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EVALUATION OF CAPACITANCE AND INDUCTANCE MATRIX OF COUPLED TRANSMISSION LINES

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The paper introduces the concept of capacitance and inductance matrices of transmission lines consisting of more than two conductors. The method of derivation of the elements of these matrices is presented for a pair of cylinders of unequal diameter with ground plane in the form of two semi infinite plates with a gap located at the plane of symmetry between the pair of mutually inverse points, which are kept fixed. Bilinear transformation expressed in terms of mutually inverse points are used to transform the cylinder pair to concentric circles and the ground plane in the form of a concentric strip. Logarithmic transformation transforms the entire structure into a strip line in which the strip is grounded. Expressions for the matrix elements including effect of dielectric are derived in terms of geometrical parameters of the line. The velocities of propagation and characteristic impedance of different eigen modes are found. The numerical data are presented for a symmetric structure.

Key Words: Capacitance and Inductance Matrix; Coupled Transmission Lines; Bilinear Transformation; Dielectric Boundary; Equipotential Contours; Cylindrical Conductors

Introduction

For a transmission line consisting of a pair of cylinders, the characteristic impedance and phase constants are expressed in terms of inductance and capacitance per unit length which are also called the per unit length parameters. In the presence of a third conductor in the form of an infinite ground plane/combination of ground planes, the flux lines between one of the principal conductors and ground plane contribute to self capacitance/inductance of the corresponding conductor. The total charge on each of the principal conductors is related to the potential difference between the principal conductors and the ground plane. A pair of simultaneous equations so obtained leads to a matrix equation.

Moment method formulation using Fourier Harmonic function as the basis function has been used in the literature[1-3] for finding the capacitance matrix of multiconductors lines in the absence and presence of dielectric insulation. This method requires derivation of a set of simultaneous equations, using either point matching[1,2] or Galerkin's technique[3] for expressing charge on the principal conductors as a function of applied potentials through inversion of a matrix of high order. The methods given in ref. [1,2] suffer from moment matrix singularly problem and hence the matrix, which leads to the value of capacitance can not be inverted. Method of ref. [3] which employs Galerkin's technique is free from this problem, but it is found difficult to apply this technique since the angular variables of the different conductors appear in the same set of equations.

From the equations (16) and (19) of ref. [3] to be used for the derivation of the simultaneous equations, it is found that the terms containing relative dielectric constant of the dielectric coating disappear, and hence there is a confusion regarding consideration of the effect of dielectric medium on the per unit length parameters of the transmission line. The authors of ref. [3] have not presented any set of simultaneous equations even for a pair of dielectric coated wires isolated in free space for the purpose of illustration. In addition, the reference points for angular locations of harmonic testing functions are not clearly indicated.

In this paper, method of using conformal transformation for finding the expression for the elements of a capacitance matrix is presented. The method is applied to the general case of two

infinite cylinders of unequal radii in the presence of a ground plane which is considered in the form of two semi-infinite plates with a gap, located in the plane of symmetry between the mutually inverse points of the cylinder pair[4,5].

A bilinear transformation expressed in terms of mutually inverse points[4,5] of the cylinder pair transforms them to concentric circles. The semi-infinite plane with a gap is transformed to an arc strip concentric with the other two concentric circles. In the transformed concentric cylindrical configuration, the conductor boundaries are equipotential surfaces. When this boundary condition is applied to Laplace's equation its solution in the region between concentric cylinders appears in the form of a logarithmic function along the radial direction, and uniform along the circumferential direction[6]. Logarithmic transformation[7] transforms the entire structure in the form of a triplate strip line[8], in which the strip is grounded and the transformed plates of the cylinders are insulated. The dielectric boundaries coincident with the equipotential contours are transformed as concentric circles through bilinear transformation. Logarithmic transformation transforms them to lines parallel to transformation of conducting boundaries. The dielectric boundaries, which are coaxial with conductors boundaries, are transformed to eccentric circles through bilinear and curved contour through logarithmic transformation.

The total charge on the two plates are expressed in terms of applied potential on them with respect to grounded strip. The coefficients of the applied potentials are the elements of the capacitance matrix.

The product of inductance and capacitance matrices is equal to square of the reciprocal of velocity of propagation of the electromagnetic wave and a unit matrix[2]. This relation enables determination of inductance matrix from capacitance matrix. Capacitance matrix in the presence of dielectric on the cylinders is determined. Analysis is carried out to find the velocities and impedances of the eigen modes in the line. Numerical data are presented for the case of structure with symmetrical configuration.

General Analysis

Consider the configuration of Fig. 1(a) which consists of two semi-infinite plates with a gap located at the plane of symmetry between the mutually inverse points of the cylinder pair.

The mutually inverse points are fixed points and equipotential contours form a system of displaced circles with their centres located on the line joining the mutually inverse points, which together with their radii can be determined using the formulation presented in ref. [6,7].

For the pair of inverse points, as shown in Figure 1(a), the transformation[4,5]

$$w=\frac{Z-Z_1}{Z-Z_2}$$

transforms Z_1 and Z_2 to zero and infinity respectively, which are mutually inverse points of a pair of concentric circles of Fig. 1(a). Hence, the circles C_1 and C_2 of Fig. 1(a), with radii R_1, R_2 and distance D between their centres are transformed to concentric circles C_1' and C_2' as shown in Fig. 1(b).

The origin of the coordinate system of Fig. 1(a) taken at $Z_1=0$. The bilinear transformation assumes the form

$$w=\frac{Z}{Z-Z_2} \qquad \ldots(1a)$$

where $Z_2=2\sqrt{h_1^2-R_1^2}$ and $h_1=\dfrac{D^2+R_1^2-R_2^2}{2D}$ $\ldots(1b)$

Using eq. (1) and following the method suggested in the literature[5], the radii of the transformed circles in Fig. 1(b) are of the form

$$\rho_1=\frac{1}{\dfrac{h_1}{R_1}+\sqrt{\left(\dfrac{h_1}{R_1}\right)^2-1}} \qquad \ldots(2)$$

for circle C_1' and

$$\rho_2=\frac{h_2}{R_2}+\sqrt{\left(\frac{h_2}{R_2}\right)^2-1} \qquad \ldots(3a)$$

where $h_2=\dfrac{D^2-R_1^2+R_2^2}{2D}$ $\ldots(3b)$

for circle C_2'

The co-ordinates of a point on the line PLMNQ representing the semi-infinite plates with a gap is of the form

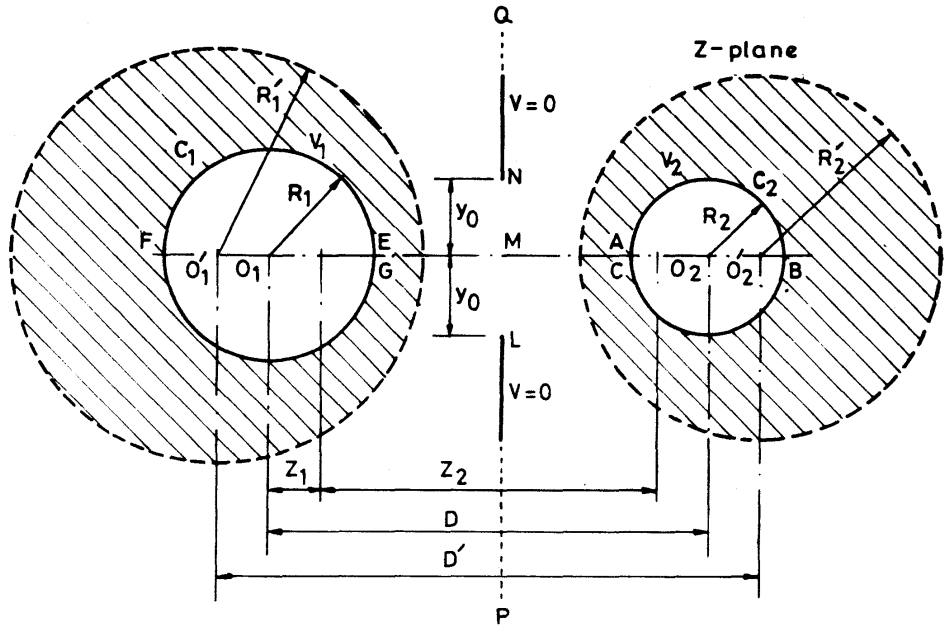

Fig. 1(a) Representation of two dielectric coated cylinders with dielectric boundaries coincident with equipotential contours, together with two semi-infinite plates with a gap in the w(=u+jv) plane

