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DYNAMICALLY ADAPTING UNSTRUCTURED TRIANGULAR GRIDS: A NEW PARADIGM FOR GEOPHYSICAL FLUID DYNAMICS MODELLING

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In the early days of computing, geophysical fluid dynamics (GFD), numerical weather prediction (NWP) in particular, was a dominant factor in the design of computer architecture and algorithms. This early work focussed initially on finite difference algorithms on a rectangular computational grid and later on spectral methods. After the initial work of Charney, von Neumann, and Arakawa, however, the focus shifted from the basic algorithms for the numerical solution of the fundamental differential equations to improvements in the model physics. Further work on fundamental numerical algorithms shifted to other disciplines – predominately the then emerging aerospace community. As a result, for 40 years, the GFD community has been using numerical techniques that are virtually unchanged. The two primary numerical methodologies that have been used for modelling the atmosphere and the ocean have been spectral methods for global modelling and structured rectilinear grids for regional modelling. In the last few years, a new paradigm for atmospheric and oceanic simulation has emerged: adaptive, unstructured, triangular grids. This paradigm has the advantage of tremendous flexibility in providing high resolution where required by either static physical properties (terrain elevation, coastlines, land use) or the evolving dynamical situation. The first application of this paradigm was the Operational Multiscale Environment model with Grid Adaptivity (OMEGA), an atmospheric simulation and forecasting tool; a more recent application of this paradigm is the more recently developed Multiscale Ocean Simulation System (MOSS). OMEGA with its embedded Atmospheric Dispersion Model (ADM) is a new atmospheric simulation system for real-time hazard prediction, conceived out of a need to advance the state-of-the-art in numerical weather prediction in order to improve our capability to predict the transport and diffusion of hazardous releases. MOSS is a nascent implementation of a similar grid structure and numerical algorithm to the simulation of ocean circulation. The purpose of this paper is to provide a description of this new paradigm and to present its use in atmospheric and oceanic simulation.

Key Words: Numerical Weather Prediction; Geophysical Fluid Dynamics; Numerical Methods; Unstructured Grids

1 Introduction

In the early days of computing, geophysical fluid dynamics (GFD), predominately numerical weather prediction (NWP), was a dominant factor in the design of computer architecture and algorithms. (The other dominant factor was projectile trajectories for artillery.) This early work focussed on solving a finite difference equation on a uniform rectilinear computational grid. After the initial work of Charney[1], von Neumann[2], and Arakawa[3], however, the focus shifted from the basic algorithms for the numerical solution of the fundamental differential equations to improvements in the model physics. Further work on fundamental numerical algorithms shifted to other disciplines – predominately the then emerging aerospace community. As a result, for 40 years, the GFD community has been using numerical techniques that are virtually unchanged. Thus, historically, spectral models have traditionally been used for global modelling and structured rectilinear grids have been used for regional modelling.

The fact that the numerical techniques has not changed does not imply, however, that progress has

not been made. Indeed, over the past forty years, numerical weather prediction has undergone a decade-by-decade advance. The 1960's saw the initial success of the barotropic model. Limited area, multi-layer primitive equation models were put into operational use in the 1970s and 1980s by the US National Weather Service (NWS) and other forecasting centres around the world. These models, first the Limited-area Fine Mesh (LFM)[4] and later the Nested Grid Model (NGM)[5] were the first operational nested models in which high resolution was applied to local regions. In parallel with this operational trend, a number of flexible modelling systems with physics intended for high resolution applications were developed. These included the Regional Atmospheric Modelling System (RAMS)[6], and the Penn State/NCAR Mesoscale Model now in its fifth version (MM5)[7]. In the 1980's increased computational power allowed the introduction of more elaborate model physics. In the 1990's mesoscale models drove toward finer and finer resolutions, mostly through the use of nested grids (*e.g.*, the ARPS model[8]) or variable horizontal resolution (*e.g.*, the GEM model[9,10]). Nested grid models, however, are not sufficient to forecast multiscale phenomena because the nests are fixed and hence unless high resolution is used everywhere, which eliminates the benefits of grid nesting, only specific scales of motion can be resolved in each nest.

