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HYDRAULIC GEOMETRY AND RESISTANCE OF GRAVEL-BED RIVERS

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The knowledge about the hydraulic geometric parameters viz. width, depth and area of the river at the bankful discharge are required for solving a variety of problems related to river training, location of bridges, and navigation. Further, the resistance offered to the flow by the boundary and air-water interface needs to be known in order to predict the average velocity of flow. This paper describes the results of the analysis of all the available gravel-bed river data, which has resulted in methods for the prediction of width, depth, area and the flow velocity or resistance coefficient.

Key Words: Gravel-Bed Rivers; Hydraulic Geometry; Resistance Relationships; Paved Beds; Mobile Bed

Introduction

Gravel-bed rivers are those rivers, which flow through very coarse material in the range of gravel, cobbles and boulders. However, they also carry certain amount of fine material along with the flow. These rivers differ significantly from the alluvial rivers flowing through sandy material. Some of the characteristics which differentiate gravel-bed rivers from the alluvial rivers are:

(i) much steeper slope (0.001 to 0.02 or even higher);

(ii) formation of a layer of relatively coarse material on the surface which is known as the pavement;

(iii) relatively high Froude number $F_r = U/\sqrt{gh}$ (The flow in some cases can be subcritical with F_r less than unity and in other cases supercritical with F_r greater than unity); here U is the average velocity and h is the average depth of flow;

(iv) absence of ripples and dunes commonly found in sand-bed rivers but large bed features such as bars, riffles and pools can occur in gravel-bed rivers;

(v) relatively less information is available about hydraulic geometry, resistance, and sediment transport in gravel-bed rivers than in sand-bed rivers.

It is only in the last three decades that some attempts have been made to collect data on gravel-bed rivers in order to study their characteristics. The two aspects of gravel-bed rivers that are of relevance to this study are their hydraulic geometry and their resistance characteristics. The hydraulic geometry refers to the geometrical characteristics of the cross-section such as the average width W, average depth h and area A $(=Wh)$ at the bankful discharge Q. The knowledge about the hydraulic geometry is required in all problems of river management such as navigation, location of bridges and barrages, and river trainings works. The works of Lacey[1] and Inglis[2] about the sand-bed rivers have shown that for such rivers

$$\text{Depth } h \text{ or hydraulic radius } R \sim \left(\frac{Q}{f_1}\right)^{\frac{1}{3}}$$

$$\text{Width } W \text{ or wetted perimeter } P \sim Q^{0.50}$$

$$\text{Area } A \sim \frac{Q^{\frac{5}{6}}}{f_1^{\frac{1}{3}}}$$

where f_1 is Lacey's silt factor and is given by $f_1 = 1.76\sqrt{d}$, d being the median size of bed material in mm. Power relations have also been suggested by Leopold and Maddock[3], and Langbein[4], whereas Williams[5] has obtained such relationships using the sediment transport and diffusion type relationship.

As regards the gravel-bed rivers, Kellerhals[6], Bray[7] and Hey[8] have related W, h, A to Q and sediment size d. Thus according to Bray,

$$W = 2.08\ Q^{0.528}\ d^{-0.70}$$

$$h = 0.256\ Q^{0.331}\ d^{-0.25}$$

All such equations are based on the analysis of limited amount of data and are not dimensionally homogeneous. Only Parker[9] has made use of dimensionless parameters W/d, h/d, and $\frac{U}{\sqrt{\Delta\gamma_s d/\rho_f}}$ and related them to the dimensionless discharge $Q/d^2\sqrt{\Delta\gamma_s d/\rho_f}$. Here $\Delta\gamma_s$ is the difference in specific weights of sediment and water and ρ_f is the mass density of water. Hence there is scope of using all the available gravel-bed river data and develop non-dimensional relationships for the hydraulic geometry.

In the analysis of river and channel problems one also needs a relationship between the average velocity U, the depth h or the hydraulic radius R, channel slope S and some coefficient which is related to the channel boundary. This is known as the resistance relationship. It may be mentioned that the hydraulic radius R is defined as the flow area A divided by the channel perimeter P and if channel is sufficiently wide $R \approx h$, which is used herein. The following are some of the forms in which the resistance relationship is expressed in dimensionless form.

Manning's equation

$$\frac{U}{\sqrt{ghs}} = \frac{h^{\frac{1}{6}}}{n\sqrt{g}} \qquad \ldots(1)$$

Chezy's equation

$$\frac{U}{\sqrt{ghS}} = \frac{C}{\sqrt{g}} \qquad \ldots(2)$$

Darcy-Weisbach equation

$$\frac{U}{\sqrt{ghS}} = \sqrt{\frac{8}{f}} \qquad \ldots(3)$$

$$\frac{U}{\sqrt{gdS}} = F\left(\frac{h}{d}\right) \qquad \ldots(4)$$

In the above equations n is Manning's roughness coefficient, C is Chezy's discharge coefficient, f is Darcy-Weisbach resistance coefficient and F is a function. These coefficients depend on the resistance offered to the flow by the channel boundary and air-water interface.

Even though some studies have been conducted to examine these aspects about gravel-bed rivers, they suffer from certain deficiencies. Firstly most of these studies are based on limited amount of data. Then except in Parker's studies about hydraulic geometry, all others have used dimensional relations. And lastly the available relations for velocity are not sufficiently accurate. Therefore, this study was undertaken in which all the available data have been analysed in a unified manner to obtain dimensionally homogeneous relationships for W, h, A and U.

Data

A summary of the data used in the analysis is given in Table I. The data were classified in two broad categories, namely bankful discharge and variable discharge. The bankful discharge data were used to study the hydraulic geometry. The variable discharge data pertain to discharges other than the bankful in any stream. Both the sets of data were used for the resistance analysis. In order to study the effect of bed condition on these relationships, each set of data were further subdivided into those with mobile bed, and those with paved bed. However it was found during the analysis that both these sets of data behaved in similar manner and there is no need to subdivide the data in this manner. This implies that the bed undulations for these data were not very pronounced.