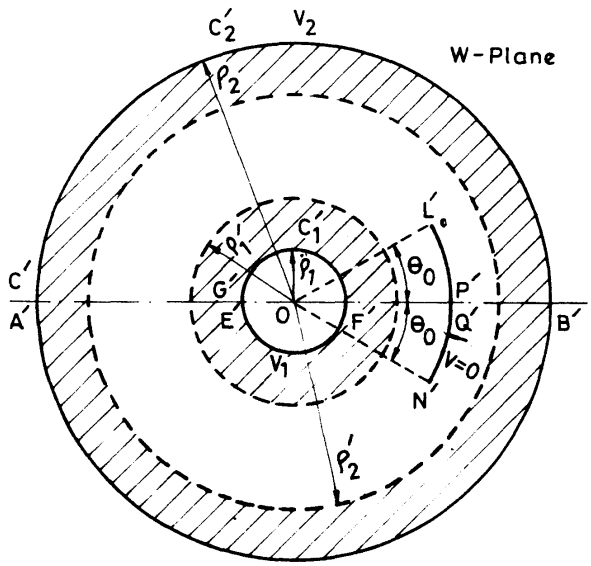

Fig. 1(b) Transformation of Fig. 1(a) to Z-plane

$$w = \frac{\frac{Z_2}{2} + jy}{\frac{Z_2}{2} + jy - Z_2} = \frac{Z_2 + j2y}{-Z_2 + j2y} \qquad \ldots (4)$$

It is found from eq. (4) that the semi-infinte ground plate with a gap is transformed to an arc of a circle of unit radius (ρ=1) with its centre at the origin of the w-plane of Fig. 1(b). If the gap of the ground plane is symmetric with respect to the line joining the centres of the cylinders, the arc strip of unit radius extends from $-\theta_0$ to $+\theta_0$

where $$\theta_0 = \pi - 2\tan^{-1}\left(\frac{2y_0}{Z_2}\right) \qquad \ldots (5)$$

and y_0 =ML=MN of Fig. 1(a).

Application of transformation[6]

$$\xi = \ln w \qquad \ldots (6)$$

transforms the structure of Fig. 1(b) to the configuration shown in Fig. 1(c). The circles C_1' and C_2' are transformed to parallel straight lines located at $\ln \rho_1$ and $\ln \rho_2$ respectively each extending from $-\pi$ to $+\pi$ in Fig. 1(c) and the arc strip is transformed to

$$\xi = \ln(1) + j\xi_{mn} = 0 + j\theta \left(-\pi + \theta_0 \le \theta \le \pi - \theta_0\right) \quad \ldots (7)$$

Dielectric Boundary Coinciding with the Equi-potential Contours

The dielectric coating on the two cylindrical conductors of Fig. 1(a) are so placed that their boundaries coincide with two equipotential contour on the two sides of line PQ having the same set of inverse points Z_1 and Z_2. If R_1' and R_2' are respectively the radii of the circles representing the boundaries of the corresponding dielectric coating and D' is the distance between their centres, the radii ρ_1' and ρ_2' of the concentric transformed circles in Fig. 1(b) are

$$\rho_1' = \frac{1}{\frac{h_1'}{R_1'} + \sqrt{\left(\frac{h_1'}{R_1'}\right)^2 - 1}} \quad \ldots (8)$$

where $h_1' = \dfrac{D'^2 + R_1'^2 - R_2'^2}{2D'}$

and $\rho_2' = \dfrac{h_2'}{R_2'} + \sqrt{\left(\dfrac{h_2'}{R_2'}\right)^2 - 1}$ $\quad \ldots (9)$

where $h_2' = \dfrac{D'^2 - R_1'^2 + R_2'^2}{2D'}$

The transformation (6) transforms the circles having radii ρ_1' and ρ_2' in Fig. 1(b) to straight lines extending from -π to +π at $\xi_{real} = \ln \rho_1'$ and $\xi_{real} = \ln \rho_2'$ as shown in the ξ-plane of Fig. 1(c).

From solution of Laplace's equation, the potential distribution in different regions are obtained for applied potentials V_1 and V_2 to the conductors with respect to ground as follows:

1 for $-\theta_0 \le \theta \le \theta_0$

(a) Region I ($\alpha L''$ P'' Q'' N'' α' shown in Fig. 1(c)).

$$\Phi_1^A = -\frac{V_1 \ln \rho}{\ln \rho_1 + (\varepsilon_r - 1)\ln \rho'_1} + V_1 + \frac{V_1 \ln \rho_1}{\ln \rho_1 + (\varepsilon_r - 1)\ln \rho'_1} \quad \ldots (10)$$

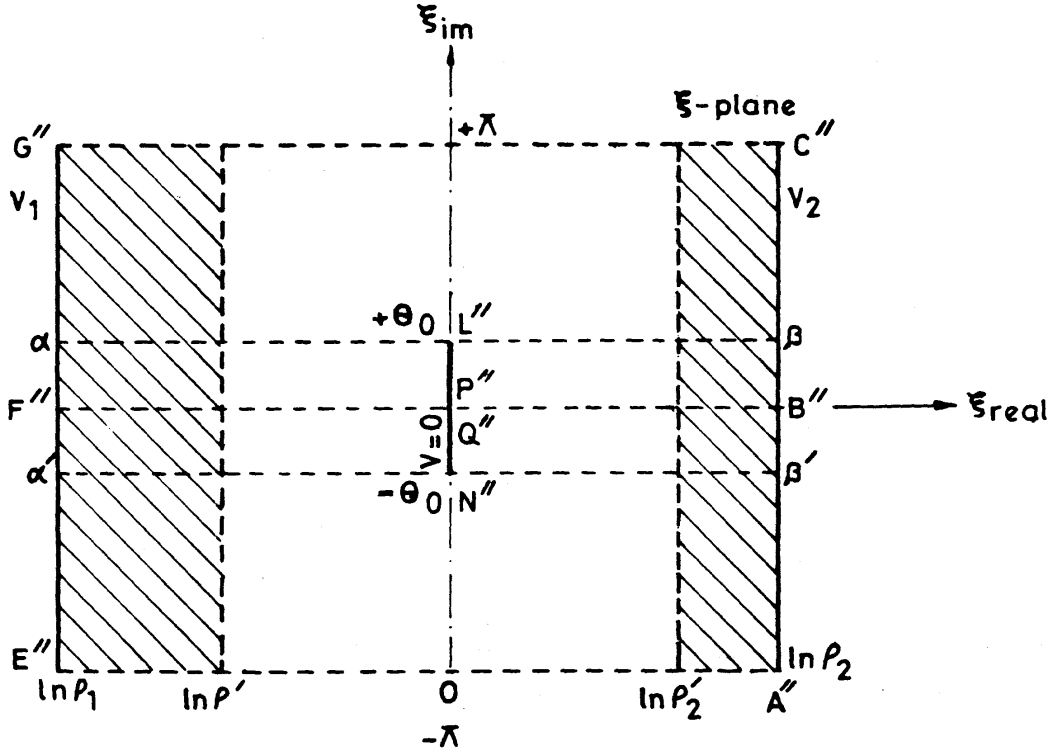

Fig. 1(c) Transformation of Fig. 1(b) to α-plane

(b) Region II ($\beta L''\ P''\ Q''\ N''\ \beta$ shown in Fig. 1(c)).

