_s\frac{2\omega_S\gamma_S}{(\omega^2-\omega_S^2)^2+4\omega_S^2\gamma_S^2}. \quad \dots (67)$$

Near the maxima corresponding to the Alfven and magnetosonic fluctuation excitations, these distributions do not differ from eq. (55); however, they do not hold for very low frequencies, $\omega << kv_A$, as distinct from eq. (55).

The components $\langle E_1^2\rangle_{\vec{k}\omega}$ and $\langle E_2^2\rangle_{\vec{k}\omega}$ of the spectral distribution of electric field fluctuations in frequency range $\omega \lesssim \omega_{B_i}$ are shown in Figs. 3 and Fig. 4 as functions of the dimensionless frequency $\tilde{\omega} \equiv \frac{\omega}{\omega_{B_i}}$ [Ref. 17]. Numerical calculations were performed for the values of the parameters which correspond to the actual conditions of real plasma with the concentration $n_0 = 1.2\cdot10^{13} cm^{-3}$, temperature $T = 10^2$ eV, and external magnetic field $B_0 = 3.4\cdot10^4$ Gs. Spectral distributions are shown for different values of the parameter $\xi \equiv \frac{kc}{\omega_{p_i}}$ (parameter ξ is proportional to the wave vector magnitude k). These values are $\xi = 0.2; 0.4; 0.7516; 1.0$, and angle $\vartheta = 0.5$ rad.

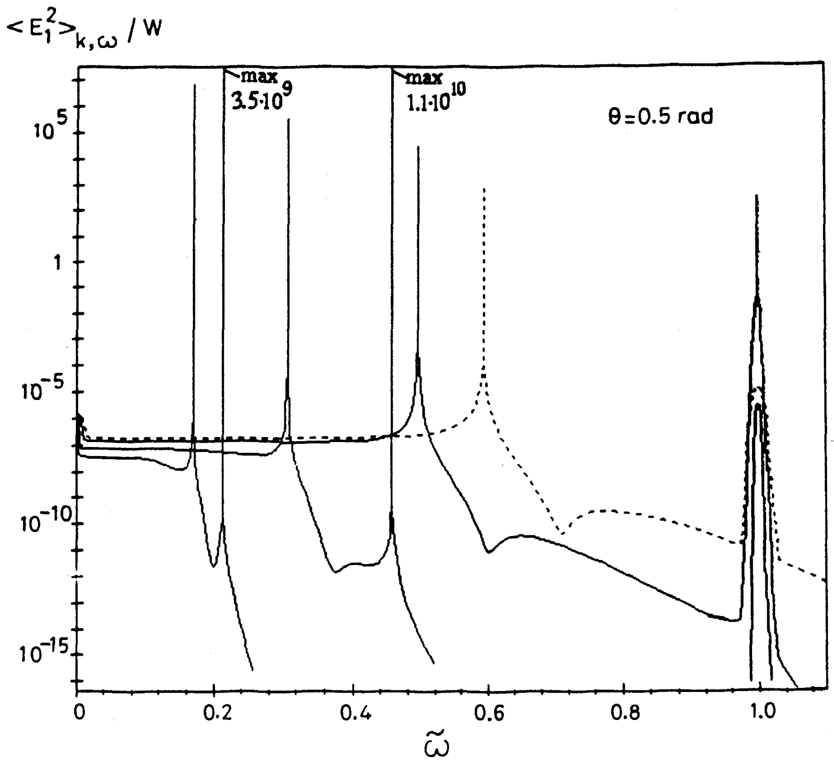


Fig. 3 The spectral distributions of the electric field fluctuations $\left\langle E_1^2 \right\rangle_{\vec{k}\omega}$ in the frequency range $\omega \lesssim \omega_{B_i}$ as functions of the dimensionless frequency $\tilde{\omega} = \dfrac{\omega}{\omega_{B_i}}$.

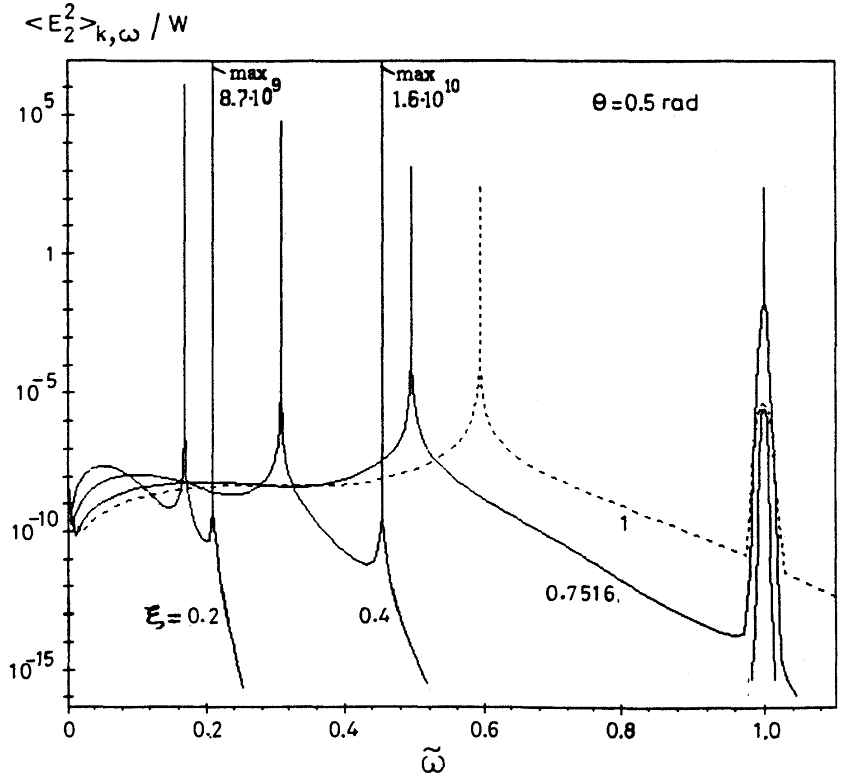


Fig. 4 The spectral distributions of the electric field fluctuations $\left\langle E_2^2 \right\rangle_{\vec{k}\omega}$ in the frequency range $\omega \lesssim \omega_{B_i}$ as functions of the dimensionless frequency $\tilde{\omega} = \dfrac{\omega}{\omega_{B_i}}$

Each spectral distribution $\langle E_1^2\rangle_{\bar{k}\omega}$ and $\langle E_2^2\rangle_{\bar{k}\omega}$ has four maxima. Two maxima in the low-frequency range $\tilde{\omega} < 0.01$ and in the range of ion cyclotron frequency $\tilde{\omega} \approx 1$ ($\omega \approx \omega_{B_i}$) are associated with incoherent fluctuations. Two narrow maxima correspond to the Alfven (at the frequency ω_A) and magnetosonic (at the frequency ω_S) collective fluctuation oscillations ($\omega_A < \omega_S$). These maxima are by several orders of magnitude higher than the maxima associated with incoherent fluctuations. Position and magnitude of the maxima which correspond to the Alfven and magnetosonic fluctuation oscillations depend on parameter ξ. When ξ increases the ω_A and ω_S also increase (to narrow maxima are shifted to the range of the higher frequency), however $\omega_A < \omega_{B_i} (\tilde{\omega}_A \equiv \frac{\omega_A}{\omega_{B_i}} < 1)$. When $\xi = 0.7516$ the maximum associated with magnetosonic collective fluctuations is located at the frequency $\tilde{\omega}_S = 1$ (incoherent maximum is also situated at this frequency). When $\xi = 1$ the maximum associated with magnetosonic collective fluctuations lies outside the shown frequency range. For the values of parameter $\xi = 0.2;0.4$ the maximum associated with magnetosonic fluctuation oscillations is by several orders of magnitude higher than maximum associated with Alfven fluctuation oscillations. The magnitude of the maximum in the range of ion cyclotron frequency $\tilde{\omega} \approx 1$ ($\omega \approx \omega_{B_i}$) which associated with incoherent fluctuations is increasing when parameter ξ is increasing (the case when parameter $\xi = 0.7516$ is an exception: two maxima have the same position). The difference between the spectral distributions $\langle E_1^2\rangle_{\bar{k}\omega}$ and $\langle E_2^2\rangle_{\bar{k}\omega}$ is significant for the small values of the parameter ξ ($\xi << 1$) when maxima associated with Alfven and magnetosonic fluctuations are located in low-frequency region ($\tilde{\omega} << 1$).

If the condition $z^2 >> z_1 >> 1$ is satisfied, i.e., in the frequency range near the ion cyclotron frequency ω_{B_i}, the damping rates γ_A and γ_S are determined only by the component R_1 contained in eq. (30). The spectral distributions eqs. (64) and (65) are applicable as well in the limiting cases when Alfven or magnetosonic perturbation frequency approaches the ion cyclotron frequency, $(\omega_{B_i} - \omega) << \omega_{B_i}$, provided the condition

$$1 >> \left(\frac{\omega_{B_i} - \omega}{\omega_{B_i}}\right)^2 >> \frac{2}{3}\frac{m}{M} q^2 \cos^2 \vartheta$$

is satisfied. In the ion cyclotron resonance range, $\left(\frac{\omega_{B_i} - \omega}{\omega_{B_i}}\right)^2 << \frac{2}{3}\frac{m}{M} q^2 \cos^2 \vartheta$, both electric field fluctuation spectra $\langle E_1^2\rangle_{\bar{k}\omega}$ and $\langle E_2^2\rangle_{\bar{k}\omega}$ have a maximum associated with incoherent cyclotron fluctuations. It is given by

$$\langle E_1^2\rangle_{\bar{k}\omega} = 2\pi\sqrt{6\pi \frac{M}{m}\frac{T}{\omega_{p_i}}\frac{\omega_{B_i}}{\omega_{p_i}}\frac{c}{s}}\,\phi(\xi) e^{-z_1^2},$$

$$\langle E_2^2\rangle_{\bar{k}\omega} = \cos^2 \vartheta \langle E_1^2\rangle_{\bar{k}\omega} \qquad \ldots (68)$$

where

$$z_1^2 = \frac{3}{2}\frac{M}{m}\frac{(\omega_{B_i} - \omega)^2}{k^2 s^2 \cos^2 \vartheta}$$

and $\phi(\xi) = \frac{1}{\xi^5 \cos^5 \vartheta}$ for $\xi^2 > 1$.