Analysis of Data and Discussion of Results

(a) Analysis of Hydraulic Geometry

Since prediction of hydraulic geometry is not amenable to theoretical analysis, one can use the dimensional analysis to advantage. The dependent variables can be any two of the four variables average width W, average depth h, area of flow A (=Wh) and the average velocity U. The independent variable related to the flow is bankful discharge Q. The sediment representing the bed and the banks will be described by the median size of the bed material d, its geometric standard deviation σ_g and the difference in the specific weights of sediment and water $\Delta\gamma_s$. The fluid is characterised by its mass density ρ_f and dynamic viscosity μ. Some thought needs to be given to the inclusion or exclusion of channel slope S and sediment

Table I

Summary of the field data used in the analysis

Sr. No.	Investigator	Country	Number of Streams/ Observations	Range of Q (m^3/s)	Range of Slope S	Range of bed size d (m)	Bed/Discharge condition
1	Williams[5]	U.S.A.	28	1.7 - 580.6	0.00085 - 0.0416	0.021 - 0.19	Mobile, Bankful Q.
2	Williams[5]	U.S.A.	23	1.7 - 580.6	0.00085 - 0.0416	0.021 - 0.19	Mobile, Bankful Q.
3	Leopold and Wolman	U.S.A.	15	0.55 - 179.3	0.0002 - 0.036	0.034 - 0.50	Mobile, Bankful Q.
4	U.S.B.R.[11]	San Luis Valley canals	14	0.5 - 42.4	0.008 - 0.00965	0.021 - 0.082	Paved, Bankful Q.
5	Kellerhals[6]	Canada	17	21.2 - 3823	0.00017 - 0.0065	0.007 - 0.265	Paved, Bankful Q.
6	Bray[12]	Alberta, Canada	67	5.5 - 8212	0.00022 - 0.015	0.027 - 0.145	Paved, Bankful Q.
7	Samide[13]	Canada	54	Variable	0.00154 - 0.00745	0.014 - 0.076	Mobile-bed, Variable Q.
8	Milhaus[14]	Oak creek, U.S.A.	17	Variable	0.0079 - 0.0125	0.008 - 0.027	Mobile-bed, Variable Q.
9	Griffiths[15]	New Zealand	52	Variable	0.00119 - 0.00714	0.012 - 0.051	Mobile-bed, Variable Q.
10	Thorne and Zevenbergen[16]	Boulder Creek, U.S.A	12	Variable	0.00143 - 00198	0.130 - 0.162	Mobile-bed, Variable Q.
11	Michalik[17]	Poland for two rivers	6	Variable	0.0002 - 0.0004	0.031 - 0.274	Mobile-bed, Variable Q.
12	Colosimo *et al*[18]	South Italy	43	Variable	0.0032 - 0.019	0.05 - 0.12	Mobile-bed, Variable Q.
13	Gladky[19]	Poland, Raba R.	30	Variable	0.00074 - 0.00087	0.013	Mobile-bed, Variable Q.
14	Church and Rood[20]		50	Variable	0.002 - 0.081	0.011 - 0.268	Mobile-bed, Variable Q.
15	Hey and Thorne[21], Hey[22]	U.K.	62	Variable	0.00119 - 0.01522	0.014 - 0.176	Mobile-bed, Variable Q.
16	Griffiths[15]	New Zealand	46	upto 1540	0.0009 - 0.011	0.013 - 0.301	Paved bed, Variable Q.
17	Bathrust[23]	U.K.	9	upto 6.0	0.0081 - 0.0116	0.212	Paved bed, Variable Q.
18	Thompson and Campbell[24]	New Zealand	26	2.7 to 411	0.0037 - 0.052	0.03 - 0.47	Paved bed, Variable Q.
19	Kellerhals[6]	Canada	3	24 - 183	0.00059 - 0.002	0.07 - 0.131	Paved bed, Variable Q.

transport rate Q_B. It is known that for a given stream the channel slope is related to the bankful discharge Q, the slope decreasing as Q increases in the downstream direction. However, when one deals with the data from different countries and basins, S and Q will not be related and hence S should be taken as an independent variable. As regards the sediment load, it is reported that even at the bankful stage with mobile bed, the gravel-bed rivers carry a small amount of sediment load. Hence Q_B can be ignored from the analysis and the data at bankful discharge with paved as well as mobile bed can be analysed together. Hence one can write

$$W, h, A, U = F(Q, d, \sigma_g, \Delta\gamma_s, \rho_f, \mu, S) \quad \ldots(5)$$

Choosing ρ_f, d and $\Delta\gamma_s$ as the repeating variables. Eq. (5) can be written in dimensionless for as

$$\frac{W}{d}, \frac{h}{d}, \frac{A}{d^2}, \frac{U}{\sqrt{\frac{\Delta\gamma_s}{\rho_f} d}} = F\left(\frac{Q}{d^2\sqrt{\frac{\Delta\gamma_s}{\rho_f} d}}, \sigma_g, \frac{\Delta\gamma_s \rho_f d^3}{\mu^2}, S\right) \quad \ldots(6)$$

As a matter of simplification it is assumed that σ_g has an insignificant effect on the hydraulic geometry. Further, since we are dealing with coarse material, the parameter involving viscosity, viz. $\Delta\gamma_s d^3 \rho_f / \mu^2$ will not have any effect on the channel adjustment process.

With these simplifications, eq. (6) can be written as

$$\frac{W}{d},\frac{h}{d},\frac{A}{d^2},\frac{U}{\sqrt{\frac{\Delta\gamma_s}{\rho_f}d}}=F\left(\frac{Q}{d^2\sqrt{\frac{\Delta\gamma_s}{\rho_f}d}},S\right) \quad ...(7)$$

In analysing the data using eq. (7), one can explore three possibilities of combining $Q\Big/d^2\sqrt{\frac{\Delta\gamma_s}{\rho_f}d}$ and S, viz. omit S and use only $Q\Big/d^2\sqrt{\frac{\Delta\gamma_s}{\rho_f}d}$ as done by Parker, use dimensionless streampower $QS\Big/d^2\sqrt{\frac{\Delta\gamma_s}{\rho_f}d}$ as suggested by Chang, or use $Q\Big/d^2\sqrt{\frac{\Delta\gamma_s}{\rho_f}dS}$ as done by the Russian investigators such as Ananian and Velikanov.

Preliminary analysis indicated that the division of data into mobile bed and paved bed categories does not increase the accuracy of relationships for the prediction of hydraulic geometry or resistance. Hence all the data at bankful stage are treated together. Initially regime type relations were studied by plotting W, h and A against Q on log - log scale which yielded almost straight lines giving equations

$$W = 4.547Q^{0.507}$$
$$h = 0.293Q^{0.332} \quad ...(8)$$
$$A = 1.330Q^{0.839}$$

It may be mentioned that the exponents of Q obtained in eq. (8) are very close to those obtained by Lacey[1] for alluvial channels; however, the coefficients are different. In order to check their accuracy two methods were used. First the per cent of data points falling within ± 50 and ± 30 per cent error lines were determined. Secondly, the square of standard error

$$SSE = \frac{\sum(x_o - x_c)^2}{N-2}$$

where x_o and x_c are the observed and computed values and N is the total number of data points was calculated. These values are tabulated in Table II. Fig. 1 shows variation of h with Q.