$$\Phi_{II}^{A} = V_2 \frac{\ln \rho}{\ln \rho_2 + (\varepsilon_r - 1)\ln \rho'_2} + V_2 - \frac{V_2 \ln \rho_2}{\ln \rho_2 + (\varepsilon_r - 1)\ln \rho'_2} \quad \ldots(11)$$

(c) For $-\pi \le \theta \le -\theta_0$ and $\theta_0 \le \theta \le \pi$, the regions are $E''\ A''\ \beta\ N''\ \varepsilon'$ and $\varepsilon\ L''\ \beta\ C''\ G''$ as shown in Fig. 1(c)

(i) With potential V_1 as reference

$$\Phi_I^B = -\frac{(V_1 - V_2)\ln \rho}{\ln \rho_1 + (\varepsilon_r - 1)\ln \frac{\rho'_1}{\rho'_2}} + V_1 + \frac{(V_1 - V_2)\ln \rho_1}{\ln \rho_1 + (\varepsilon_r - 1)\ln \frac{\rho'_1}{\rho'_2}} \quad \ldots(12)$$

(ii) With potential V_2 as reference

$$\Phi_2^B = \frac{(V_1 - V_2)\ln \rho}{\ln \rho_1 + (\varepsilon_r - 1)\ln \frac{\rho'_1}{\rho'_2}} + V_2 - \frac{(V_1 - V_2)\ln \rho_2}{\ln \rho_1 + (\varepsilon_r - 1)\ln \frac{\rho'_1}{\rho'_2}} \quad \ldots(13)$$

Total charge on the conductor at potential V_1 of Fig. 1(a) is given by

$$Q_1 = -\varepsilon_0 \varepsilon_r \left[\frac{\partial}{\partial \rho}\Phi_I^A\right]_{\rho=\rho_1} \rho_1 2\theta_0 - \varepsilon_0 \varepsilon_r \left[\frac{\partial}{\partial \rho}\Phi_I^B\right]_{\rho=\rho_1} \rho_1 2(\pi - \theta_0). \quad \ldots(14)$$

Total charge on the conductor at potential V_2 of Fig. 1(a) is obtained from

$$Q_2 = -\varepsilon_0 \varepsilon_r \left[\frac{\partial}{\partial \rho}\Phi_{II}^A\right]_{\rho=\rho_2} \times \rho_2 (2\theta_0) - \varepsilon_0 \varepsilon_r \left[\frac{\partial}{\partial \rho}\Phi_{II}^B\right]_{\rho=\rho_2} \rho_2 2(\pi - \theta_0) \quad \ldots(15)$$

The elements of the capacitance matrix are obtained using the relations

$$C_{11} = \left.\frac{Q_1}{V_1}\right|_{V_2=0};\ C_{12} = \left.\frac{Q_1}{V_2}\right|_{V_1=0};\ C_{21} = \left.\frac{Q_2}{V_1}\right|_{V_2=0};$$

$$C_{22} = \left.\frac{Q_2}{V_2}\right|_{V_1=0} \quad \ldots(16)$$

It is found that Q_1 and Q_2 depend upon V_1 and V_2. From eqs. (10)-(16), it is found that the expressions for the elements of the capacitance matrix for the structure of Fig. 1(a) are obtained as

$$C_{11} = \varepsilon_0 \varepsilon_r \left[\frac{2\theta_0}{\ln \frac{1}{\rho_1} + (\varepsilon_{r-1})\ln \frac{1}{\rho'_1}} + \frac{2(\pi - \theta_0)}{\ln \rho_2 + \ln \frac{1}{\rho_1} + (\varepsilon_{r-1})\ln \frac{\rho'_2}{\rho'_1}}\right] \quad \ldots(17a)$$

$$C_{22} = \varepsilon_0 \varepsilon_r \left[\frac{2\theta_0}{\ln \frac{1}{\rho_2} + (\varepsilon_{r-1})\ln \frac{1}{\rho'_2}} + \frac{2(\pi - \theta_0)}{\ln \rho_2 + \ln \frac{1}{\rho_1} + (\varepsilon_{r-1})\ln \frac{\rho'_2}{\rho'_1}}\right] \quad \ldots(17b)$$

$$C_{12} = C_{21} = -\varepsilon_0 \varepsilon_r \frac{2\theta_0}{\ln \rho_2 + \ln \frac{1}{\rho_1} + (\varepsilon_{r-1})\ln \frac{\rho'_2}{\rho'_1}} \quad \ldots(17c)$$

The inductance matrix is obtained using the following condition (2)

$$\begin{bmatrix} L_{11} & L_{12} \\ L_{12} & L_{22} \end{bmatrix} \begin{bmatrix} C'_{11} & -C'_{12} \\ -C'_{12} & C'_{22} \end{bmatrix} = \mu_0 \varepsilon_0 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \ldots(18)$$

where C'_{11}, C'_{12}, C'_{22} are obtained from eq. (17) on substituting $\varepsilon_r = 1$. μ_0, ε_0 are respectively the permeability and dielectric constant of free space. From the reciprocity theorem mutual inductance and capacitance $L_{12} = L_{21}$, $C_{12} = C_{21}$ are identical.

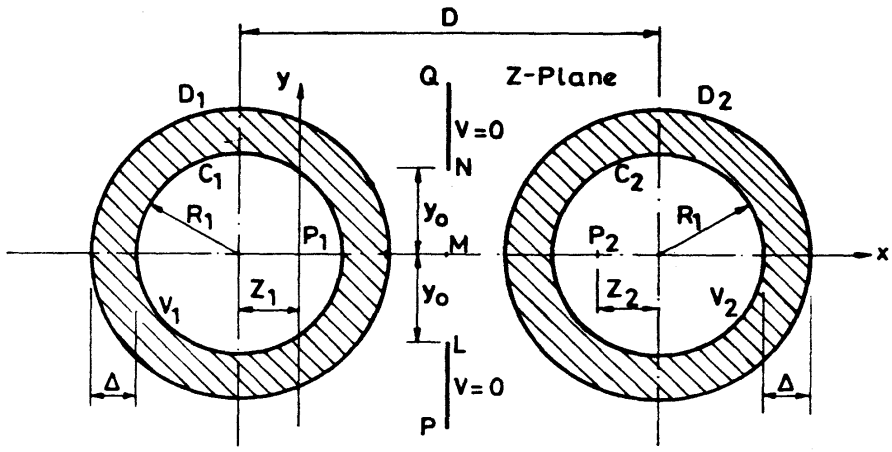

Fig. 2(a) Representation of two dielectric coated cylinders with dielectric boundaries concentric with conductor boundaries together with two semi-infinite plates with a gap in the w (= u+jv) plane