6 Spectral Distribution of Magnetic Field Fluctuations

Making use of the relation eq. (52) between the distributions of electric and magnetic field fluctuations and the spectral distribution of electric field fluctuations eq. (55), we find the spectral distribution of long-wavelength fluctuations ($kv_A << \omega_{B_i}$) of magnetic field in a plasma with strong external magnetic field to be given by

$$\langle B_i B_j\rangle_{\bar{k}\omega} = 8\pi \frac{T}{\omega} k^2 v_A^2 \{b_i^{(A)} b_j^{(A)}$$
$$\times \frac{\omega^2 P'' \cos^2 \vartheta}{\omega^2 - k^2 v_A^2 \cos^2 \vartheta) + (\omega^2 P'')^2}$$

$$+b_i^{(S)}b_j^{(S)}\frac{\omega^2\psi''}{\omega^2-k^2v_A^2)+(\omega^2\psi'')^2}\}\omega << \omega_{B_i} \quad \dots (69)$$

When deriving eq. (69), we introduced the magnetic field polarization vectors for the Alfven and magnetosonic waves, i.e.,

$$\vec{b}^{(A)}=(0,1,0), \quad \vec{b}^{(S)}=(-\cos\vartheta,0,\sin\vartheta) \quad \dots (70)$$

Similarly to the spectral distribution of electric field fluctuations, the spectral distribution of magnetic field fluctuations (69) has two maxima given rise to by Alfven and magnetosonic fluctuation oscillations and a low-frequency maximum associated with incoherent fluctuations. Like in the case of electric field fluctuations, differing polarizations of Alfven and magnetosonic waves are responsible for the fact that relevant magnetic field fluctuation maxima are manifested in different components of the spectral distribution tensor. The maximum associated with the Alfven excitations appears in the spectral distribution of the $\langle B_2^2\rangle_{\vec{k}\omega}$ component whereas the maximum corresponding to the magnetosonic fluctuations is manifested in the spectral distributions of $\langle B_1^2\rangle_{\vec{k}\omega}$, $\langle B_3^2\rangle_{\vec{k}\omega}$, $\langle B_1B_3\rangle_{\vec{k}\omega}$ and $\langle B_3B_1\rangle_{\vec{k}\omega}$.

As distinct from the case of electric field fluctuations, the maximum of incoherent fluctuations of the magnetic field corresponds to the zero frequency. Bearing in mind that ion motion should be allowed for in the case of very low frequencies $\left(z^2 << \frac{m}{M}\right)$, we obtain the expressions for the spectral distributions of incoherent Alfven and magnetosonic fluctuations, i.e.,

$$\langle B_2^2\rangle_{\vec{k}\omega}^{(A)}=2\pi\sqrt{\frac{\pi}{6}\frac{m}{M}}\frac{T}{ks}q^2\sin\vartheta tg\vartheta e^{-\frac{M}{m}z^2},$$

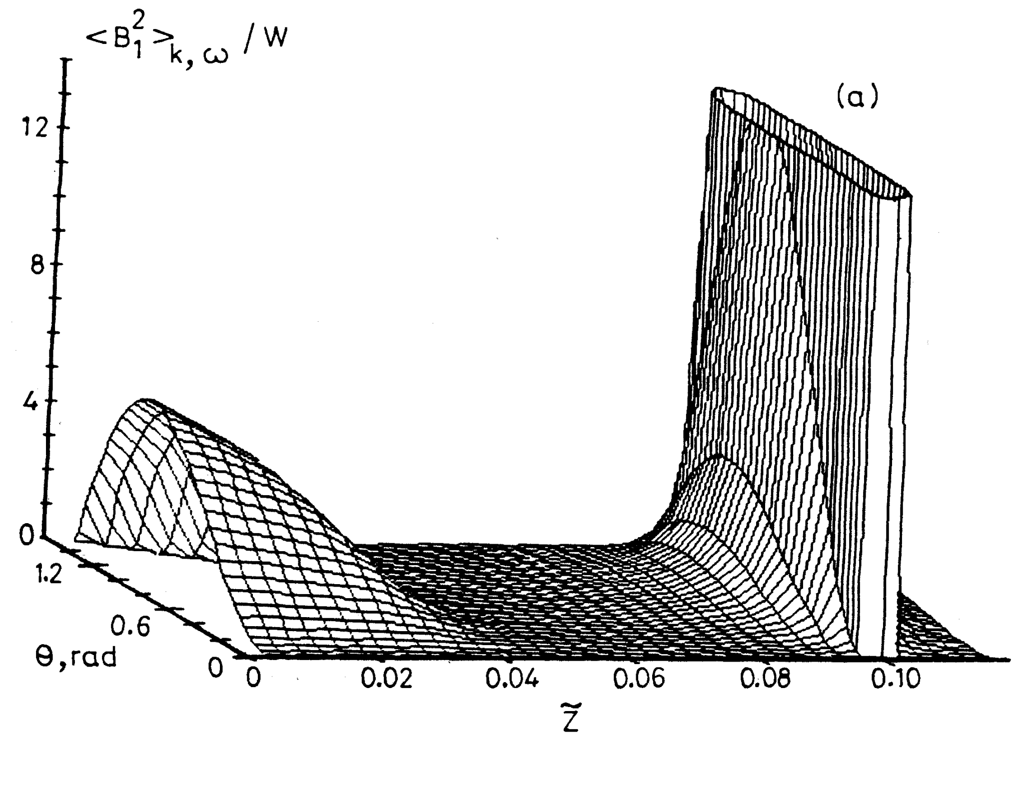
$$\omega << kv_A\cos\vartheta \quad \dots (71)$$

$$\langle B_iB_j\rangle_{\vec{k}\omega}^{(S)}=b_i^{(S)}b_j^{(S)}8\pi$$

$$\times\sqrt{\frac{2\pi}{3}\frac{m}{M}}\frac{T}{kv_A}\sin\vartheta tg\vartheta e^{-\frac{M}{m}z^2}, \omega << kv_A \quad (72)$$

The widths of the distributions eqs. (71) and (72) are determined by the ion thermal velocity.

The components $\langle B_1^2\rangle_{\vec{k}\omega}$, $\langle B_2^2\rangle_{\vec{k}\omega}$ and $\langle B_3^2\rangle_{\vec{k}\omega}$ of the spectral distribution of magnetic field fluctuations are shown in Fig. 5 (a-c) as functions



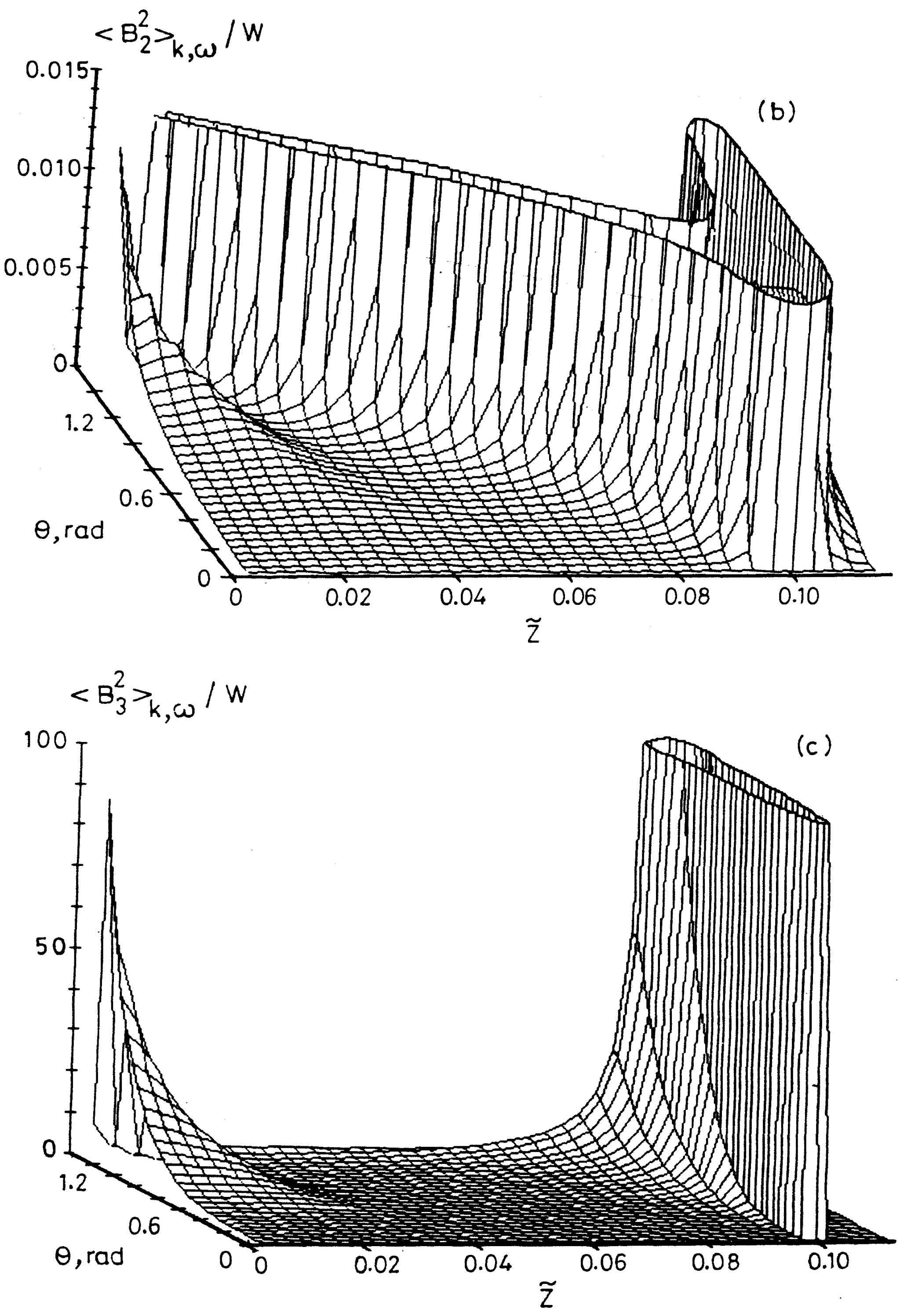


Fig. 5 The spectral distributions of the magnetic field fluctuations $\left\langle B_1^2 \right\rangle_{\bar{k}\omega}$ (a), $\left\langle B_2^2 \right\rangle_{\bar{k}\omega}$ (b) and $\left\langle B_3^2 \right\rangle_{\bar{k}\omega}$ (c) in the frequency range $\omega << \omega_{B_i}$ as functions of the dimensionless frequency $\tilde{z} = \sqrt{\dfrac{3}{2}\dfrac{\omega}{ks}}$ and the angle ϑ.