Similar exercise was carried out using W/d, h/d and A/d^2 and determining their variation with $Q_1 = Q/d^2\sqrt{\frac{\Delta\gamma_s}{\rho_f}d}$ by plotting on log-log scales. The

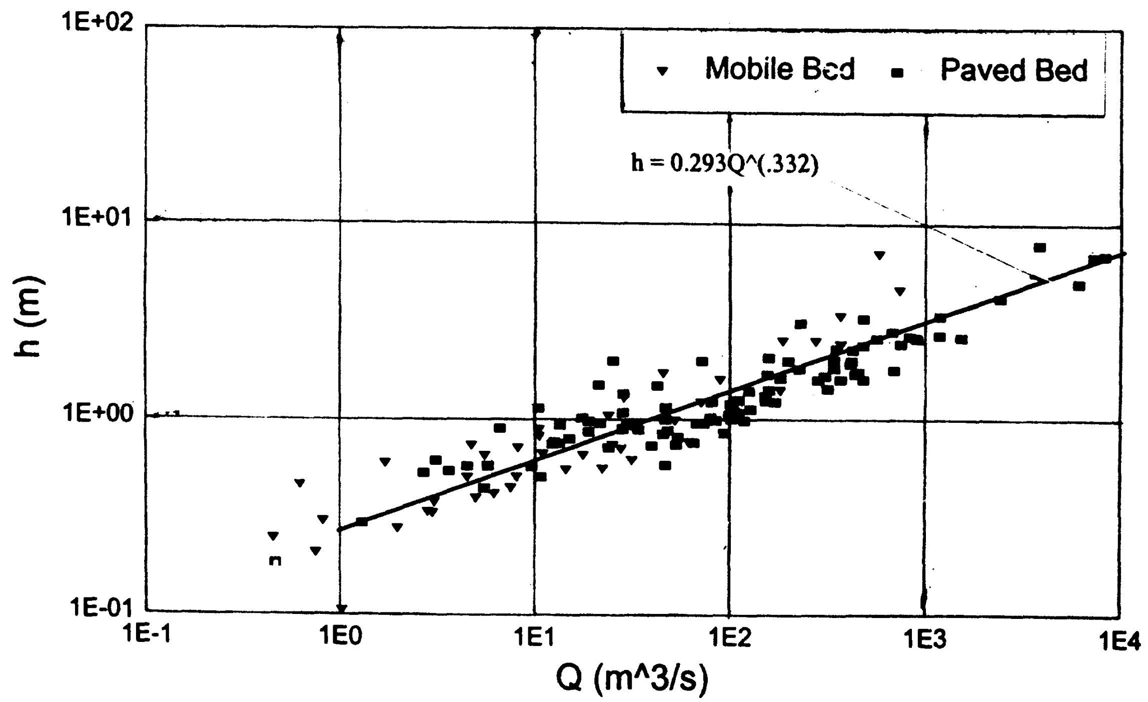

Fig. 1 Variation of h with Q Bankful Discharge

relationships obtained were

$$\frac{W}{d} = 7.675Q_1^{0.448}$$
$$\frac{h}{d} = 0.504Q_1^{0.373} \qquad \ldots(9)$$
$$\frac{A}{d^2} = 3.872Q_1^{0.821}$$

Fig. 2 shows variation of h/d with Q_1. Similar exercise was carried out using $Q_2 = QS\Big/\sqrt{\frac{\Delta\gamma_s}{\rho_f}d}$ as the independent parameter and the corresponding equations obtained. These are

$$\frac{W}{d} = 88.928Q_2^{0.505}$$
$$\frac{h}{d} = 3.888Q_2^{0.420} \qquad \ldots(10)$$
$$\frac{A}{d^2} = 345Q_2^{0.921}$$

Typical variation of A/d^2 with Q_2 is shown in Fig. 3. Lastly, relationships were established between W/d, h/d, A/d^2 and $Q_3 = Q\Big/d^2\sqrt{\frac{\Delta\gamma_s}{\rho_f}dS}$. The equations obtained are

$$\frac{W}{d} = 3.872Q_3^{0.396}$$
$$\frac{h}{d} = 0.308Q_3^{0.330} \qquad \ldots(11)$$
$$\frac{A}{d^2} = 1.108Q_3^{0.726}$$

Figs. 4, 5, 6 show these relationships. Table II summarises the accuracy aspect of eqs. (8), (9), (10) and (11).