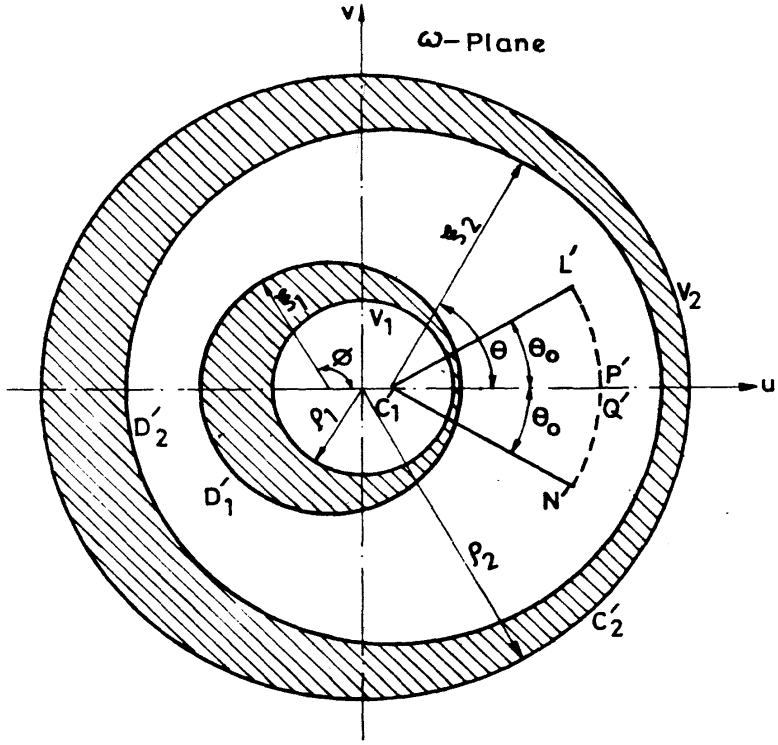

Fig. 2(b) Transformation of Fig. 2(a) to Z-plane

Dielectric Cylinders Concentric with the Cylindrical Conductors

Dielectric boundary concentric with conductor boundaries is shown in Fig. 2(a). In this case, transformation of the dielectric boundaries are eccentric to the transformed configuration as shown in Fig. 2(b).

The transformed boundary of dielectric of width Δ on R_1 is a circle having centre at

$$u_1 = \frac{2R_1\Delta + \Delta^2}{\left(h_1 + \sqrt{h_1'^2 - R_1^2}\right)^2 - (R_1 + \Delta)^2} \qquad \ldots(19)$$

$$v_1 = 0$$

and radius equal to

$$\xi_1 = \left[\left\{\frac{2R_1\Delta + \Delta^2}{\left(h_1' + \sqrt{h_1'^2 - R_1^2}\right)^2 - (R_1 + \Delta)^2}\right\}\right]^2$$

$$+\frac{(R_1+\Delta)^2-\left(h_1'+\sqrt{h_1'^2-R_1^2}\right)^2}{\left(h_1'+\sqrt{h_1'^2-R_1^2}\right)^2-(R_1+\Delta)^2}\Bigg]^{\frac{1}{2}} \quad \ldots(20)$$

The transformed boundary of dielectric of width Δ on R_2 is a circle having centre at

$$u_2=\frac{2R_1\Delta+\Delta^2}{\left(h_1'+\sqrt{h_2'^2-R_1^2}\right)^2-(R_1+\Delta)^2} \quad \ldots(21)$$

$$v_2=0$$

and radius equal to

$$\xi_2=\Bigg[\left\{\frac{2R_2\Delta+\Delta^2}{\left(h_2'+\sqrt{h_2'^2-R_2^2}\right)^2-(R_2+\Delta)^2}\right\}^2$$

$$+\frac{(R_2+\Delta)^2-\left(h_1'+\sqrt{h_1'^2-R_1^2}\right)^2}{\left(h_2'+\sqrt{h_2'^2-R_1^2}\right)^2-(R_2+\Delta)^2}\Bigg]^{\frac{1}{2}} \quad \ldots(22)$$

Since, the dielectric boundaries in the transformed plane are not concentric with the conductor boundaries, it is convenient to apply logarithmic transformation of the form

$\alpha=\alpha_r+j\alpha_i=\ln W$.

By this transformation the conductor boundaries are transformed to parallel straight lines located at $\ln\rho_1$ and $\ln\rho_2$ each of length 2π along the imaginary axis of the α-plane as shown in Fig. 2(c). The dielectric boundaries on the conductors in the W-plane are transformed to curved contours. The location of the ground plane in Fig 2 (c) is the same as that in Fig. 1(c).

The coordinates of the points on the curved contour representing the transformation of the dielectric boundary on the cylinder of radius R_1 are obtained as

$$\alpha_{r1}=\ln\sqrt{(u_1+\xi_1\cos\phi)^2+(\xi_1\sin\phi)^2} \quad \ldots(23)$$

$$\alpha_{i1}=\tan^{-1}\frac{\xi_1\sin\phi}{u_1+\xi_1\cos\phi} \quad \ldots(24)$$

The coordinates of the points on the curved contour representing the transformation of the dielectric boundary on the cylinder of radius R_2 are obtained as

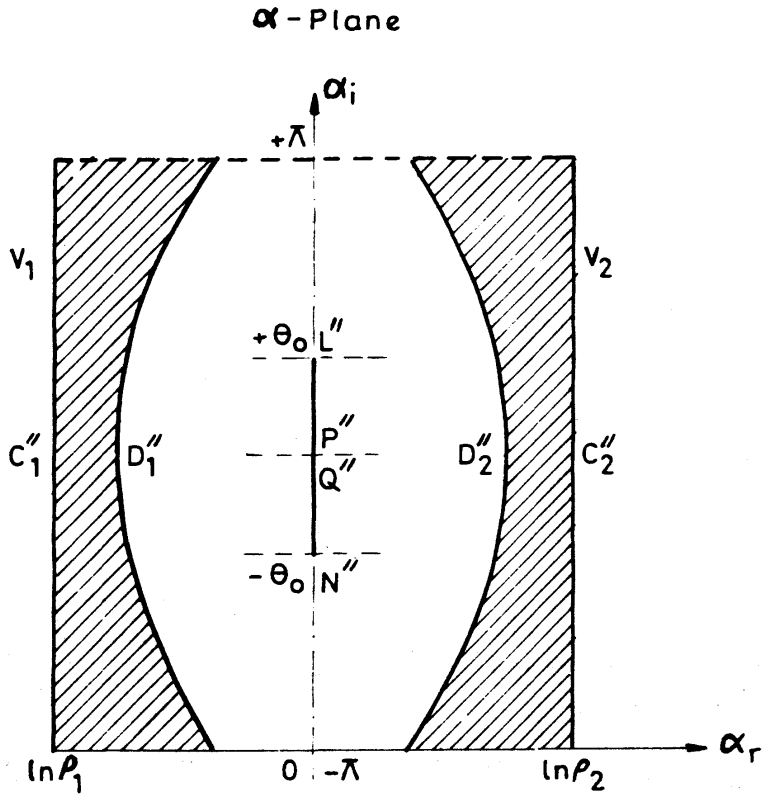

Fig. 2(c) Transformation of Fig. 2(b) to α-plane

$$\alpha_{r2} = \ln\sqrt{(u_2 + \xi_2\cos\phi)^2 + (\xi_2\sin\phi)^2} \quad \ldots(25)$$

$$\alpha_{i2} = \tan^{-1}\frac{\xi_2\sin\phi}{u_{21} + \xi_2\cos\phi} \quad \ldots(26)$$

The symbol ϕ and θ appearing in above equations represent the angles of the radius vectors in the W-plane with the u-axis of Fig. 2(b). Functional dependence of α_{r1}, α_{r2}, α_{i1}, α_{i2} in eqs. (23) and (25) on ϕ and θ is obtained from eqs. (24) and (26) respectively. Representing the above continuous curves by a number of discrete steps and considering the series and parallel combination of the resulting capacitances[4,8] expression for the elements of the capacitance matrix of the structure of Fig. 2(a).