At the same time meteorology was benefitting from this research and technology boom, computational fluid dynamics (CFD) researchers were creating new innovative numerical techniques designed to model fluid flows around complex boundaries. In the 1970s and early 1980s the models developed for aerospace engineering and plasma physics were surprisingly similar to their counterparts in the atmospheric sciences. The grids were composed of regular, rectangular cells extending from no-slip or free-slip surfaces. As more computational power became available and atmospheric modellers were pushing more physics into their models, CFD practitioners were busy refining complex gridding techniques around irregular surfaces. One of the methodologies developed was the use of unstructured triangular grids.

In the last few years, this new paradigm of unstructured adaptive grids has been applied to atmospheric and oceanic simulation. This paradigm has the advantage of tremendous flexibility in providing high resolution where required by either static physical properties (terrain elevation, coastlines, land use) or the changing dynamical situation. The first application of this paradigm was the Operational Multiscale Environment model with Grid Adaptivity (OMEGA), an atmospheric simulation and forecasting tool[11]; a more recent application of this paradigm is the recently developed Multiscale Ocean Simulation System (MOSS). OMEGA with its embedded Atmospheric Dispersion Model (ADM) is a new atmospheric simulation system for real-time hazard prediction, conceived out of a need to advance the state-of-the-art in numerical weather prediction in order to improve our capability to predict the transport and diffusion of hazardous releases. MOSS is a nascent implementation of a similar grid structure and numerical algorithm to the simulation of ocean circulation. In this article, we provide a description of this new numerical paradigm and present its use in atmospheric and oceanic simulation.

2 Numerical Methods – A Primer

The basis for all computational methods is the assumption that a function $f(x)$ is known at a set of discrete points x_i. The function is then expanded in a Taylor series as:

$$f(x) = f_j + (x - x_j) f_j' + \frac{1}{2}(x - x_j)^2 f'' + \frac{1}{3!}(x - x_j)^3 f''' + \ldots \quad (1)$$

where $f_j = f(x_j)$. Applying to the discrete points x_{j-1}, x_{j+1} yields:

$$f_{j-1} = f_j - \Delta_- f_j' + \frac{1}{2}\Delta_-^2 f'' + O(\Delta_-^3) \quad \ldots(2)$$

$$f_{j+1} = f_j + \Delta_+ f_j' + \frac{1}{2}\Delta_+^2 f'' + O(\Delta_+^3) \quad \ldots(3)$$

where $\Delta_- \equiv x_j - x_{j-1}$ and $\Delta_+ \equiv x_{j+1} - x_j$, and we also define $\alpha \equiv \Delta_+ / \Delta_-$.

Taking linear combinations of f_{j-1}, f_{j+1}, we find that eqs. (2) and (3) yield:

$$A f_{j+1} + B f_{j-1} = (A + B) f_j + (A\alpha - B) \times \Delta_- f' + \frac{1}{2}(A\alpha^2 + B)\Delta_-^2 f'' + O(\Delta_-^3) \quad \ldots(4)$$

This can be simplified, using the relationships $B = -A\alpha^2$ and $B = A\alpha$, respectively, to yield second-

order accurate definitions for the first and second derivatives of f:

$$f'_j = \frac{f_{j+1} - (1-\alpha^2) f_j - \alpha^2 f_{j-1}}{\alpha(1+\alpha)\Delta_-} + O(\Delta_-^3) \quad ...(5)$$

$$f''_j = \frac{f_{j+1} - (1+\alpha) f_j + \alpha f_{j-1}}{\frac{1}{2}\alpha(1+\alpha)\Delta_-^2} + O(\Delta_-^3) \quad ...(6)$$