of the dimensionless frequency $\tilde{z} = \frac{\omega}{ks}$ and the angle ϑ. Numerical calculations were available[18] for the values of the parameters $\Theta^2 = 0{,}01$ and $q^2 = 0{,}1$. Each spectral distribution $\langle B_1^2 \rangle_{\vec{k}\omega}$ and $\langle B_3^2 \rangle_{\vec{k}\omega}$ has two maxima (Fig. 5 a,c). The broad maximum in the low-frequency range is associated with incoherent fluctuations. The narrow maximum at the frequency $\tilde{z} = \Theta$ $(\omega = kv_A)$ corresponds to the magnetosonic fluctuation oscillations. This maximum is by several orders of magnitude higher than the broad low-frequency maximum. The spectral distribution $\langle B_2^2 \rangle_{\vec{k}\omega}$ (Fig. 5b) shows a broad maximum produced by the low-frequency incoherent fluctuations along with the narrow maximum at the frequency $\tilde{z} = \Theta\cos\vartheta$ $(\omega = kv_A\cos\vartheta)$ that is given rise to by the Alfven fluctuation oscillations. The frequency associated with the latter maximum depends on the angle ϑ. In $\langle B_2^2 \rangle_{\vec{k}\omega}$, a maximum at the frequency $\tilde{z} = \Theta$ $(\omega = kv_a)$ is also observed; it is given rise to by the magnetosonic fluctuation oscillations. The value of this maximum is by several orders of magnitude lower than the Alfven fluctuation maximum in the spectrum of $\langle B_2^2 \rangle_{\vec{k}\omega}$ and the magnetosonic fluctuation maxima in the spectral distributions of $\langle B_1^2 \rangle_{\vec{k}\omega}$ and $\langle B_3^2 \rangle_{\vec{k}\omega}$. The values of the magnetosonic fluctuation maxima in the spectral distributions $\langle B_1^2 \rangle_{\vec{k}\omega}$ and $\langle B_3^2 \rangle_{\vec{k}\omega}$ are given in Fig. 6 (a,b) as functions of the angle ϑ. The minimum values of these maxima lie in the angle range $\vartheta \approx 1{,}45$ rad; the widths of the maxima attain their greatest values in this range. In the spectral distribution of $\langle B_2^2 \rangle_{\vec{k}\omega}$, the minimum value of the maximum associated with Alfven fluctuation oscillations and the maximum value of its width are attained for $\vartheta \approx 0{,}04$ rad (Fig. 7); the value of the maximum (as distinct from the dependence in Fig. 6a) steeply increases for small values of the angle ϑ and less steeply growth towards the angle $\vartheta = \pi/2$.

7 Fluctuations in Nonequilibrium Plasmas

To find the spectral distributions of electric and magnetic field fluctuations in a nonequilibrium plasma, we have to know the spectral distribution of the Langevin current along with the dielectric permittivity tensor. The spectral distribution of current density fluctuations in the plasma disregarding the Coulomb interaction of charged particles is usually taken for the spectral distribution of the Langevin current. If the charged particle distribution is axially symmetric with respect to the external magnetic field, the spectral distribution is given by[21]:

$$\langle j_i j_j \rangle^0_{\vec{k}\omega} = e^2 \int_{-\infty}^{\infty} dt e^{i\omega t} \int d\vec{v} \int d\vec{v}' v_i v_j W_{\vec{k}}(\vec{v},\vec{v}';t) f_0(\vec{v}), \quad \dots (73)$$

where $W_{\vec{k}}(\vec{v},\vec{v}';t)$ is the probability density in the phase space for the particle transition from the point $\vec{v}$ to the point $\vec{v}'$ for the time t. If the particle interactions are neglected, the transition probability density is given by

$$W_{\vec{k}}(\vec{v},\vec{v}';t) = e^{-ia[\sin(\omega_B t+\alpha)-\sin\alpha]-ik_\parallel v_\parallel t} \times \delta(v'_x - v_\perp\cos(\omega_B t+\alpha)) \times \delta(v'_y + v_\perp \sin(\omega_B t+\alpha))\delta(v'_\parallel - v_\parallel), \quad \dots (74)$$

where $a = \frac{k_\perp v_\perp}{\omega_B}$ and the initial velocity is determined as

$$\vec{v} = (v_\perp\cos\alpha, -v_\perp\sin\alpha, v_\parallel).$$

The spectral distribution (73) may be written in the form

$$\langle j_i j_j \rangle^0_{\vec{k}\omega} = e^2 \int d\vec{v} W_{ij}(v_\perp, v_\parallel; \omega, \vec{k}) f_0(v_\perp, v_\parallel), \quad \dots (75)$$

where

$$W_{ij}(v_\perp, v_\parallel; \omega, \vec{k}) = \int_{-\infty}^{\infty} dt e^{i\omega t} \int_0^{2\pi} \frac{d\alpha}{2\pi} \int d\vec{v}' v_i v'_j W_{\vec{k}}(\vec{v},\vec{u}';t). \quad \dots (76)$$

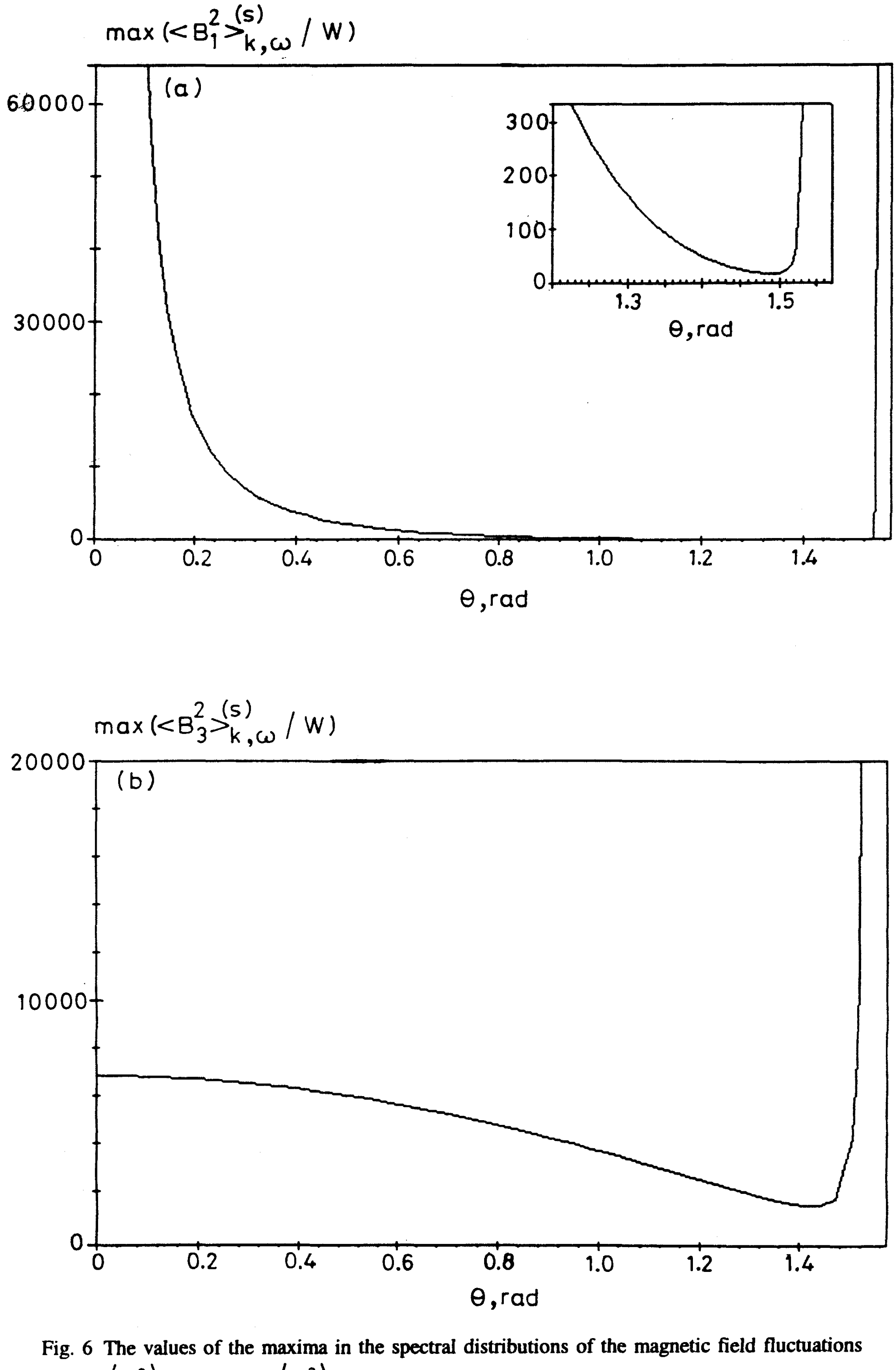


Fig. 6 The values of the maxima in the spectral distributions of the magnetic field fluctuations $\left\langle B_1^2 \right\rangle_{\bar{k}\omega}$ (a) and $\left\langle B_3^2 \right\rangle_{\bar{k}\omega}$ (b) which are associated with magnetosonic fluctuation oscillations as function of the angle ϑ.