$$C_{11} = \frac{\varepsilon_0\varepsilon_r}{\ln\frac{\rho_2}{\rho_1}}\left[\int_{-\pi}^{-y_0}\frac{d\alpha_i}{1+(\varepsilon_r-1)\ln\left[\frac{\alpha_{r_2(\alpha_i)}}{\alpha_{r_1(\alpha_i)}}\right]\Big/\left[\ln\frac{\rho_2}{\rho_1}\right]} + \int_{y_0}^{\pi}\frac{d\alpha_i}{1+(\varepsilon_r-1)\ln\left[\frac{\alpha_{r_2(\alpha_i)}}{\alpha_{r_1(\alpha_i)}}\right]\Big/\left[\ln\frac{\rho_2}{\rho_1}\right]}\right] + \frac{\varepsilon_0\varepsilon_r}{\ln\rho_1}\int_{-y_0}^{y_0}\frac{d\alpha_i}{1+(\varepsilon_r-1)\ln\left[\frac{\alpha_{r_2(\alpha_i)}}{\alpha_{r_1(\alpha_i)}}\right]\Big/\left[\ln\frac{\rho_2}{\rho_1}\right]} \quad \ldots(27)$$

$$C_{12} = \frac{\varepsilon_0\varepsilon_r}{\ln\frac{\rho_2}{\rho_1}}\left[\int_{-\pi}^{-y_0}\frac{d\alpha_i}{1+(\varepsilon_r-1)\ln\left[\frac{\alpha_{r_2(\alpha_i)}}{\alpha_{r_1(\alpha_i)}}\right]\Big/\left[\ln\frac{\rho_2}{\rho_1}\right]} + \int_{y_0}^{\pi}\frac{d\alpha_i}{1+(\varepsilon_r-1)\ln\left[\frac{\alpha_{r_2(\alpha_i)}}{\alpha_{r_1(\alpha_i)}}\right]\Big/\left[\ln\frac{\rho_2}{\rho_1}\right]}\right] \quad \ldots(28)$$

where y_0 is given by eq. (5).

General Expression for Impedance and Velocities of Eigen Modes

The equation satisfied by voltage column matrix V] and current column matrix I] in a three conductor transmission line is of the form[9].

$$\frac{\partial}{\partial x}V] = -[Z_p]I]$$

$$\frac{\partial}{\partial x}I] = -[Y_p]V]$$

where square matrices $[Z_p]$ and $[Y_p]$ are of the form

$$[Z_p] = j\omega\begin{bmatrix} L_{11} & L_{12} \\ L_{12} & L_{22}\end{bmatrix} \quad \ldots(29)$$

$$[Y_p] = j\omega\begin{bmatrix} C_{11} & -C_{12} \\ -C_{12} & C_{22}\end{bmatrix} \quad \ldots(30)$$

From relation between column matrices for voltage and current and eqs. (29) and (30) it is found that the column matrices for voltage and current satisfy the following equations

$$\frac{\partial^2}{\partial x^2}V] = [Z_p][Y_p]V] = \gamma^2 V] \quad \ldots(31)$$

$$\frac{\partial^2}{\partial x^2}I] = [Z_p][Y_p]I] = \gamma^2 I] \quad \ldots(32)$$

where V and I have x variation as $e^{-\gamma x}$.

The eqs. (31) and (32) are eigenvalue equations. The eigenvalues γ are found from the solution of characteristic equation

$$\det\left([Z_p][Y_p] - \gamma^2[U]\right) = 0 \quad \ldots(33)$$

where [U] is unit matrix.

Substituting eqs. (29) and (30) in eq. (33), the eigen values are obtained as

$$\gamma_1 = \frac{j\omega}{\sqrt{2}}\left[(a+d) + \sqrt{(a+d)^2 - 4(ad-bc)}\right]^{\frac{1}{2}} = j\beta_1 \quad \ldots(34)$$

$$\gamma_2 = \frac{j\omega}{\sqrt{2}}\left[(a+d) - \sqrt{(a+d)^2 - 4(ad-bc)}\right]^{\frac{1}{2}} = j\beta_2 \quad \ldots(35)$$

where $a = L_{11}C_{11} - L_{12}C_{12}$, $b = L_{12}C_{22} - L_{11}C_{12}$, $c = L_{12}C_{11} - L_{22}C_{12}$, $d = L_{12}C_{12} - L_{22}C_{22}$.

Substituting γ_1 and γ_2 in eqs. (31) and (32) solution of homogeneous equations, which are the eigenvectors corresponding to eigenvalues γ_1 and γ_2 are found using the method suggested in the literature [7]. If [M] is the matrix of the eigen vectors also called the modal matrix [7], the transformation to normal co-ordinates V'] and I'] is obtained as

$V]=[M]V']$ and $I]=[M]I']$

and their substitution in the transmission line equations relating the column matrices for voltage and current leads to the relation[7].

$$\begin{bmatrix} V_1' \\ V_2' \end{bmatrix} = \begin{bmatrix} \gamma_1 & 0 \\ 0 & \gamma_2 \end{bmatrix}^{-1} [M]^{-1} [Z_p] [M] \begin{bmatrix} I_1' \\ I_2' \end{bmatrix} \quad \ldots(36)$$

In deriving eq. (36) use is made of voltage and current variation as $e^{-\gamma x}$.

It will be shown that in cases of practical interest the matrix product $[M]^{-1}[Z_p][M]$ leads to a diagonal matrix. In such a situation the product of diagonal and square matrices on the right hand side of eq. (36) is a diagonal matrix, whose diagonal elements are the modal impedances Z_{m1} and Z_{m2}, and it assumes the form

$$\begin{bmatrix} V_1' \\ V_2' \end{bmatrix} = \begin{bmatrix} Z_{m1} & 0 \\ 0 & Z_{m2} \end{bmatrix} \begin{bmatrix} I_1' \\ I_2' \end{bmatrix} \quad \ldots(37)$$

The modal velocities V_{m1} and V_{m2} are found from eqs. (34) and (35) as

$$V_{m1} = \frac{\omega}{\beta_1} = \frac{\sqrt{2}}{\sqrt{a+d+\sqrt{(a+b)^2-4(ad-bc)}}} \quad \ldots(38)$$

$$V_{m2} = \frac{\omega}{\beta_2} = \frac{\sqrt{2}}{\sqrt{a+d-\sqrt{(a+b)^2-4(ad-bc)}}} \quad \ldots(39)$$

Modal Parameters of Symmetrical System

For cylinders of identical radii coated with identical dielectric width

$$R_1 = R_2 = R_0 \qquad R_1' = R_2' = R_0' \quad \ldots(40)$$

and the structure is symmetrical with respect to semi-infinite plates with a gap. Under this condition

$$C_{11} = C_{22} \text{ and } L_{11} = L_{22} \quad \ldots(41)$$

When condition as in eq. (41) is satisfied it is found that [L], [C] and matrix product [L] [C] commute with the geometrical matrix[10]

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \ldots(42)$$

If two matrices commute, they have the same eigenvectors.

Hence, the eigenvectors of [L], [C] and hence [Z_p], [Y_p] matrices are obtained from the eigenvectors of matrix (42), which is of the form

$$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \quad \ldots(43)$$

The first column of eq. (43) represents the even mode excitation while the second column represents the odd mode excitation. From eqs. (31), (32), (35-40) and (43) it can be shown that for $\varepsilon_r \neq 1$

$$V_{even} = \frac{1}{\sqrt{(L_{11}+L_{12})(C_{11}-C_{12})}} = \frac{1}{\sqrt{L_{even}C_{even}}} \quad \ldots(44)$$

$$V_{odd} = \frac{1}{\sqrt{(L_{11}-L_{12})(C_{11}+C_{12})}} = \frac{1}{\sqrt{L_{odd}C_{odd}}} \quad \ldots(45)$$

and $$Z_{even} = \sqrt{\frac{L_{11}+L_{12}}{C_{11}-C_{12}}} = \sqrt{\frac{L_{even}}{C_{even}}} \quad \ldots(46)$$

$$Z_{odd} = \sqrt{\frac{L_{11}-L_{12}}{C_{11}+C_{12}}} = \sqrt{\frac{L_{odd}}{C_{odd}}} \quad \ldots(47)$$

where $L_{even}, L_{odd}, C_{even}, C_{odd}$ are respectively even and odd mode inductance and capacitance pre unit length respectively.