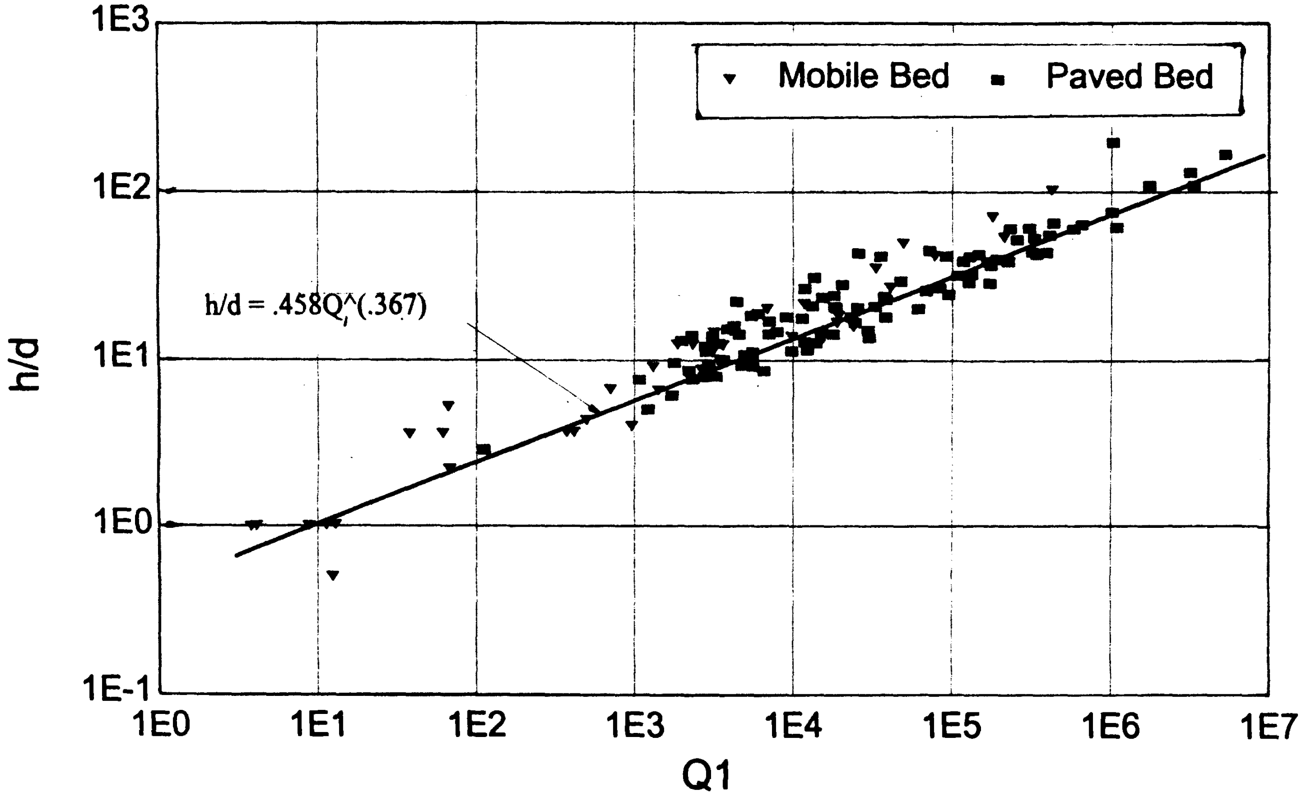

Fig. 2 Variation of h/d with Q_1 Bankful Discharge

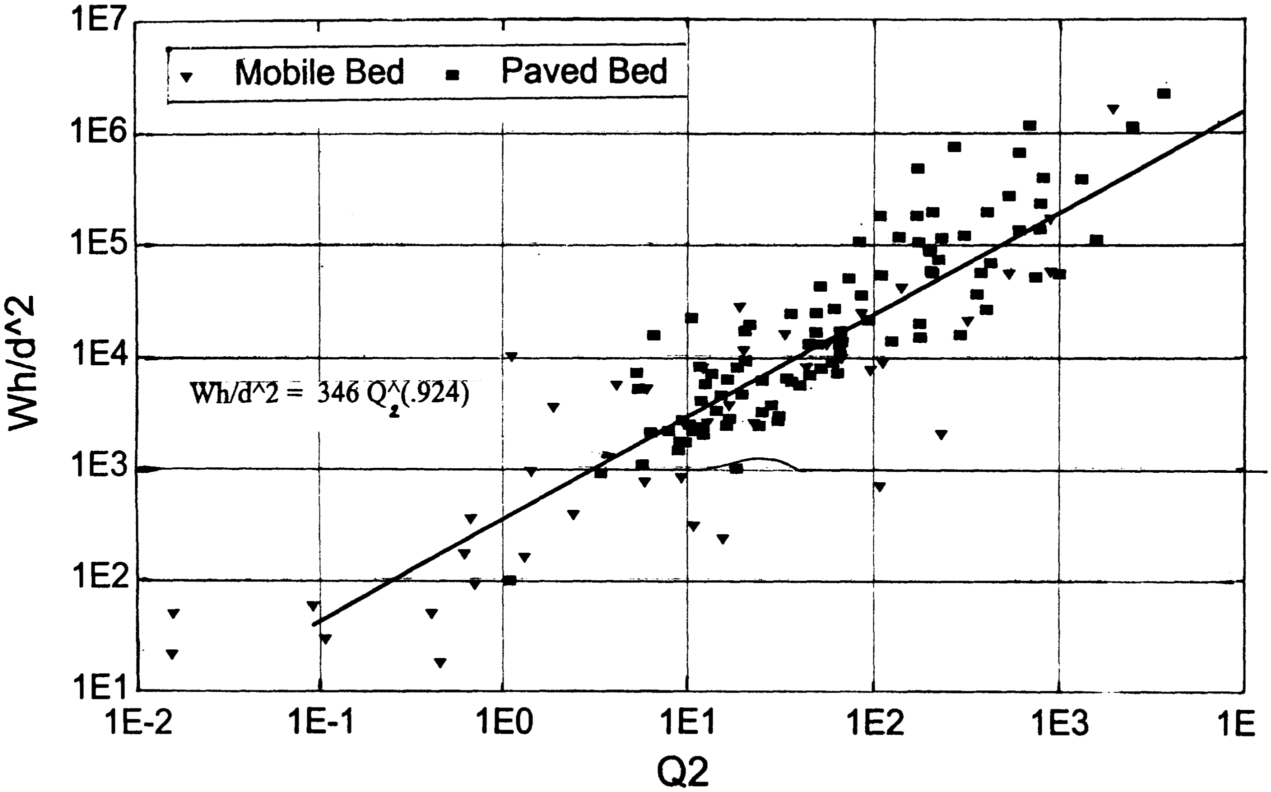

Fig. 3 Variation of *Wh/d^2* with Q_2 Bankful Discharge

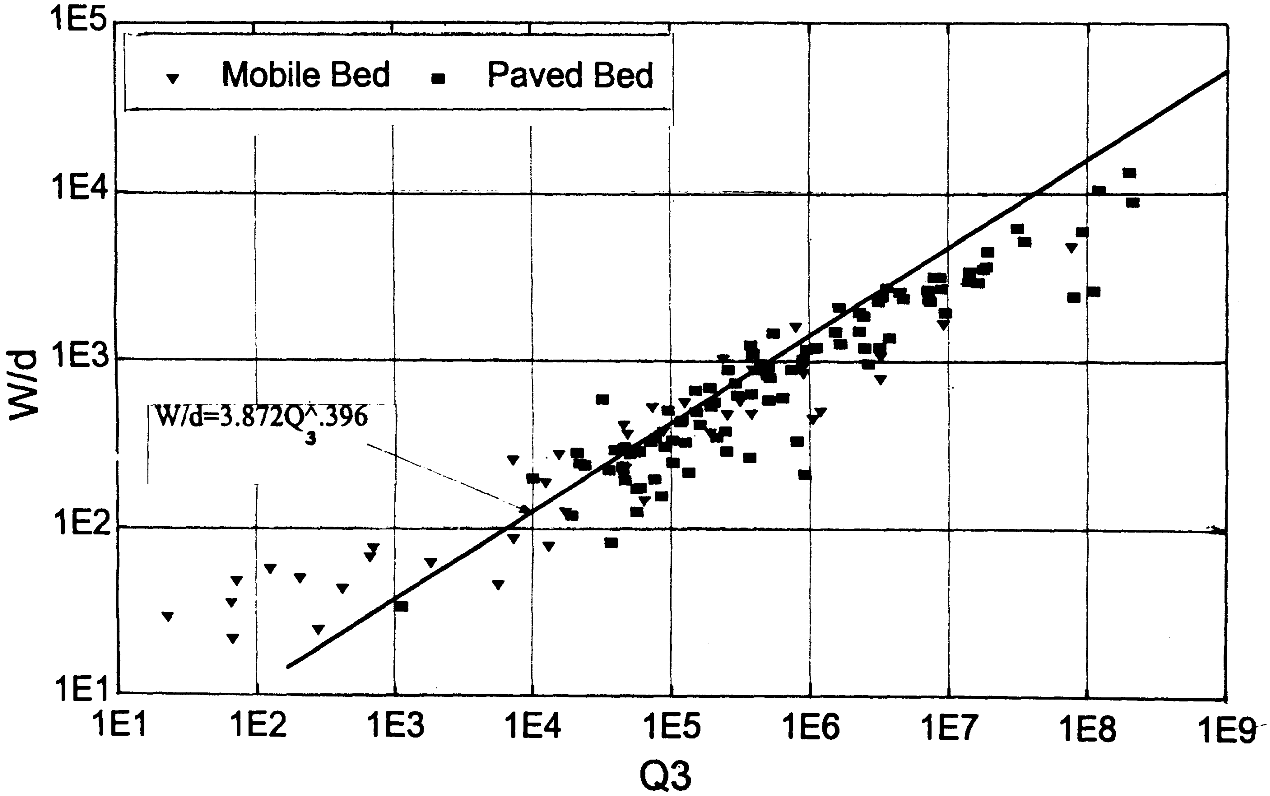

Fig. 4 Variation of *W/d* with Q_3 Bankful Discharge

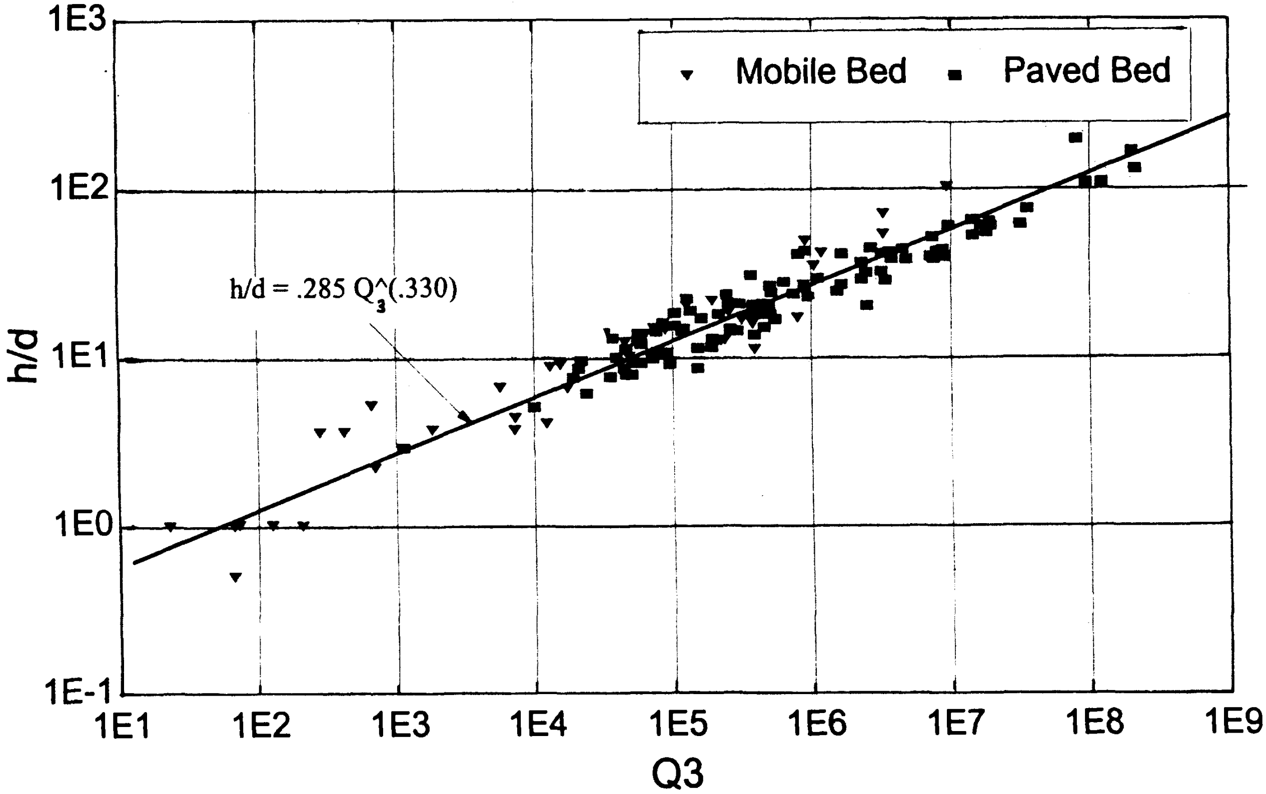

Fig. 5 Variation of h/d with Q_3 Bankful Discharge

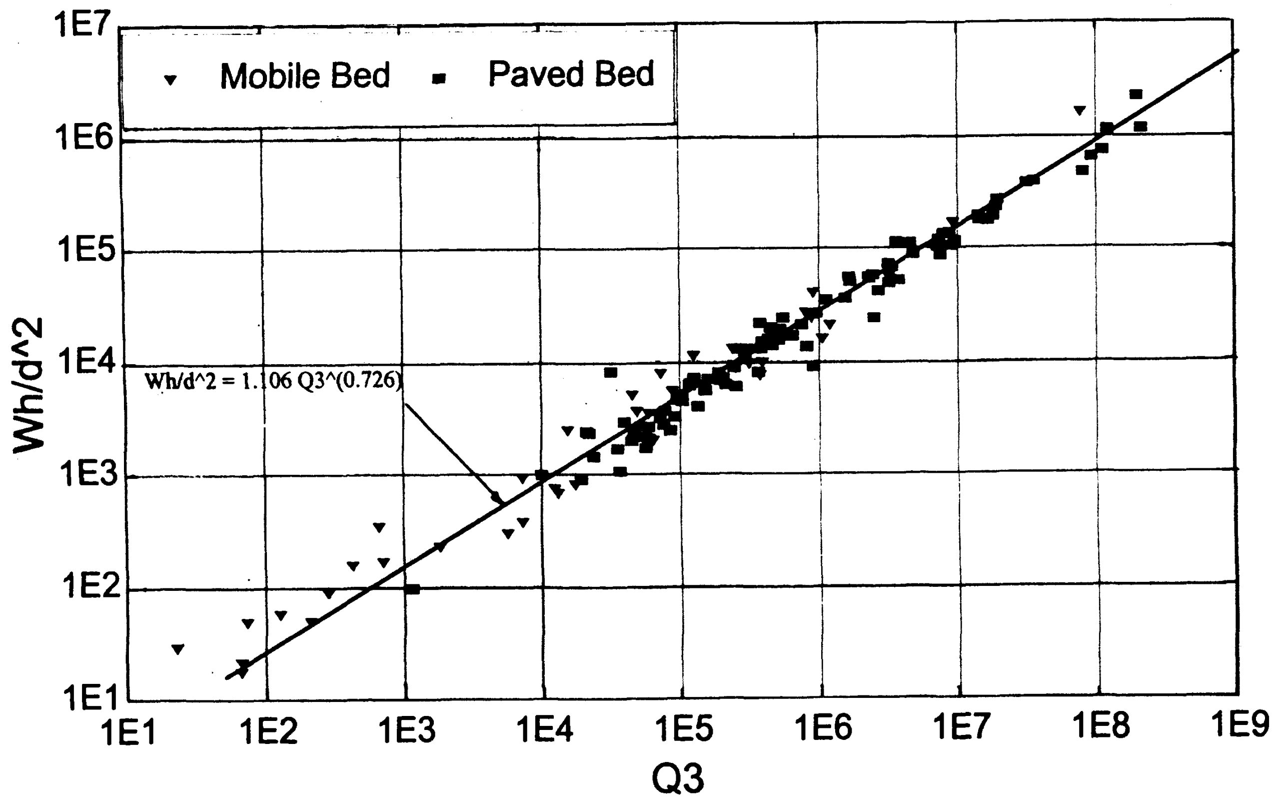

Fig. 6 Variation of Wh/d^2 with Q_3 Bankful Discharge

Table II

Accuracy of different equations

Eq. No.	Equation	SSE	% of data between ±50%	% of data between ± 30 %
8	$W = 4.547\ Q^{0.507}$	0.183	79.28	59.30
	$h = 0.293\ Q^{0.332}$	0.091	90.00	78.57
	$A = 1.330\ Q^{0.838}$	0.193	82.86	65.00
9	$W/d = 7.675\ Q_1^{0.446}$	0.214	84.29	65.71
	$h/d = 0.504\ Q_1^{0.373}$	0.105	94.29	76.43
	$A/d^2 = 3.872\ Q_1^{0.821}$	0.196	84.29	58.27
10	$W/d = 88.928\ Q_2^{0.507}$	0.522	62.14	52.14
	$h/d = 3.888\ Q_2^{0.420}$	0.322	77.86	50.00
	$A/d^2 = 346\ Q_2^{0.924}$	1.23	53.57	31.00
11	$W/d = 3.872\ Q_3^{0.396}$	0.199	81.43	57.74
	$h/d = 0.308\ Q_3^{0.330}$	0.308	94.23	85.00
	$A/d^2 = 1.108\ Q_3^{0.726}$	0.142	92.86	71.43

Careful scrutiny of Table II shows that in general the relationships with Q and Q_3 are superior to those with Q_1 or Q_2 since for them the SSE is smaller and relatively more data fall within ± 50 percent and ± 30 percent