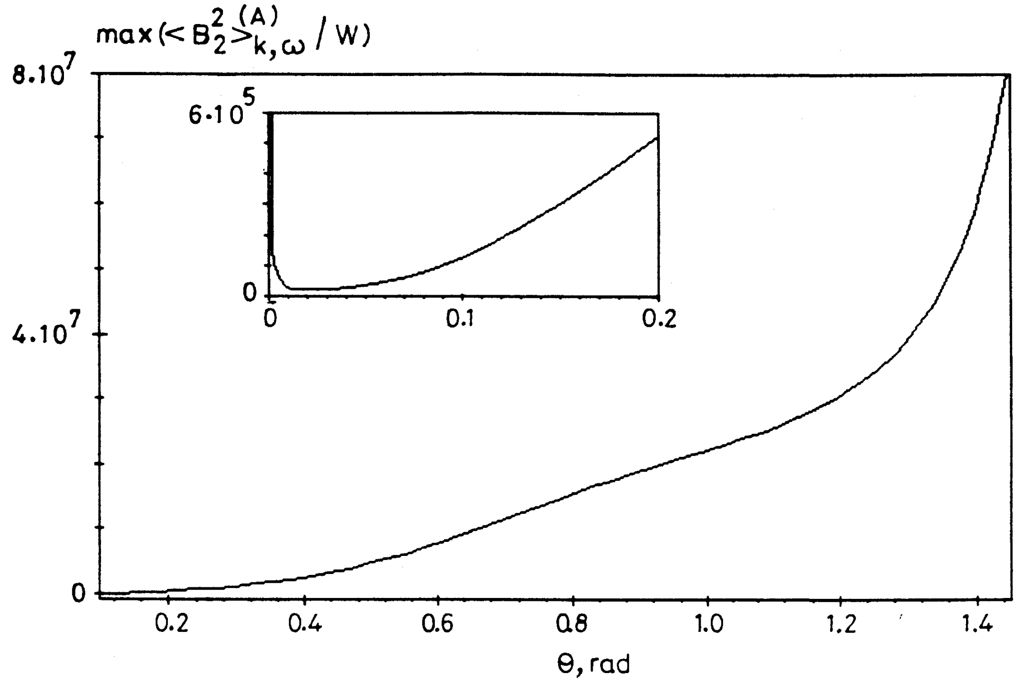


Fig. 7 The values of the maxima in the spectral distributions of the magnetic field fluctuations $\left\langle B_2^2\right\rangle_{\vec{k}\omega}$ which are associated with Alfven fluctuation oscillations as function of the angle ϑ.

Expanding the exponents in the expression (74) according to

$$e^{ia\sin\varphi} = \sum_n J_n(a)e^{in\varphi},$$

where $J_n(a)$ is the n th order Bessel function, and performing the integration in (76), we obtain

$$W_{ij}(v_\perp, v_\parallel; \omega, \vec{k}) = 2\pi\sum_n \prod_{ij}^{(n)}(v_\perp, v_\parallel)\delta(\omega - k_\parallel v_\parallel - n\omega_B) \quad \ldots (77)$$

where the tensor $\prod_{ij}^{(n)}(v_\perp, v_\parallel)$ is defined by

$$\prod_{ij}^{(n,\alpha)}(v_\perp, v_\parallel) = \begin{Bmatrix} \frac{n\omega_{B_\alpha}}{k_\perp} J_n(a_\alpha) \\ -iv_\perp J_n'(a_\alpha) \\ v_\parallel J_n(a_\alpha) \end{Bmatrix} \times \left(\frac{n\omega_{B_\alpha}}{k_\perp} J_n(a_\alpha), iv_\perp J_n'(a_\alpha), v_\parallel J_n(a_\alpha)\right)$$

$$a_\alpha = \frac{k_\perp v_\perp}{\omega_{B_\alpha}}. \quad \ldots (78)$$

Thus, the spectral distribution of current density fluctuations in an ensemble of non-interacting charged particles, the distribution function of which is axisymmetric relative to the magnetic field, is as follows:

$$\left\langle j_i h_j \right\rangle_{\vec{k}\omega}^{0\alpha} = 2\pi\sum_\alpha e_\alpha^2 \int d\vec{v} \sum_n \prod_{ij}^{(n\alpha)}(v_\perp, v_\parallel) \times \delta(\omega - k_\parallel v_\parallel - n\omega_{B_\alpha}) f_{0\alpha}(v_\perp, v_\parallel). \quad \ldots (79)$$

In the case of a nonequilibrium plasma the dielectric permittivity tensor may be found on the basis of inverting the fluctuation-dissipation relationship[6]:

$$\varepsilon_{ij}(\omega, \vec{k}) = \delta_{ij} + 4\pi\sum_\alpha \kappa_{ij}^\alpha(\omega, \vec{k}) = \left(1 - \sum_\alpha \frac{\omega_{p\alpha}^2}{\omega^2}\right)\delta_{ij}, - \frac{2}{\omega^2}\sum_\alpha \int d\omega' \frac{\omega' \frac{\partial}{\partial E_\alpha}\left\langle j_i j_j \right\rangle_{\vec{k}\omega'}^{0\alpha}}{\omega' - \omega - i0} \quad \ldots (80)$$

where $\frac{\partial}{\partial E_\alpha}\langle j_i j_j\rangle_{\vec{k}\omega}^{0\alpha}$ is the correlation function averaged over the derivative of the energy distribution:

$$\omega\frac{\partial}{\partial E_\alpha}\langle j_i j_j\rangle_{\vec{k}\omega}^{0\alpha}=\frac{2\pi e_\alpha^2}{m_\alpha}\sum_n\int d\vec{v}\prod_{ij}^{(n,\alpha)}(\vec{v})\delta\left(\omega-n\omega_{B_\alpha}-k_\parallel v_\parallel\right)\times\left(\frac{n\omega_{B_\alpha}}{v_\perp}\frac{\partial}{\partial v_\perp}+k_\parallel\frac{\partial}{\partial v_\parallel}\right)f_{0\alpha}(v_\perp,v_\parallel) \quad \dots (81)$$

For the potential (longitudinal electric) field, the spectral distribution of the Langevin source and the dielectric permittivity of a magnetoactive plasma with the distribution function $f_0(\vec{v})$ being axially symmetric with respect to $\vec{B}_0$ are given by

$$\langle\delta n^2\rangle_{\vec{k}\omega}^0=2\pi\sum_n\int d\vec{v}J_n^2\left(\frac{k_\perp v_\perp}{\omega_B}\right)\times\delta(\omega-n\omega_B-k_\parallel v_\parallel)f_0(v_\perp,v_\parallel), \quad \dots (82)$$

$$\varepsilon(\omega,\vec{k})=1+\sum_\alpha\frac{4\pi e^2}{mk^2}\sum_n\int d\vec{v}\frac{J_n^2\left(\frac{k_\perp v_\perp}{\omega_B}\right)}{\omega-n\omega_B-k_\parallel v_\parallel+i0}\times\left(\frac{n\omega_B}{v_\perp}\frac{\partial}{\partial v_\perp}+k_\parallel\frac{\partial}{\partial v_\parallel}\right)f_0(v_\perp,v_\parallel) \quad \dots (83)$$

For the strong external magnetic field, in the low-frequency range ($\omega \ll \omega_{B_i}$) the terms with $n=0$ are significant only in eq. (82) and in the contribution from the second term in the brackets in eq. (83). The contribution from the first term to the latter equation includes all terms with $n\neq 0$. Thus, eqs. (82) and (83) may be approximated by the formula[19, 20]:

$$\langle\delta n^2\rangle_{\vec{k}\omega}^0=2\pi\int d\vec{v}J_0^2\left(\frac{k_\perp v_\perp}{\omega_B}\right)\delta(\omega-k_\parallel v_\parallel)f_0(v_\perp,v_\parallel) \quad \dots (84)$$

$$\varepsilon(\omega,\vec{k})=1+4\pi\sum_\alpha\kappa_\alpha(\omega,\vec{k})=\varepsilon_0(\vec{k})+\sum_\alpha\frac{4\pi e^2}{mk^2}\int d\vec{v}\frac{J_0^2\left(\frac{k_\perp v_\perp}{\omega_B}\right)}{\omega-k_\parallel v_\parallel+i0}k_\parallel\frac{\partial}{\partial v_\parallel}f_0(v_\perp,v_\parallel) \quad \dots (85)$$

where $\varepsilon_0(\vec{k})$ is result after the summation over n

$$\varepsilon_0(\vec{k})=1+\sum_\alpha\frac{4\pi e^2}{mk^2}\int d\vec{v}\left\{J_0^2\left(\frac{k_\perp v_\perp}{\omega_B}\right)-1\right\}\times\frac{1}{v_\perp}\frac{\partial}{\partial v_\perp}f_0(v_\perp,v_\parallel) \quad \dots (86)$$

When particle distributions are Maxwellian with temperatures $T_\perp$ and $T_\parallel$ with respect to the external magnetic field direction, eqs. (84) and (85) yield

$$\langle\delta n^2\rangle_{\vec{k}\omega}^0=\sqrt{2\pi}\frac{n_0}{k_\parallel s_\parallel}e^{-\beta}I_0(\beta)e^{-z_\parallel^2}, \quad \dots (87)$$

$$\varepsilon(\omega,\vec{k})=\varepsilon_0(\vec{k})\times\left\{1+\frac{1}{\tilde{a}^2k^2}\left[\begin{array}{l}1-\varphi(z_\parallel)+i\sqrt{\pi}z_\parallel e^{-z_\parallel^2}\\+Z_i(1-\varphi(\mu z_\parallel)+i\sqrt{\pi}\mu z_\parallel e^{-\mu^2 z_\parallel^2}\end{array}\right]\right\}, \quad \dots (88)$$

where Z_i is the ion charge number,

$$\varepsilon_0(\vec{k})=1+\sum_\alpha\frac{1}{a_\perp^2k^2}\left(1-e^{-\beta}I_0(\beta)\right),$$

$$a_\perp^2=\frac{T_\perp}{4\pi n_0e^2},\ a_\parallel^2=\frac{T_\parallel}{4\pi n_0e^2},$$

$$\tilde{a}^2=\frac{\varepsilon_0(\vec{k})}{e^{-\beta}I_0(\beta)}a_\parallel^2,\quad \beta=\frac{k_\perp^2T_\perp}{m\omega_B^2},$$

$$s_\parallel^2=\frac{T_\parallel}{m},\ z_\parallel=\frac{1}{\sqrt{2}}\frac{\omega}{k_\parallel s_\parallel}$$