When $\varepsilon_r = 1$, it is found from eq. (18)

$$L_{12}C_{11}' - L_{11}C_{12}' = 0 \text{ or } \frac{L_{12}}{L_{11}} = \frac{C_{12}'}{C_{11}'} \quad \ldots(48)$$

and $$V_{even} = V_{odd} = V_c = \frac{1}{\sqrt{L_{11}C_{11} - L_{12}C_{12}}}$$

where V_c is the velocity in the coupled lines. From eqs. (46)-(48) it is found that

$$Z_{even} = \sqrt{\frac{L_{11}}{C_{11}'}} \sqrt{\frac{1+\frac{L_{12}}{L_{11}}}{1-\frac{C_{12}'}{C_{11}'}}}$$

$$Z_{odd} = \sqrt{\frac{L_{11}}{C_{11}'}} \sqrt{\frac{1-\frac{L_{12}}{L_{11}}}{1+\frac{C_{12}'}{C_{11}'}}}$$

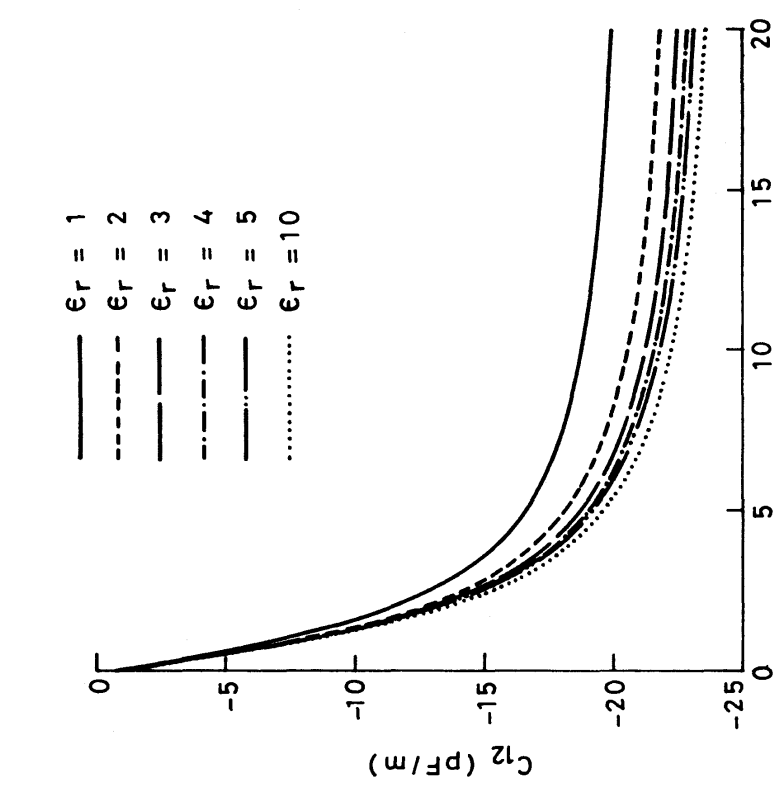

Fig. 4 Variation of C_{12} of the structure of Fig. 1(b) with semi-width of the gap for R_1=1, R'_1=2, D=4, D'= $2\sqrt{7}$, ε_r =1,2,3,4,5,10

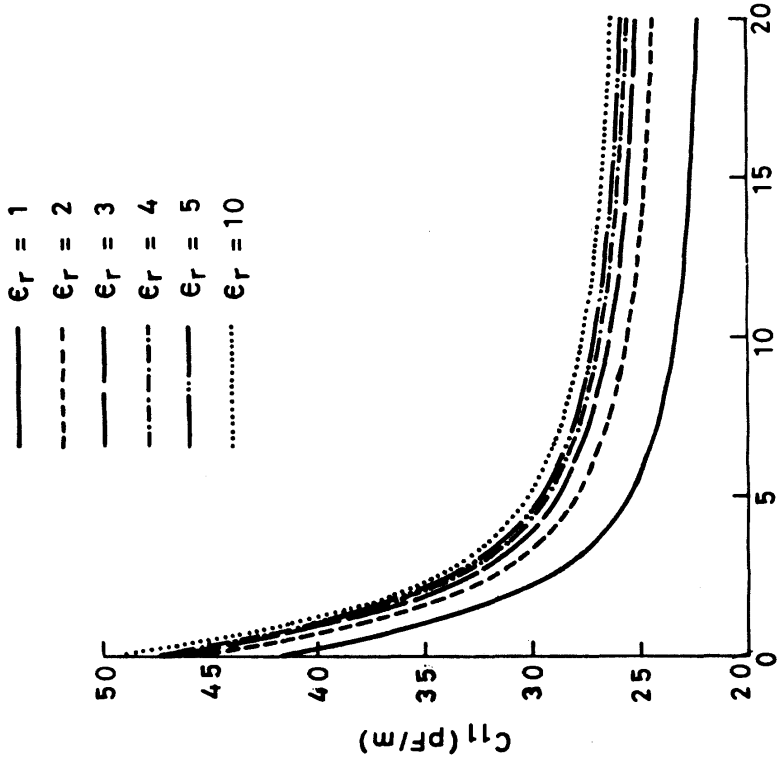

Fig. 3 Variation of C_{11} of the structure of Fig. 1(a) with semi-width of the gap for R_1=1, R'_1=2, D=4, D'= $2\sqrt{7}$, ε_r =1,2,3,4,5,10

$$Z_{even} \times Z_{odd} = \frac{L_{11}}{C_{11}} = Z_0^2 .$$

Numerical Results

Using eqs. (1)-(9) and (17), the variation of $C_{11}(=C_{22})$ and $C_{12}(=C_{21})$ is evaluated as a function of y_0 for the structure of Fig. 1(a) for $R_1 = R_2 = R_0 = 1, D=4, R_1' = R_2' = R_0' = 2, D' = 2\sqrt{7}$, $\varepsilon_r = 1,2,3,4,5,10$. The results are presented in Figs. 3 and 4. Using the data so obtained for $\varepsilon_r = 1$ in eq. (18), the elements of the inductance matrix are evaluated. It is worthwhile to mention that the elements of the inductance matrix do not depend upon ε_r.

Using eqs. (5), (19)-(28), the variation of elements of capacitance matrix for concentric dielectric are evaluated as a function of y_0 for the structure of Fig. 2(a) for $R_1 = R_2 = R_0 = 1$, D=4, Δ=1 (touching dielectric). The results are presented in Figs. 5 and 6.

Using the relations appearing in eqs. (44)-(47), and the data of Figs. 3 and 4, the variation of $L_{even}, L_{odd}, C_{even}, C_{odd}$ with y_0 are evaluated for the structure of Fig. 1(a) and the results are presented in Figs. 7, 8 and 9. Using the same set of equations and the data of Figs. 5 and 6, the variation of even and odd mode parameters and evaluated for the structure Fig. 2(a). The results are presented in Figs. 10 and 11.

Discussion

The results presented in Figs. 3 to 6 reveal that when the width of the gap in large, the self and mutual capacitances are almost equal in magnitude. The variation of the elements of the capacitance matrix with change in the width of the gap and ε_r is more pronounced when the dielectric boundaries are concentric with the metallic boundaries.