bands. Further, since S occurs in the denominator in Q_3, W and h will reduce as S increases which one would expect logically. In addition the relationships between Q_3 and W/d, h/d and A/d^2 are non-dimensional. Hence it is recommended that eq. (11) be used to predict W, h and A for known Q, d and S. In order to see the effect of d on W, h and A, eq. (11) can be expressed in the following form:

$$W \sim Q^{0.396} S^{-0.198} d^{0.01}$$

$$h \sim Q^{0.330} S^{-0.165} d^{0.175} \qquad \ldots(12)$$

$$A \sim Q^{0.726} S^{-0.363} d^{0.185}$$

It may be seen that the effect of S on W and A is more pronounced than that of d. Further the effect of variation in d on h and A is such that W $(=A/h)$ is almost unaffected by variation in d.

Resistance to Flow in Gravel-Bed Rivers

A detailed review of literature on resistance indicated that when viscous force is not important, the following forms of equations have been used by different investigators to predict Manning's n, Darcy-Weisbach friction factor f or the velocity U.

$$n = aS^b$$

$$\frac{n}{d^{\frac{1}{6}}} = F\left(\frac{h}{d}\right)$$

$$\frac{h}{d^{\frac{1}{6}}} = 10.27 + 17.7\log\left(\frac{h}{d}\right)$$

$$\frac{1}{\sqrt{f}} = a\left(\frac{h}{d}\right)^b$$

$$\frac{1}{\sqrt{f}} = a\log\left(\frac{h}{d}\right) + b$$

$$U = a(h^2S)^b$$

$$\frac{U}{\sqrt{gd}} = a\left(\frac{h}{d}\right)^b S^e$$