We note that the spectral distribution of fluctuation sources in this case (strong magnetic field) is determined by the longitudinal temperature only. According to the general theory [6], the correlation function for the electron density fluctuations in the case of potential interactions between particles takes the form

$$\langle \delta n_e^2 \rangle_{\vec{k}\omega} = \left|1 - \frac{4\pi\kappa_e(\omega,\vec{k})}{\varepsilon(\omega,\vec{k})}\right|^2 \langle \delta n_e^2 \rangle_{\vec{k}\omega}^0 + \left|\frac{4\pi\kappa_e(\omega,\vec{k})}{\varepsilon(\omega,\vec{k})}\right|^2 Z_i^2 \langle \delta n_i^2 \rangle_{\vec{k}\omega}^0 \quad \dots (89)$$

Here, plasma discreteness reveals itself in the spontaneous density fluctuations $\langle \delta n_e^2 \rangle_{\vec{k}\omega}^0$ and $\langle \delta n_i^2 \rangle_{\vec{k}\omega}^0$, due to the random motion of individual non-interacting particles. The factors near the correlation function $\langle \delta n_e^2 \rangle_{\vec{k}\omega}^0$ and $\langle \delta n_i^2 \rangle_{\vec{k}\omega}^0$, in eq. (89) govern plasma polarization by the shielding cloud of electrons and ions around the test particle. Comparing eqs. (87) and (88) with those for a nonmagnetized plasma, we observe that these equations are related by the scaling transformation

$$z \to z_\parallel, \quad a^2 \to \tilde{a}^2, \quad n_0 \to \tilde{n}_0 \equiv n_0 e^{-\beta} I_0(\beta)$$

and $\kappa(\omega,\vec{k}) \to \kappa_0(k) + \kappa(\omega,\vec{k})$.

This makes it possible to reproduce the above analysis using the appropriate results for a nonmagnetized plasma[22].

The spectral distribution of electric field fluctuations in a nonequilibrium plasma is determined by the expression[21]

$$\langle E_i E_j \rangle_{\vec{k}\omega} = \frac{16\pi}{\omega^2} \frac{1}{|\Lambda|^2} \lambda_{ik}^* \lambda_{jl} \langle j_k j_l \rangle_{\vec{k}\omega}^0 . \quad \dots (90)$$

and the spectral distribution of magnetic field fluctuations in a nonequilibrium plasma is given by

$$\langle B_i B_j \rangle_{\vec{k}\omega} = \frac{16\pi^2}{\omega^2} \frac{\eta^2}{|\Lambda|^2} \in_{ikm} \times \in_{jln} \frac{k_k k_l}{k^2} \lambda_{mr}^* \lambda_{ns} \langle j_r j_s \rangle_{\vec{k}\omega}^0 \quad \dots (91)$$

For equilibrium plasmas, we have

$$\langle j_i j_j \rangle_{\vec{k}\omega}^0 = i\frac{\omega}{4\pi} T(\Lambda_{ij}^* - \Lambda_{ji}) \quad \dots (92)$$

and thus the spectral distribution eq. (91) reduces to eq. (48). In the plasma transmittance range, the imaginary part Λ'' of the dispersion determinant is much smaller than the real part Λ' ($\Lambda'' << \Lambda'$), and the algebraic complement may be treated as Hermitian, i.e.,

$$\lambda_{ij} = \lambda_{ji}^* .$$

The spectral distribution in the vicinity of the eigenoscillation frequency in the equilibrium case reduces then to

$$\langle E_i E_j \rangle_{\vec{k}\omega} = \frac{8\pi^2}{\omega^2} T \lambda_{ji} \delta(\Lambda') . \quad \dots (93)$$

We note that in the plasma transmittance range in the vicinity of the eigenoscillation frequency a relation occurs

$$\lambda_{ki}\lambda_{jl} = \lambda_{kl}\lambda_{ji} .$$

Then the expression for the spectral distribution of electric field fluctuations in the transmittance range of a nonequilibrium plasma reduces to the form similar to eq. (93), i.e.,

$$\langle E_i E_j \rangle_{\vec{k}\omega} = \frac{8\pi^2}{\omega} \tilde{T}(\omega,\vec{k}) \lambda_{ji} \delta(\Lambda') , \quad \dots (94)$$

where $\tilde{T}(\omega,\vec{k})$ is the effective temperature,

$$\tilde{T}(\omega,\vec{k}) = \frac{2\pi^2}{\omega} \frac{\lambda_{kl}}{\Lambda''} \langle j_k j_l \rangle_{\vec{k}\omega}^0 . \quad \dots (95)$$

Near the eigenfrequencies, the algebraic complements λ_{ij} are directly related to the electric field polarization vector,

$$\lambda_{ji} = e_i^* e_j Sp\lambda .$$

Thus we have

$$\langle E_i E_j \rangle_{\vec{k}\omega} = \frac{8\pi^2}{\omega} \tilde{T}(\omega,\vec{k}) e_i^* e_j Sp\lambda \delta(\Lambda') , \quad \dots (96)$$

The spectral distribution of magnetic field fluctuations near the eigenoscillation frequencies is given by

$$\langle B_i B_j \rangle_{\vec{k}\omega} = \frac{8\pi^2}{\omega} \tilde{T}(\omega,\vec{k}) \eta^2 b_i^* b_j Sp\lambda \delta(\Lambda') . \quad \dots (97)$$

We note that the effective temperature $\tilde{T}(\omega,\vec{k})$ depends on the frequency and the wave vector. It is obvious that the value of the effective temperature for the frequency equal to the eigenoscillation frequency should be regarded as the temperature of the relevant eigenoscillations,

$$T_A \equiv \tilde{T}(kv_A \cos\vartheta, k), T_S \equiv \tilde{T}(kv_A, k). \quad \dots (98)$$

The values of T_A and T_B can differ considerably. The condition of plasma stability with respect to collective excitations reduces to the requirement that the damping rate $\gamma_{A,S}$ of the relevant collective excitation must be greater than zero. The effective temperature of relevant excitations grows infinite at the stability boundary.

The spectral distribution of incoherent fluctuations of the magnetic field is maximum for zero frequency. We multiply the total spectrum eq. (91) by $b_i^{(A)}b_j^{(A)}$ and $b_i^{(S)}b_j^{(S)}$ and thus separate the incoherent fluctuations of Alfven and magnetosonic types, respectively. We thus have

$$b_i^{(A)}\langle B_i B_j\rangle_{\vec{k}\omega} b_j^{(A)} = 16\pi^2 \frac{\eta^2}{\omega^2|\Lambda|^2}\cos^2\vartheta L_k^* L_l \langle j_k j_l\rangle_{\vec{k}\omega}^{(0)},$$

$$b_i^{(S)}\langle B_i B_j\rangle_{\vec{k}\omega} b_j^{(S)} = 16\pi^2 \frac{\eta^2}{\omega^2|\Lambda|^2}\lambda_{2k}^*\lambda_{2l}\langle j_k j_l\rangle_{\vec{k}\omega}^{(0)}, \quad \dots (99)$$

where

$$L_1 = \left[(\eta^2 - \varepsilon_{22})\varepsilon_{33} - \varepsilon_{23}^2\right]\cos\vartheta + \varepsilon_{12}\varepsilon_{23}\sin\vartheta,$$
$$L_2 = \varepsilon_{12}\varepsilon_{33}\cos\vartheta + \varepsilon_{11}\varepsilon_{23}\sin\vartheta,$$
$$L_3 = -\left[(\eta^2 - \varepsilon_{22})\varepsilon_{11} - \varepsilon_{12}^2\right]\sin\vartheta - \varepsilon_{12}\varepsilon_{23}\cos\vartheta.$$

We employ expression (3) for the permittivity tensor components in the limiting case $q^2 << 1$ and thus find that

$$b_i^{(A)}\langle B_i B_j\rangle_{\vec{k}\omega} b_j^{(A)} = 16\pi^2 \frac{\omega_{B_i}^2}{\omega_{p_i}^2}k^2 v_A^2 \times \cos^4\vartheta \frac{\langle j_1^2\rangle_{\vec{k}\omega}^{(0)}}{\left(\omega^2 - k^2v_A^2\cos^2\vartheta\right)^2 + (\omega^2 P'')^2}, \quad \dots (100)$$

$$b_i^{(S)}\langle B_i B_j\rangle_{\vec{k}\omega} b_j^{(S)} = 16\pi^2 \frac{\omega_{B_i}^2}{\omega_{p_i}^2}k^2 v_A^2 \frac{\langle j_2^2\rangle_{\vec{k}\omega}^{(0)}}{\left(\omega^2 - k^2v_A^2\right)^2 + (\omega^2\Psi'')^2} \quad \dots (101)$$

The ratios of the intensities of these spectral distributions for zero frequencies to the quantities $b_i^{(A)}\langle B_i B_j\rangle_{\vec{k}\omega} b_j^{(A)}/T$ and $b_i^{(S)}\langle B_i B_j\rangle_{\vec{k}\omega} b_j^{(S)}/T$ in terms of which the spectral distribution corresponds to eq. (69), may be regarded as effective temperatures of the relevant incoherent fluctuations, i.e.,

$$T_A^N = 8\sqrt{6\pi}\sqrt{\frac{M}{m}\frac{1}{q^2}\frac{\omega_{B_i}^2}{\omega_{p_i}^2}\frac{s}{v_A}}\frac{\langle j_1^2\rangle_{\vec{k}\omega}^{(0)}}{kv_A}\frac{1}{\sin^2\vartheta\cos\vartheta}, \quad \dots (102)$$

$$T_S^N = \sqrt{6\pi}\sqrt{\frac{M}{m}\frac{\omega_{B_i}^2}{\omega_{p_i}^2}}\frac{\langle j_2^2\rangle_{\vec{k}\omega}^{(0)}}{ks}\frac{\cos\vartheta}{\sin^2\vartheta}. \quad \dots (103)$$

We note that effective temperatures of incoherent fluctuations are always finite, as distinct from the temperatures of collective Alfven and magnetosonic fluctuations. Thus, it is convenient to describe the state of a nonequilibrium plasma in terms of the sets of temperatures T_A, T_S and T_A^N, T_S^N; the diagnostics of states of a none quilibrium plasma with strong external magnetic field may be reduced to the calculation of these temperatures.