The odd mode inductances and capacitances are independent of the width of the gap between the semi-infinite plates. This can be attributed to the fact that in the case of odd mode excitation, the electric wall or short circuit appears in the plane of symmetry. The corresponding electrical parameters do not, therefore, depend upon the width of

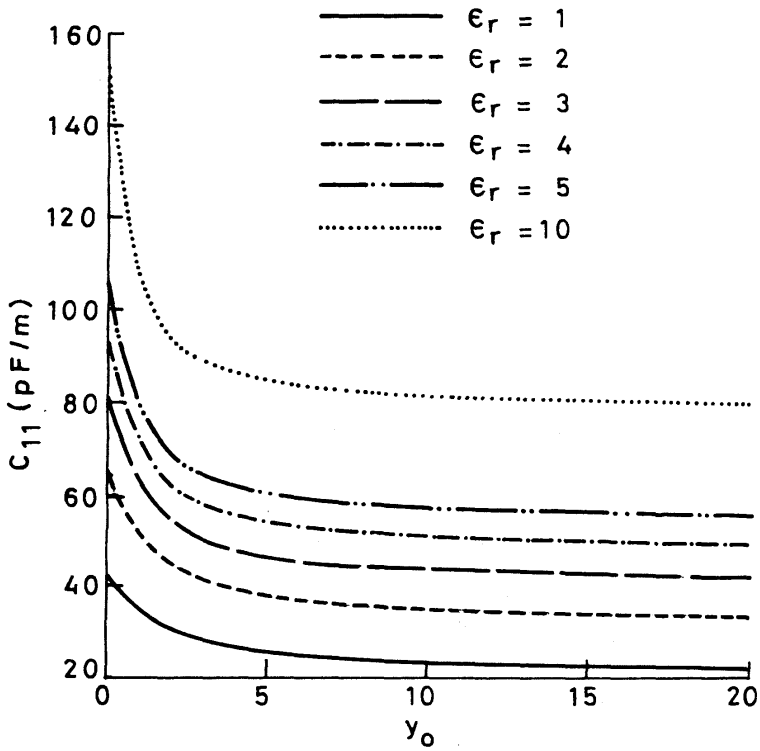

Fig. 5 Variation of C_{11} of the structure of Fig. 2(a) with semi-width of the gap for R_1=1, Δ=1, D=4, ε_r=1,2,3,4,5,10

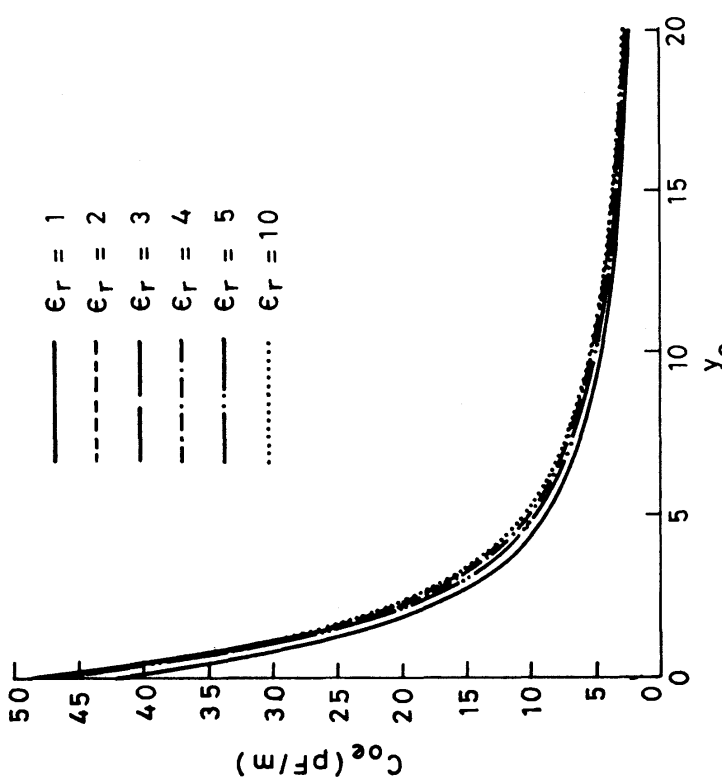

Fig. 7 Variation of L_{even} and L_{odd} for the structure considered with semi-width of the gap for R_1=1, D=4, ε_r=1 (Inductance matrix parameters are independent of dielectric and hence are identical of Figs. 1(a) and 2(a)

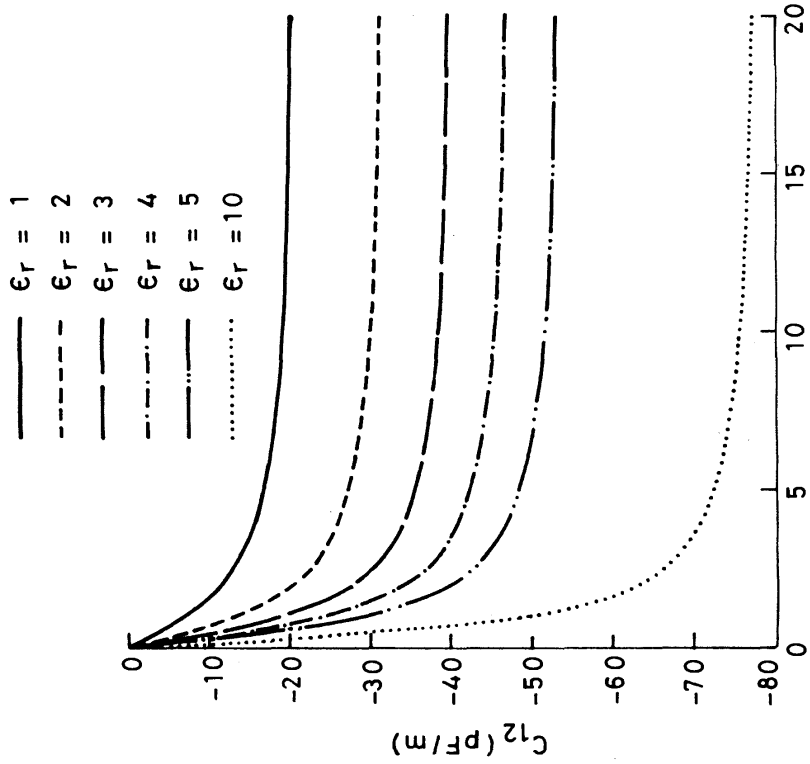

Fig. 6 Variation of C_{12} of the structure of Fig. 2(b) with semi-width of the gap for R_1=1, Δ=1, D=4, ε_r=1,2,3,4,5,10

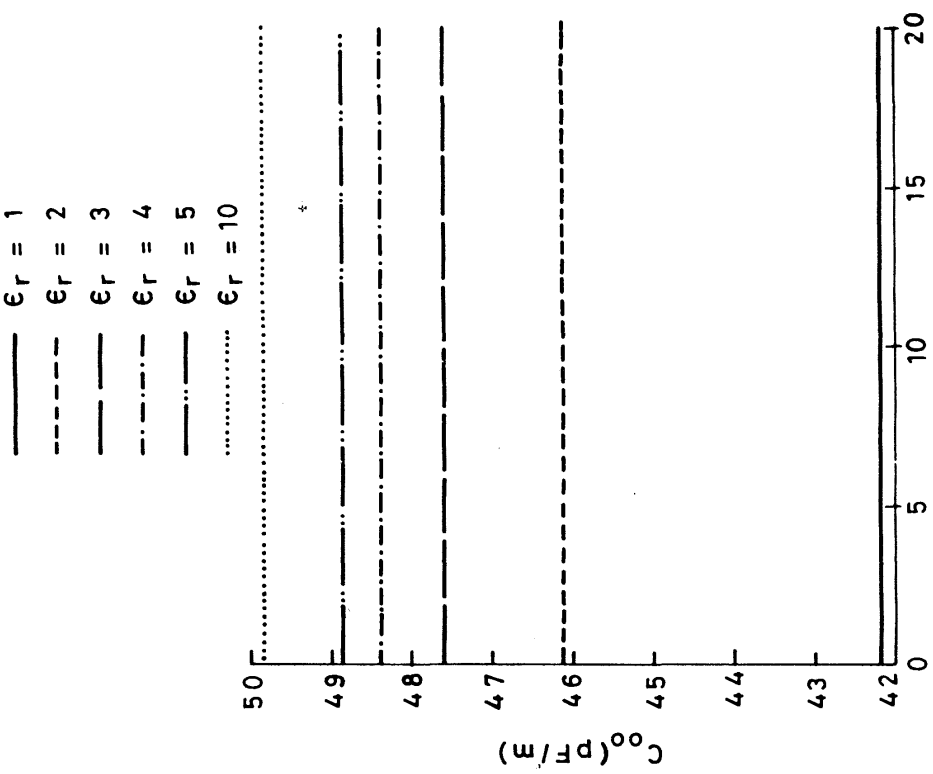

Fig. 8 Variation of even mode capacitance with semi-width of the gap for the structure of Fig. 1(a) for R_1=1, R'_1=2, D=4, $D'=2\sqrt{7}$, ε_r=1,2,3,4,5,10