Eqs. (5) and (6) are valid for all grid configurations; for the special case of a uniform grid ($\Delta_+ = \Delta_- = \Delta; \alpha = 1$), eqs. (5) and (6) reduce to the familiar (*cf.*, Haltiner and Williams[12]) expressions:

$$f'_j = \frac{f_{j+1} - f_{j-1}}{2\Delta} + O(\Delta^3) \quad ...(7)$$

$$f''_j = \frac{f_{j+1} - 2f_j + f_{j-1}}{\Delta^2} + O(\Delta^3) \quad ...(8)$$

Where the traditional analysis looks only at the number of terms of the Taylor series that are retained, adaptive grids look at the terms themselves. Thus, where the slope is constant, the adaptive grid represents the function by only a few points. The benefit of non-uniform grids is the ability to replicate a function more efficiently (in a computational sense) than using a uniform mesh. For example, the simple Gaussian function $f(x) = Ae^{-x^2}$ can be represented with a fraction of the number of grid points without sacrificing accuracy. A demonstration of this is shown in Fig. 1 where we compare a Gaussian using a 21-point uniform (0.5) grid with an 11-point non-uniform grid. Both of these discrete representations provide an integral accuracy of 0.5%.

The real benefit of adaptive grids arises when the breadth of scales of the physical system is large. In the case depicted in Fig. 1, the range of scales was 10 (the domain) to 1 (sigma); in the atmosphere, the range of scales can be far greater.

Unstructured triangular grids are the logical extension of the adaptive grid discussed above into two dimensions. (In three dimensions, the grid is tetrahedral.) OMEGA creates an atmospheric grid by tessellating the surface of the Earth using triangular tiles (*cf.*, Fig. 2) and then constructing radii through each of the surface vertices. A set of surfaces is then constructed that are terrain following near the surface and spherical at high altitude. While completely unstructured three-dimensional (tetrahedral) meshes have been used for other purposes[13, 14], the benefit of having a structured vertical dimension

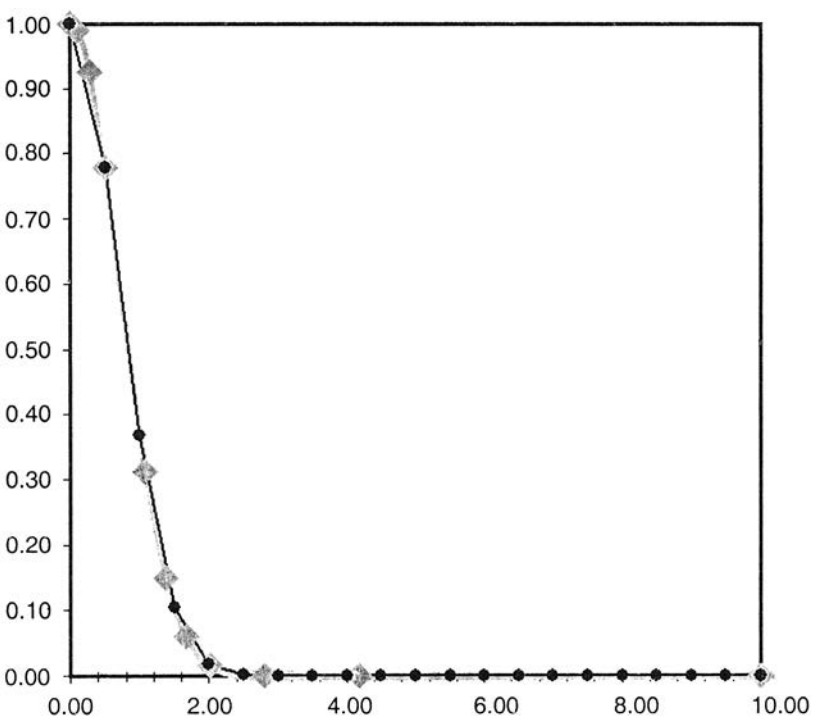

Fig. 1 Two representations of a Gaussian. The gray curve represents a 501-point baseline representation, the black curve with dots shows a 21-point representation using a uniformly spaced (0.5) grid, and the diamonds show an 11-point adaptive grid representation.