Let us generalize the transition probability eq. (76) for the case of a turbulent plasma state[19]. We consider plasmas with developed turbulence and assume that there occur large-scale turbulent pulsations. This means that microscopic motion of noninteracting particles reduces to the motion of particles under the influence of the field averaged over a small macroscopic volume, and the stochastic motion of the latter volume. We assume thermal motion of individual particles and chaotic turbulent large-scale motions to occur independently. Therefore, the transition probability density for a turbulent system is given by

$$W_{\vec{k}\omega}^T(\vec{v},\vec{v}') = \int\frac{d\omega'}{2\pi}\int d\vec{v}^T \times P_{\vec{k}\omega-\omega'}(\vec{v}^T)W_{\vec{k}\omega'}(\vec{v},\vec{v}'+\vec{v}^T), \quad \dots (104)$$

where $P_{\vec{k}\omega}(\vec{v}^T)$ is the factor determined by the stochastic Brownian motion of small macroscopic volumes. If the elementary volume is involved in the diffusion-drift motion [19], then

$$P_{\vec{k}\omega} \equiv \int d\vec{v}^T P_{\vec{k}\omega}(\vec{v}^T) = \frac{2k^2 D}{(\omega - \vec{k}\vec{u}_D)^2 + k^4 D^2}, \quad \dots (105)$$

where $\vec{u}_D$ is the drift velocity and D is the diffusion coefficient. The more pragmatic model was proposed[20] in which one imagines that small macroscopic plasma volumes move chaotically across the magnetic field and the characteristic function is a Gaussian

$$P_{\vec{k}\omega} = \frac{\sqrt{2\pi}}{\gamma_F} \exp\left\{\frac{(\omega - \omega_F)^2}{2\gamma_F^2}\right\} \quad \dots (106)$$

where the mean (drift) velocity $\vec{u}_D$ and the root mean square velocity u, associated with fluid-like motion, determine the Doppler frequency $\omega_F = \vec{k}\vec{u}_D$ (at which the spectrum has its maximum) and the spectral width $\gamma_F = k_\perp u$, respectively.

The spectral distribution of the Langevin current for a turbulent system is defined by

$$\langle j_i j_j \rangle^T_{\vec{k}\omega} = \int \frac{d\omega'}{2\pi} P_{\vec{k}\omega-\omega'} \langle j_i j_j \rangle^0_{\vec{k}\omega'}. \quad \dots (107)$$

The dielectric permittivity tensor for a turbulent plasma is determined by the formula (80) in which the correlation function $\langle j_i j_j \rangle^0_{\vec{k}\omega}$ has to be changed to $\langle j_i j_j \rangle^T_{\vec{k}\omega}$.

The detailed consideration of the spectral distributions of the potential fluctuations in a turbulent plasma with large-scale random motions was done in papers [Ref. nos.19, 20, 22, 23]. In the case of potential field the spectral distribution of the fluctuation source and the electric susceptibility for a turbulent plasma are given by

$$\langle \delta n^2 \rangle^T_{\vec{k}\omega} = \int \frac{d\omega'}{2\pi} P_{\vec{k}\omega-\omega'} \langle \delta n^2 \rangle^0_{\vec{k}\omega}, \quad \dots (108)$$

$$\kappa^T(\omega, \vec{k}) = \int \frac{d\omega'}{2\pi} \tilde{P}_{\vec{k}\omega-\omega'} \kappa(\omega', \vec{k}), \quad \dots (109)$$

where

$$i\tilde{P}_{\vec{k}\omega} = \int \frac{d\omega'}{2\pi} \frac{P_{\vec{k}\omega}}{\omega' - \omega - i0}.$$

Making use of the characteristic function in eq. (106), we find the spectral distribution for spontaneous fluctuations[20]:

$$\langle \delta n^2 \rangle^T_{\vec{k}\omega} = \sqrt{\frac{2\pi}{C}} \frac{\tilde{n}_0}{\gamma_T} e^{-\frac{(\omega-\omega_F)^2}{2C\gamma_T^2}} \quad \dots (110)$$

and the dielectric permittivity for an electron plasma

$$\varepsilon(\omega, \vec{k}) = \varepsilon(k)\left\{1 + \frac{1}{C}\frac{1}{\tilde{a}^2 k^2}\left[1 - \varphi\left(\frac{\omega - \omega_F}{\sqrt{2C}\gamma_T}\right) + i\sqrt{\pi}\frac{\omega - \omega_F}{\sqrt{2C}\gamma_T} e^{-\frac{(\omega-\omega_F)^2}{2C\gamma_T^2}}\right]\right\}.$$

$\gamma_T = ks_\parallel$ is the frequency spectral width for spontaneous microscopic density fluctuations in a macroscopically steady plasma. The shape parameter

$$C \equiv 1 + \left(\frac{\gamma_F}{\gamma_T}\right)^2 = 1 + \left(\frac{k_\perp}{k}\frac{u}{s_\parallel}\right)^2$$

changes in the range from unity to infinity, as it depends on the ratio between the magnitudes of the wavevector perpendicular and parallel components and the ratio between the root mean square turbulent velocity and the particle thermal velocity. The spectral distribution of electron density fluctuations for a turbulent plasma is related to result for a nonmagnetized plasma by the scaling transformations:

$$T \to CT \geq T, \ \omega \to \frac{\omega - \omega_F}{\sqrt{C}}, \ a^2 \to C\tilde{a}^2.$$

The characteristic length scale, which separates incoherent and collective fluctuations according to $C\tilde{a}^2 k^2 \varepsilon(\vec{k}) \sim 1$ depends on the effects of fluid motion $(C > 1)$, particle polarization drift $(\varepsilon(\vec{k}) > 1)$ and finite Larmor radius. The characteristic scale length is large when these effects are important. The spectra are broader for larger C, with collective features being less pronounced. The frequency scale $\sqrt{2C}\gamma_T$ depends on particle thermal motion along the magnetic field and on fluid, and it decreases with $k_\parallel$. In Fig. 8, the normalized spectrum $\frac{\varepsilon(k)^2}{\tilde{n}}\langle\delta n^2\rangle^{\cdot}_{\bar{k}\omega}$ is plotted as functions of $z_\parallel = \frac{\omega}{\sqrt{2}\gamma_T}$ and various values of $y = \tilde{a}^2k^2$. This figure shows the well-known sharp peaks in the spectrum due to collective electron plasma oscillations.

8 Electromagnetic Wave Scattering and Conversion in Magnetized Plasma

The main problem in calculating the cross-sections of electromagnetic wave scattering in plasmas is to find the current produced by the nonlinear interaction of the incident wave with the fluctuations of electron density and their velocity, and the fluctuations of electric and magnetic fields. This current determines the scattered wave field. The nonlinear constitutive equation for the plasma, and hence the scattered-wave-inducing current, may be derived from kinetic or hydrodynamic equations for the electron and ion plasma components. Since the difference between electron and ion masses is very large, the consideration may be restricted to the electron component only. The results of the fluid-approximation study of nonlinear processes in plasmas, obtained within its applicability range, are shown to be in accordance with the results of the kinetic treatment[10,11]. In the cold plasma, when thermal effects in the dispersion of incident and scattered waves may be completely

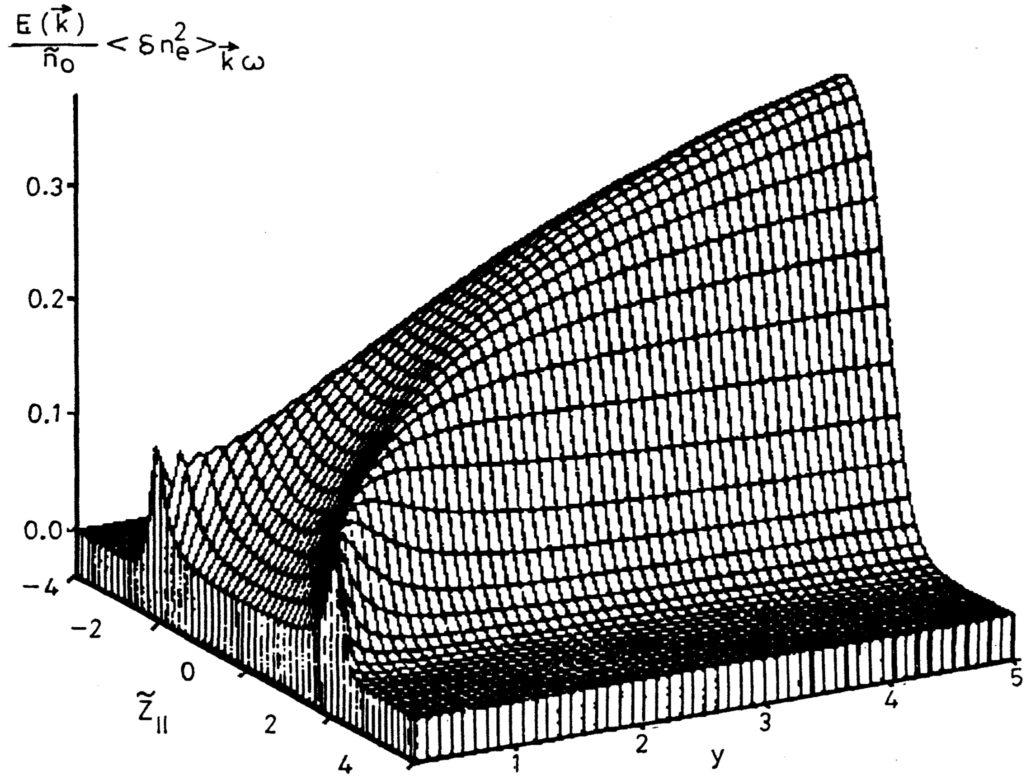


Fig. 8 The spectral distributions of the electron density fluctuations $\frac{\varepsilon(\vec{k})}{\tilde{n}_0}\langle\delta n_e^2\rangle_{\bar{k}\omega}$ as functions of the dimensionless frequency $\tilde{z}_\parallel = \frac{1}{\sqrt{2}}\frac{\omega}{ks_\parallel}$ and various values of $y = \tilde{a}^2k^2$ (Ref. 20).