$$\frac{U}{\sqrt{gds}} = F\left(\frac{h}{d}\right) \qquad \ldots(13)$$

From these relationships it can be seen that the most important parameter governing n, f or dimensionless velocity is h/d and then S. Since it is important to predict the velocity not only at bankful stage but also at other stages, all the data have been used to study the variation of n, f and dimensionless velocity. The important results of this analysis are given below.

When Manning's n was plotted against S on log-log scale it was found that, even though there is considerable scatter, n increases slowly as S increases and the relationship between the two is

$$n = 0.074 S^{0.134}\,. \qquad \ldots(14)$$

However the scatter of data points about the mean line being large, prediction n using eq. (14) is likely to introduce large errors. For hydrodynamically rough plane surface $n/d^{1/6}$ is found to depend as h/d. For gravel-bed river data it is found that

$$\frac{n}{d^{\frac{1}{6}}} = 0.082\left(\frac{h}{d}\right)^{-0.011} \qquad \ldots(15)$$

Since $n/d^{1/6}$ is a weak function of (h/d), $\left(h^{\frac{1}{6}}/n\right) = \left(h^{\frac{1}{6}}/d^{\frac{1}{6}}\right) \times \left(d^{\frac{1}{6}}/n\right)$ will also depend on h/d as indicated by Limersinos[25]. Fig. 7 shows variation of $h^{1/6}/n$ with h/d on semi-log scale. The equation of the mean line is