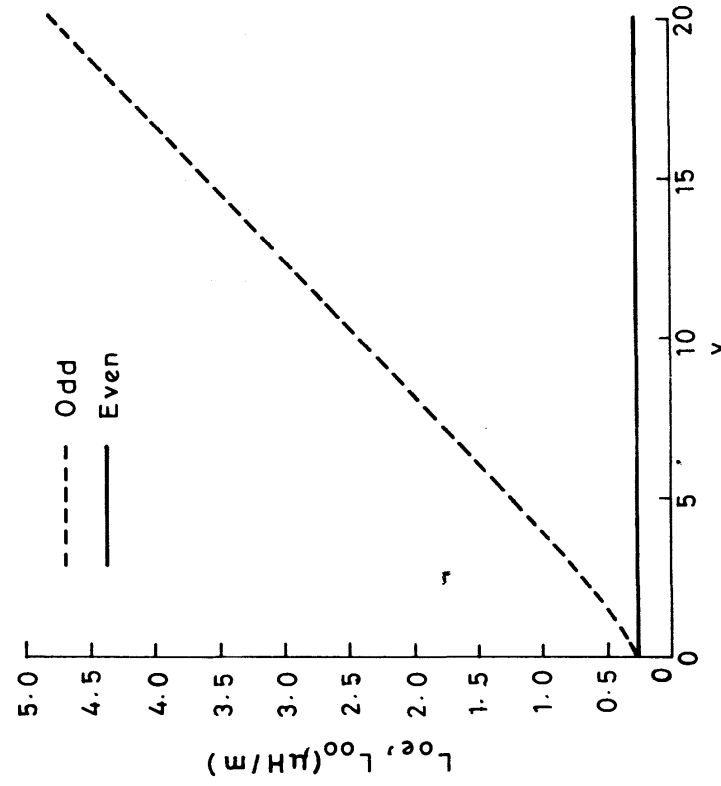

Fig. 9 Variation of odd mode capacitance with semi-width of the gap for the structure of Fig. 1(a) for R_1=1, R'_1=2, D_1=4, $D_2=2\sqrt{7}$, ε_r=1,2,3,4,5,10

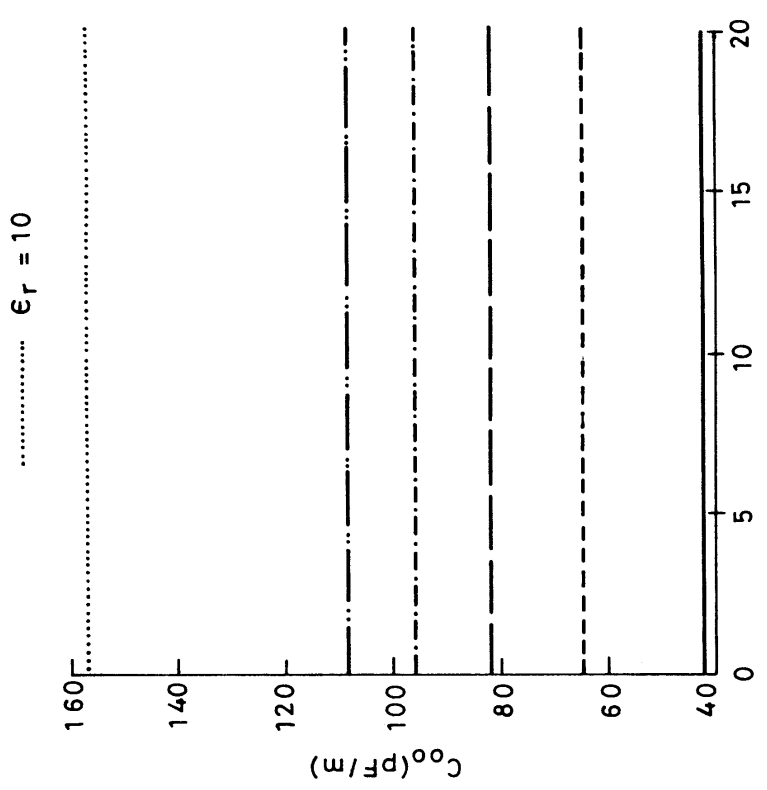

Fig. 11 Variation of odd mode capacitance with semi-width of the gap for the structure of Fig. 2(a) for $R_1=1$, $\Delta=1$, $D=4$, $\varepsilon_r=1,2,3,4,5,10$

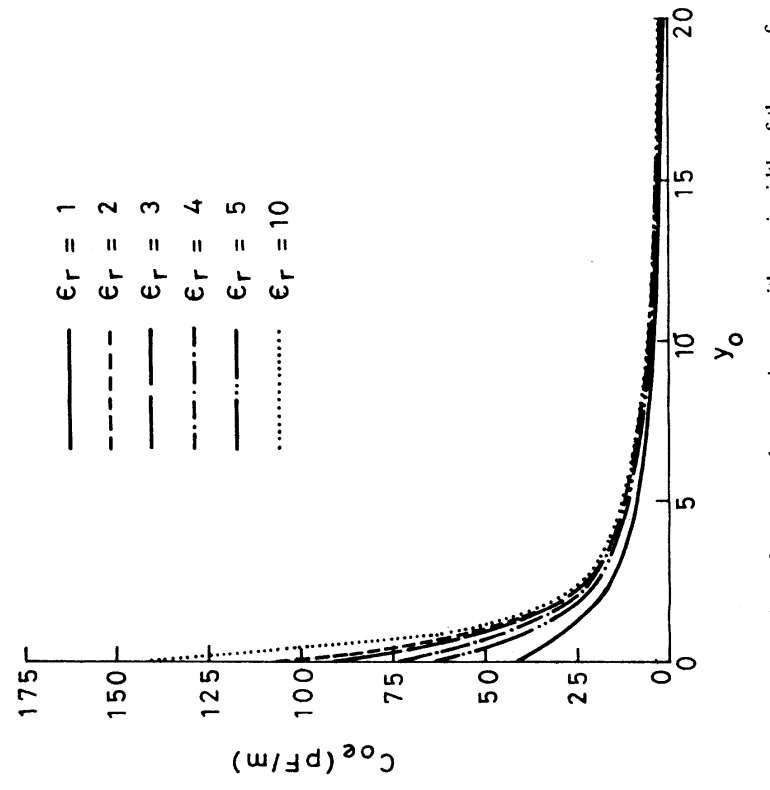

Fig. 10 Variation of even mode capacitance with semi-width of the gap for the structure of Fig. 2(a) for $R_1=1$, $\Delta=1$, $D=4$, $D'=2\sqrt{7}$, $\varepsilon_r=1,2,3,4,5,10$

the gap. The odd mode velocity and impedance are consequently independent of the width of the gap between the semi-infinite plates. They depend upon the value of the dielectric constant.

From the examination of the data presented in Figs. 7 to 11, it is found that even and odd mode velocities are identical for all values of the width of the gap when the dielectric boundary coincides with equipotential boundary. The even and odd mode impedances are, however, different. When the dielectric boundary is concentric with the conductor boundary, the even and odd mode impedances and velocities are different.

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