disregarded, the scattered-waves-inducing current is given by the expression:

$$\vec{J}_{\vec{k}\omega} = -i\{ \frac{\omega_0}{n_0}\delta n_{\vec{q}\Delta\omega}\hat{\kappa}(\omega_0)\vec{E}^0 + \delta\vec{u}_{\vec{q}\Delta\omega}(\vec{k}_0\hat{\kappa}(\omega_0)\vec{E}^0)$$
$$+ \omega\hat{\kappa}(\omega)[\frac{1}{c}\left(\left[\delta\vec{u}_{\vec{q}\Delta\omega},\vec{B}_0\right]+\left[\vec{u}^0,\delta\vec{B}_{\vec{q}\Delta\omega}\right]\right)$$
$$- 4\pi\frac{\omega_0}{\omega^2_{p_e}}\left(\left(\vec{k}_0\delta\vec{u}_{\vec{q}\Delta\omega}\right)\hat{\kappa}(\omega_0)\vec{E}^0 + \delta\vec{u}_{\vec{q}\Delta\omega}\left(\vec{q}\hat{\kappa}(\omega_0)\vec{E}^0\right)\right)]\} \quad \ldots (111)$$

Here we introduced following denotations: ω_0 and $\vec{k}_0$ are a frequency and a wave vector of the incident wave; ω and $\vec{k}$ are a frequency and a wave vector of the scattered wave; $\Delta\omega = \omega - \omega_0$ and $\vec{q} = \vec{k} - \vec{k}_0$; E^0 and $\vec{B}^0$ are the electric and magnetic fields of the incident wave; $\delta n_{\vec{q}\Delta\omega}$ and $\delta\vec{u}_{\vec{q}\Delta\omega}$ are fluctuations of the electron density and velocity; $\delta\vec{B}_{\vec{q}\Delta\omega}$ is fluctuations of the magnetic field; $\hat{\kappa}(\omega)$ is the dielectric permittivity tensor of the cold magnetoactive plasma, $\hat{\varepsilon}(\omega) = 1 + 4\pi\hat{\kappa}(\omega)$,

$$4\pi\kappa_{ij}(\omega) = \sum_{e,i}\frac{\omega_p^2}{\omega_B^2 - \omega^2}\left\{\delta_{ij} + i\frac{\omega_B}{\omega}\in_{ijk} b_k - \frac{\omega_B^2}{\omega^2}b_i b_j\right\}, \quad \ldots (112)$$

$\vec{b}$ is the unit vector in a direction of the external magnetic field $\vec{B}_0$. Using the linear connections between fluctuations of physical quantities in a plasma and the linearity of incident wave, we can present eq. (111) in the form:

$$J_i(\omega,\vec{k}) = -i\omega\kappa_{ij}(\omega)\left\{\alpha_{jk}\frac{\delta n_{\vec{q}\Delta\omega}}{n_0} + \gamma_{jkl}\frac{\delta B_{l\vec{q}\Delta\omega}}{B_0}\right\}E_k^0, \quad \ldots (113)$$

where the fluctuations of all quantities are expressed in terms of the longitudinal fluctuations of charge density and the transversal magnetic fluctuations. Let us consider the scattering of waves with a small change of frequency ($\Delta\omega << \omega_0$). In this case, we can use the approximate expression

$$\alpha_{jk} \approx \delta_{jk} - \frac{4\pi}{q^2}q_j k_{0l}\kappa_{lk}(\omega_0),$$

$$\gamma_{jkl} \approx -4\pi\frac{\omega_0\omega_B}{\omega_p^2}\in_{jml}\kappa_{mk}(\omega_0), \Delta\omega << \omega_0. \quad \ldots (114)$$

Ordinary and extraordinary electromagnetic waves with equal frequencies but different refraction index and polarization can propagate in a plasma with the external magnetic field. The polarization vectors of the ordinary and extraordinary waves $\vec{e}_0$ and $\vec{e}_X$ respectively satisfy the orthogonality condition

$$\varepsilon_{ij}e_{0i}e^*_{Xj} = 0. \quad \ldots (115)$$

For the waves that propagate along the magnetic field ($\vartheta = 0$),

$$\eta^2_{0,X} = \varepsilon_1 \mp \varepsilon_2, \quad \vec{e}_{0,X} = (1, \mp i, 0), \quad \ldots (116)$$

in the other words, these waves are transversal with the circular polarization. For the waves that propagate in the direction perpendicular to the magnetic field ($\vartheta = \frac{\pi}{2}$):

$$\eta_0^2 = \varepsilon_3, \vec{e}_0 = (0,0,1);$$

$$\eta_X^2 = \varepsilon_1 - \frac{\varepsilon_2^2}{\varepsilon_1}, \quad \vec{e}_X = (1, -i\frac{\varepsilon_1}{\varepsilon_2}, 0). \quad \ldots (117)$$

If the incident electromagnetic wave propagates along the magnetic field $\vec{k}_0 \parallel \vec{B}_0$, then $\alpha_{ij} \to \delta_{ij}$; at the same time the scattering of waves on the longitudinal charge density fluctuations takes place. Because orthogonality of the polarization vectors $\vec{e}_0$ and $\vec{e}_X$, it is impossible the wave conversion on the charge density fluctuations.

If the incident wave (let $\vec{e}_0$) propagates perpendiculary to the direction of magnetic field $\vec{k}_0 \perp \vec{B}_0$, then $\vec{k}_0\hat{\kappa}(\omega_0)\vec{e}_0 = 0$ and again we get $\alpha_{jk} = \delta_{jk}$. Therefore, the wave conversion or transformation of the ordinary electromagnetic wave in the extraordinary one occurs due to the interaction with the transversal magnetic fluctuations only.

The general expression for the differential cross-section of electromagnetic wave scattering in magnetoactive plasmas was derived in Ref. [6]

from the hydrodynamic equations. In the case of electromagnetic wave scattering by electron density fluctuations, the differential cross-section is given by

$$d\Sigma = \frac{1}{2\pi}\left(\frac{e^2}{mc^2}\right)^2 \frac{\omega_0^2\omega^2}{\omega_{p_e}^4} R|\xi^2|\langle \delta n_e^2 \rangle_{\vec{q}\Delta\omega} d\omega d0, \quad \dots (118)$$

where

$$R = \frac{\eta^3}{\eta_0\left(|\vec{e}_0|^2 - \frac{|\vec{e}_0\vec{k}_0|^2}{k_0^2}\right)e_i^*\varepsilon_{ij}e_j}, \quad \xi = e_i^*(\varepsilon_{ij}^0 - \delta_{ij})e_j^0$$

This formula determines the plasma parameters (density, temperature, etc.) in terms of observed electromagnetic wave scattering spectra.

The differential cross-section of electromagnetic wave conversion under scattering can be calculated in the same manner as in [Ref. 6]. The importance of electromagnetic wave scattering by magnetic fluctuations was worked out by Thompson[24]. We restrict the consideration to the electromagnetic wave conversion with small frequency shift ($\Delta\omega << \omega_0$), then the differential cross-section of conversion is described by the expression

$$d\Sigma = \frac{1}{2\pi}\frac{\omega^4}{c^4B_0^2} \times R\sum_{l,l'}\left(e_i^*\kappa_{ij}(\omega)\gamma_{jkl}e_k^0\right)\left(e_i^*\kappa_{ij}(\omega)\gamma_{jkl'}e_k^0\right) \times \langle \delta B_l \delta B_{l'}^* \rangle_{\vec{q}\Delta\omega} d\omega d0 \quad \dots (119)$$

We note that

$$\langle \delta B_l \delta B_{l'}^* \rangle_{\vec{q}\Delta\omega} = \delta_{ll'}\langle \delta B_l^2 \rangle_{\vec{q}\Delta\omega},$$

and make use of the expression for the coefficient γ_{jkl}, then the differential cross-section reduces to the form

$$d\Sigma = 32\pi^2\left(\frac{e^2}{mc^2}\right)^2 R\frac{n_0}{mc^2}\frac{\omega^6}{\omega_{p_e}^6} \times \sum_l \left|e_i^*\kappa_{ij}(\omega)\in_{jml}\kappa_{mk}(\omega_0)e_k^0\right|^2 \langle \delta B_l^2 \rangle_{\vec{q}\Delta\omega} d\omega d0 \quad \dots (120)$$