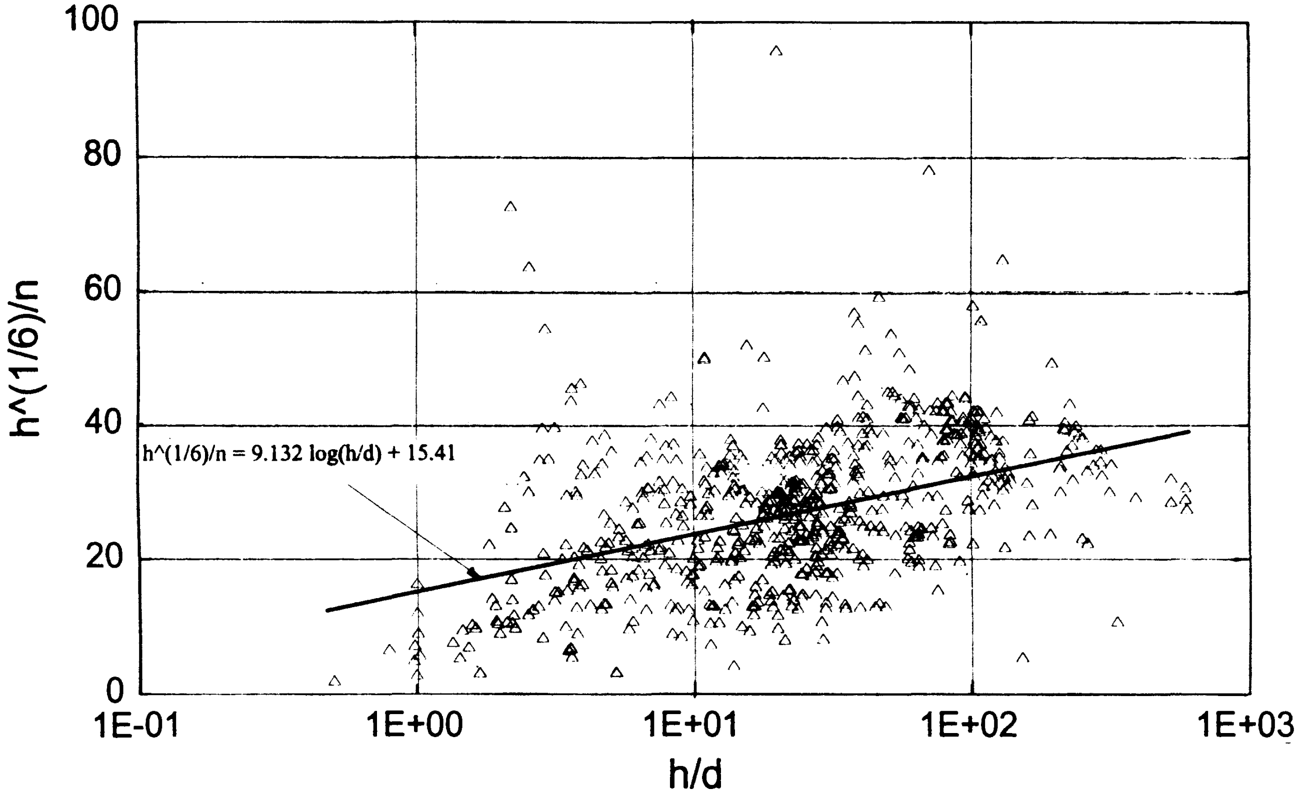

Fig. 7 Variation of h^(1/6)/n vs h/d all data

$$\frac{h^{\frac{1}{6}}}{n} = 9.132\log\frac{h}{d} + 15.14 \qquad \ldots(16)$$

It is found that the SSE is 0.170 while 5 percent of the data points show larger than ± 30 percent error in $h^{1/6}/n$. This equation seems to be superior to eq. (14) and eq. (15). Among the two relations for f tested (see eq. (13)) the power law for $1/\sqrt{f}$ seems to better when all the data are plotted (see Fig. 8). The equation of the mean line obtained is

$$\frac{1}{\sqrt{f}} = 1.557\left(\frac{h}{d}\right)^{0.13} \qquad \ldots(17)$$

Here SSE for $1/\sqrt{f} = 0.170$. Since $n/d^{1/6}$ is nearly constant and is almost independent of h/d, it is concluded that bed undulations do not contribute to the roughness. The relationships for dimensionless velocity were more promising. Fig. 9 shows variation of $Frs\left(= U/\sqrt{gdS}\right)$ with h/d. It can be seen that one gets a straight line on log -log scales with the equation

$$\frac{U}{\sqrt{gdS}} = 4.403\left(\frac{h}{d}\right)^{0.693} \qquad \ldots(18)$$

with SSE of 0.170 in the prediction of $U/\sqrt{gdS}$. An attempt was also made to change the power of S so as to reduce the scatter. To do this the above equation was written as

$$\frac{U}{\sqrt{gd}} = a\left(\frac{h}{d}\right)^{b} S^{c}$$

and the values of a, b and c were optimised so that the equation gave minimum error in $U/\sqrt{gd}$. The equation obtained in this manner was

$$\frac{U}{\sqrt{gd}} = 2.586\left(\frac{h}{d}\right)^{0.631} S^{0.372} \qquad \ldots(19)$$

Lastly it was decided to verify the Russian approach of obtaining stage-discharge curve in non-dimensional form. Grishanin[26] through dimensional reasoning has shown that, as a first approximation $h(gW)^{0.25}/\sqrt{Q}$ remain constant for alluvial rivers. He found that for a wide variation in Q this parameter assumes a constant value of 0.904 with a standard deviation of 0.158. This approach was applied to the