We consider the case when the incident ordinary electromagnetic wave is converted into an extraordinary electromagnetic wave. Suppose the process occurs in the plane perpendicular to the external magnetic field $\vec{B}_0$. The polarization vector of the ordinary wave $\vec{e}_O \equiv \vec{e}^0 = (0,0,1)$ is directed along the magnetic field $\vec{B}_0$, hence the azimuthal angle of the wave vector $\vec{k}_0$ of the incident wave is insignificant. The polarization vector of the extraordinary wave, $\vec{e}_X$, lies in the plane perpendicular to $\vec{B}_0$ and depends on the azimuthal angle φ of the wave vector $\vec{k}$, i.e.,

$$\vec{e}_X \equiv \vec{e} = \left(\cos\varphi + i\frac{\varepsilon_1}{\varepsilon_2}\sin\varphi,\ \sin\varphi - i\frac{\varepsilon_1}{\varepsilon_2}\cos\varphi,\ 0\right)$$

We consider the case when the wave vector change under conversion $\vec{q} = \vec{k} - \vec{k}_0$ is small, then we can assume that $\vec{q} \perp \vec{k}$. We choose the coordinate system in a way that the vector $\vec{q}$ is directed along the x-axis, then the azimuthal angle of the converted wave vector is $\varphi = -\frac{\pi}{2}$. Therefore, the polarization vector of the converted wave is given by

$$\vec{e}_X \equiv \vec{e} = \left(-i\frac{\varepsilon_1}{\varepsilon_2}, -1, 0\right). \quad \dots (121)$$

We do not normalize the converted wave polarization vector since the cross-section in eq. (120) does not depend on the normalization condition). Thus we obtain an expression for the differential cross-section of the electromagnetic wave conversion in magnetized plasma, i.e.,

$$d\Sigma_{0\to X} = \frac{1}{8\pi^2}\left(\frac{e^2}{mc^2}\right)^2 R\frac{n_0}{mc^2} \times \left\{\frac{\omega^2}{\omega_{p_e}^2}\langle \delta B_1^2 \rangle_{\vec{q}\Delta\omega} + \frac{(\omega^2 + \omega_{p_e}^2)^2}{\omega_{p_e}^2\omega_{B_e}^2}\langle \delta B_2^2 \rangle_{\vec{q}\Delta\omega}\right\} d\omega\, d0, \quad \dots (122)$$

where

$$R = \frac{\eta^3}{\eta_0 e_i^*\varepsilon_{ij}e_j} = \sqrt{\frac{\varepsilon_1^2 - \varepsilon_2^2}{\varepsilon_1\varepsilon_2}\frac{\varepsilon_2^2}{\varepsilon_1^2}}.$$

We remind the reader that in this Section the notation for the dielectric permittivity tensor of a cold plasma is

$$\varepsilon_{ij}(\omega) \equiv 1 + 4\pi\kappa_{ij}(\omega) = \begin{pmatrix} \varepsilon_1 & -i\varepsilon_2 & 0 \\ i\varepsilon_2 & \varepsilon_1 & 0 \\ 0 & 0 & \varepsilon_3 \end{pmatrix} \dots \quad (123)$$

$$\varepsilon_1(\omega) = 1 - \sum_\alpha \frac{\omega_{p_\alpha}^2}{\omega^2 - \omega_{B_\alpha}^2},$$

$$\varepsilon_2(\omega) = \sum_\alpha \frac{\omega_{B_\alpha}}{\omega} \frac{\omega_{p_\alpha}^2}{\omega^2 - \omega_{B_\alpha}^2}, \varepsilon_3(\omega) = 1 - \sum_\alpha \frac{\omega_{p_\alpha}^2}{\omega^2}.$$

The first term within the curly braces in eq. (122) describes the conversion of an ordinary electromagnetic wave into an extraordinary one due to the interaction with magnetosonic fluctuation excitations in the plasma, the second term is associated with Alfven fluctuation excitations. We note that, for an equilibrium plasma, the integral contribution of Alfven excitations in the conversion is much greater than the contribution of magnetosonic excitations,

$$\frac{\sum^A}{\sum^S} = \frac{(\omega^2 + \omega_{p_e}^2)^2}{\omega^2 \omega_{B_e}^2} > 1.$$

In nonequilibrium plasmas, this ratio can be both greater and smaller than one for various plasma states. In the case of conversion by incoherent magnetic fluctuations with very small frequency shift, the value of this ratio is determined by the inverse ratio of the Langevin sources, i.e.,

$$\frac{\sum^A}{\sum^S} = \frac{\left\langle j_1^2 \right\rangle_{\vec{k}0}^0}{\left\langle j_2^2 \right\rangle_{\vec{k}0}^0}.$$

It should be noted that the analysis of experimental data must take into account the fluid-like turbulent motion in the plasma which can considerably influence the character of the converted wave spectrum, similarly to the effect of such motions on the spectra of wave scattering by particle density fluctuations[20,25]. Various aspects of plasma diagnostics are considered in detail in Ref. (26).

9 Conclusions

The main purpose of this review is to consider the low-frequency properties of a plasma with strong magnetic field, in particular, the specifics of collective excitations, dispersion, polarization, and damping of Alfven and magnetosonic waves. Low-frequency fluctuations of charge density, longitudinal and transverse electric field, and magnetic field in a magnetized equilibrium plasma are considered in detail; spectral distributions of such fluctuations are found; incoherent fluctuations caused by chaotic motion of individual particles are separated as well as the collective fluctuations associated with Alfven and magnetosonic plasma excitations. In the range of very small frequencies (much smaller than the ion cyclotron frequency) the Alfven and magnetosonic long-wave fluctuations are manifested in different components of the fluctuation spectral distribution tensor, i.e., Alfven and magnetosonic fluctuations are separated due to different characters of polarization of these excitations. The positions of the maxima of fluctuation spectra are determined by the frequencies of relevant plasma eigenoscillations. As the wave number increases, the maxima of fluctuation spectra are shifted to the ion cyclotron frequency range and separation of Alfven and magnetosonic waves disappears; the maxima associated with Alfven and magnetosonic fluctuations are manifested in all components of the spectral distribution; the magnetosonic maximum rapidly approaches the ion cyclotron frequency as the wave number grows. In the vicinity of the ion cyclotron frequency, a cyclotron resonance occurs which leads to the damping of collective excitations. In the ion cyclotron frequency range, incoherent cyclotron fluctuations occur; their spectral distribution is also found in the paper.

Much attention is paid to the fluctuations in nonequilibrium plasmas. The Langevin fluctuation sources are introduced in order to find the effective temperatures which are responsible for the collective fluctuation levels for the electric and magnetic fields. The effective temperatures are found as well for the incoherent magnetic field fluctuations associated with Alfven and magnetosonic excitations. The temperatures thus introduced make it possible to describe the nonequilibrium plasma states quantitatively. Fluctuations are also considered in a plasma with large-scale turbulent pulsations. The influence of such fluid-like chaotic motions on the fluctuation spectra is analyzed. Such motions must be taken into account when the plasma state is studied in terms of fluctuation spectra and electromagnetic wave scattering in the plasma.

Electromagnetic wave scattering is one of the most efficient methods of plasma diagnostics both in controlled fusion devices and in ionospheric and space plasmas. The study of electromagnetic wave conversion resulting from wave-plasma interaction provides additional possibilities to obtain information on the plasma state, in particular the character of magnetic field fluctuations.

10 Acknowledgements

The author is deeply thankful to A V Kharchenko for the numerical calculation of spectral distributions quoted in the review. He is also thankful to V M Khryapa for his help in preparing this review for publication. The author acknowledges the financial support of the International Science Education Program (grant N QSU082182).

References

1 A I Akhiezer, I G Prohoda and A G Sitenko *Sov Phys JETP* **6** (1958) 576
2 J P Dougherty and O T Farley Proc Roy Soc A259, (1960) 79; O T Farley, J P Dougherty and D W Barron *Proc Roy Soc* **A263**, (1961) 238
3 A I Akhiezer, I A Akhiezer and A G Sitenko *Sov Phys JETP* **14** (1962) 462
4 E E Salpeter *Phys Rev* **120** (1960) 1528; *ibid* 122 (1961) 1663,
5 M N Rosenbluth and N Rostoker *Phys Fluids* **5** (1962) 776
6 A G Sitenko *Electromagnetic Fluctuations in Plasma* Academic Press New York (1967)
7 A G Sitenko and Yu A Kirochkin *Sov Phys Uspekhi* **9** (1966) 430
8 D E Evans and J Katzenstein *Rep Prog Phys* **32** (1969) 207
9 J Sheffield *Plasma Scattering of Electromagnetic Radiation* Academic Press New York (1975)
10 A G Sitenko *Plasma Phys Control Fusion* **37** (1995) 163
11 A G Sitenko and S M Zinevych *Plasma Phys Control Fusion* **38** (1996) 627
12 T Lehner, J M Rax and X L Zou *Europhys Lett* **8** (1989) 759
13 L Vahala, G Vahala and N Bretz *Phys Fluids* **B4** (1992) 619
14 A G Sitenko and K N Stepanov *Sov Phys JETP* **4** (1957) 512
15 K N Stepanov *Sov Phys JETP* **7** (1958) 892; *ibid* **11** (1960) 192
16 T H Stix *The Theory of Plasma Waves* McGraw-Hill New York (1962)
17 A G Sitenko and A V Kharchenko *Plasma Phys Control Fusion* (in press)
18 A G Sitenko and A V Kharchenko *Ukr J Phys* **43** (1998) 545
19 A G Sitenko and A G Zagorodny *Ukr J Phys* **40** (1995) 390
20 P P Sosenko, N Maafa and D Gresillon *J Atmospheric and Terrestrial Physics* **58** (1996) 1047
21 A G Sitenko *Fluctuations and Non-Linear Wave Interactions in Plasmas* Pergamon Press Oxford (1982)
22 P P Sosenko *Quasiparticles in Macroscopically Agitated Magnetized Plasma—Palaiseau* (PMI 2827) (1993)
23 P P Sosenko, A G Sitenko and D Gresillon Ukr J Phys 42 38 1997
24 W B Thompson *J Plasma Physics* **5** (1971) 225
25 D Gresillon and B Cabrit *Proc 1989 Int Conf on Plasma Phys* New Delhi India Invited Papers Indian Academy of Sciences Bangalore (1991) 173
26 I H Hutchinson *Principles of Plasma Diagnostics* Cambridge University Press Cambridge (1987)