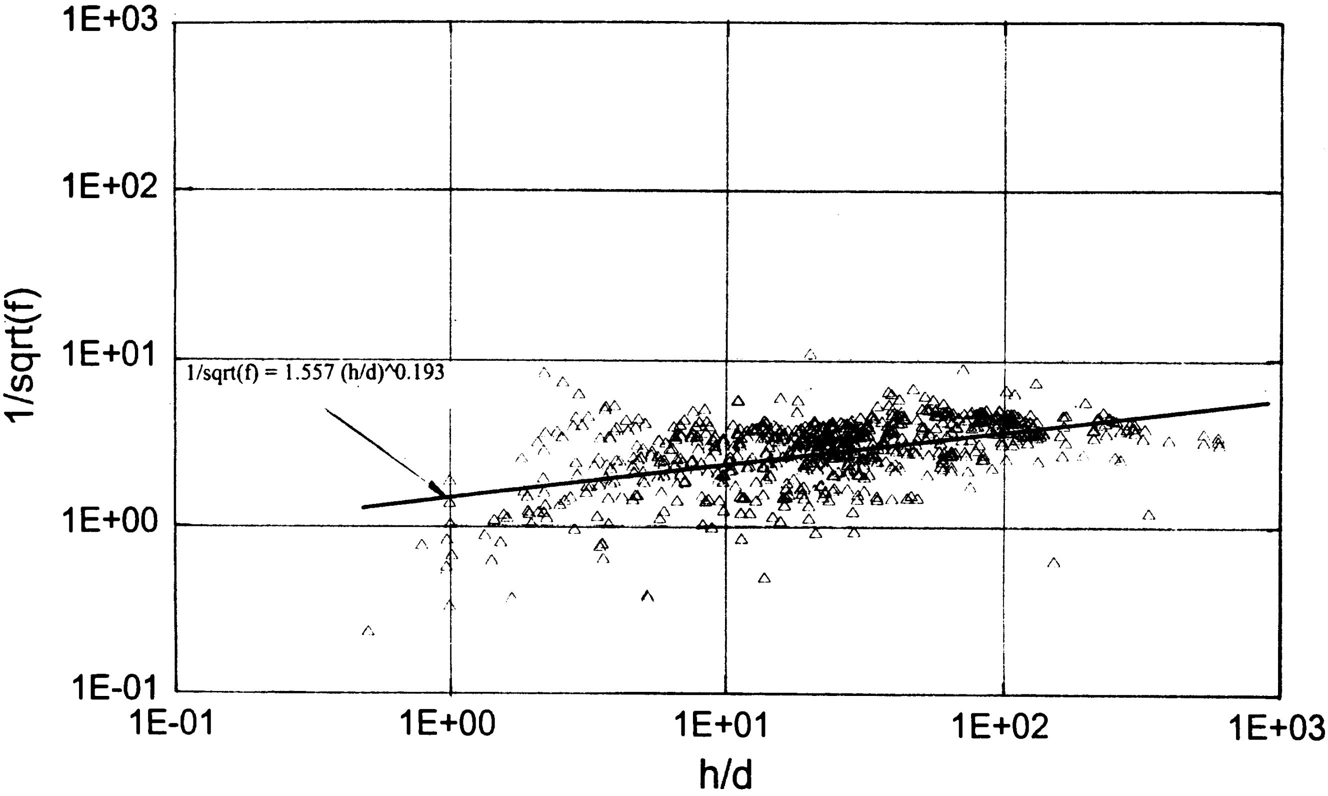

Fig. 8 Variation of 1/sqrt(*f*) vs *h*/*d* all data

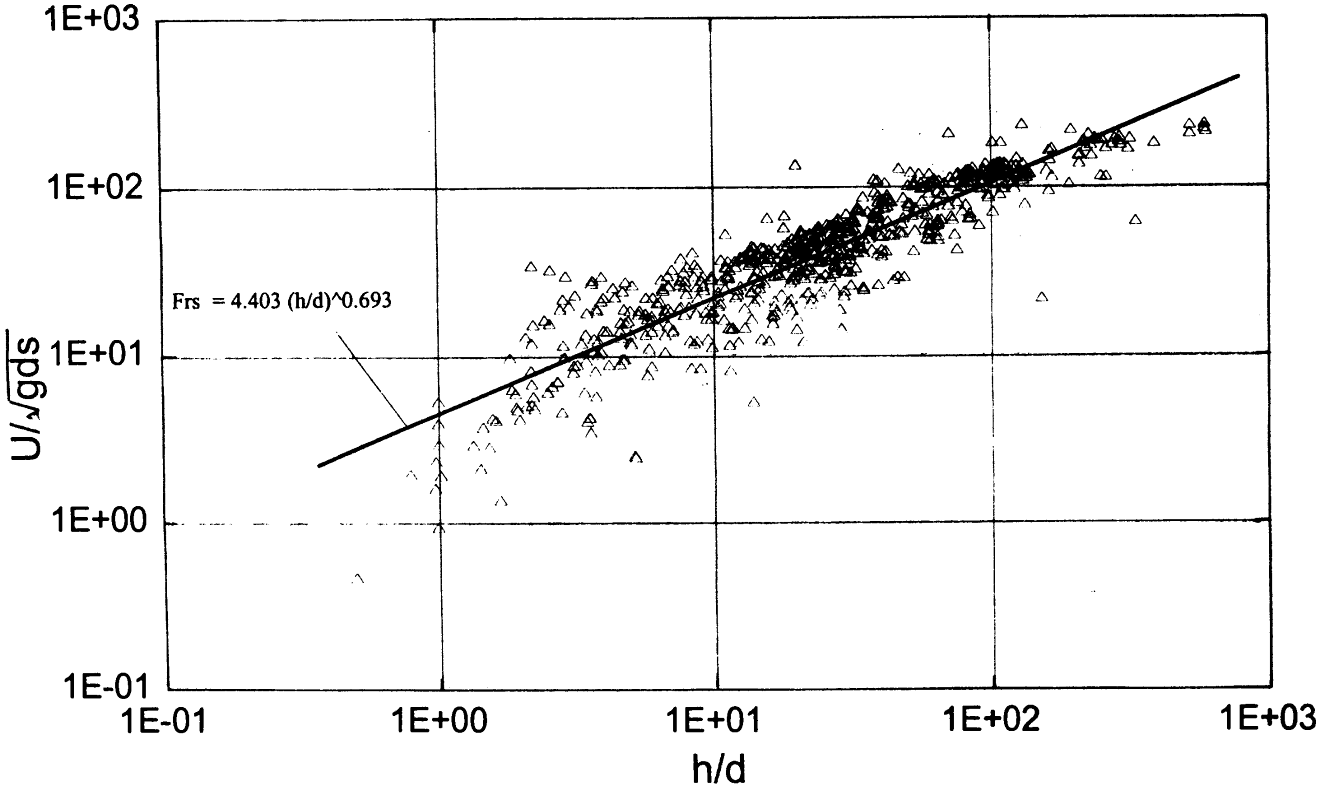

Fig. 9 Variation of Frs vs *h*/*d* all data

gravel-bed river data. It was found that $h(gW)^{0.25}/\sqrt{Q}$ is weakly dependent on h/d and increases with increase in h/d. The variation can be expressed by the equation

$$\frac{h(gw)^{0.25}}{\sqrt{Q}} = 0.459\left(\frac{h}{d}\right)^{0.117} \qquad \ldots(20)$$

It is realised that there is considerable scatter on this graph, however the correlation is significant. This equation can be written in the following form

$$Q = 14.863W^{0.50}h^{1.766}d^{0.234} \qquad \ldots(21)$$

Concluding Remarks

Gravel-bed river data from various sources have been compiled and a total number of 706 data points have been tabulated. These data have been used to study the hydraulic geometry of gravel-bed rivers at bankful discharge and resistance to flow. The major conclusions drawn from this study are the following:

1. The data with paved as well as mobile bed can be analysed together without loss of accuracy.
2. The hydraulic geometry of gravel-bed rivers at the bankful discharge can be conveniently studied using the dimensionless parameters W/d, h/d, A/d^2 and $Q/d^2\sqrt{\frac{\Delta\gamma_s}{\rho_f}d_s}$. Eq. (11) gives these relationships.
3. The resistance coefficients n and f are primarily dependant on h/d while the Froude number $U/\sqrt{gdS}$ depends on h/d and S. Eq. (16) and eq. (17) show the variation of $h^{1/6}/n$ and $1/\sqrt{f}$ with h/d.
4. Eqs. (18) and (19) give reasonably good prediction of average velocity of flow U.

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