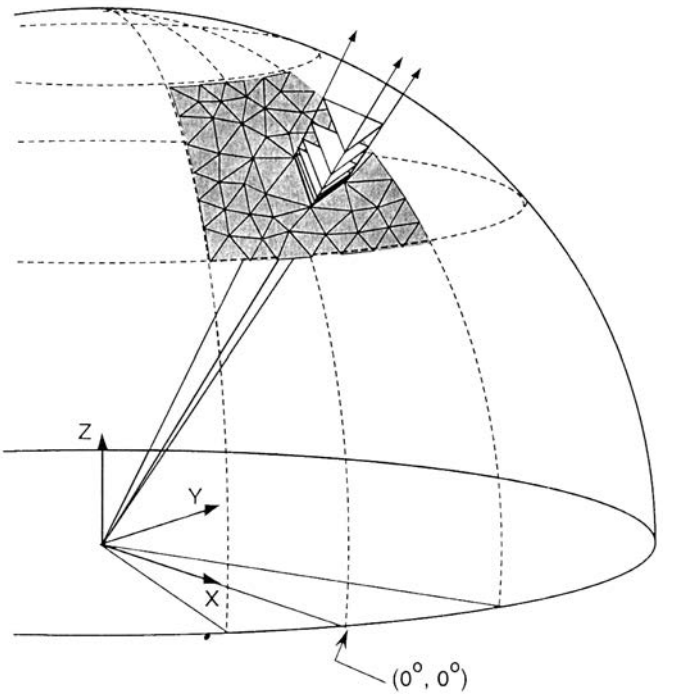

Fig. 2 OMEGA coordinate system and vertical alignment of OMEGA grid

in an atmospheric grid is a significant reduction in the computational requirements of the model. Specifically, the structured vertical grid enables the use of a tri-diagonal solver to perform implicit solution of both vertical advection and vertical diffusion. Since in many larger scale applications the vertical grid spacing is one or more orders of magnitude smaller than the horizontal grid spacing, the ability to perform vertical operations implicitly relaxes the limitation on the timestep.

The flexibility of unstructured grids facilitates the meshing of arbitrary surfaces and volumes in three dimensions. In particular, unstructured grid cells in the horizontal dimension can increase local resolution to better capture topography or the important physical features of atmospheric circulation flows and cloud dynamics. In many fields of application there is recognition that these methods are more efficient and accurate than the structured logical grid approach used in more traditional codes[15,16].

3 Unstructured Triangular Grids – Application to Atmospheric Simulation

As mentioned in the introduction, the first application of adaptive, unstructured grids to atmospheric simulation is the OMEGA modelling system. A complete description of this system can be found in Bacon *et al.*[11] Boybeyi *et al.*[17] presents details of the OMEGA Atmospheric Dispersion Model (ADM) and the results of an extensive model evaluation against data from the ETEX experiment[18, 19]. Gopalakrishnan *et al.*[20] presents the application of OMEGA, including dynamic adaptation, to hurricane track forecasting and a comparison against observations for 20 forecasts covering 8 storms. Accordingly, this paper presents just an overview of the modelling system and focuses our attention on the advantages of the adaptive unstructured grid paradigm.

An overview of the OMEGA system is shown in Table I. Briefly, OMEGA is a fully non-hydrostatic, three-dimensional prognostic model. It is based on an adaptive, unstructured triangular prism grid that is referenced to a rotating Cartesian coordinate system. The model uses a finite-volume flux-based numerical advection algorithm derived from Smolarkiewicz[21]. OMEGA has a detailed physical model for the planetary boundary layer (PBL) with a 2.5 level Mellor and Yamada closure scheme[22]. OMEGA uses a cumulus parameterization scheme developed by Kain and Fritsch[23]; a modified Kuo scheme[24-26] also exists. OMEGA also incorporates an extensive bulk-water microphysics package derived from Lin *et al.*[27] OMEGA computes the shortwave absorption by water vapour and longwave emissivities of water vapour

Table I
An Overview of OMEGA

Governing equations	3D fully non-hydrostatic, prognostic
Grid structure	Unstructured triangular prisms, terrain following
Grid adaptivity	To surface properties and evolving weather
Coordinate system	Rotating Cartesian coordinates
Numerics	Finite volume with Smolarkiewicz corrective scheme
PBL	Treated as viscous sublayer, surface, and transition layers
Turbulence closure	1.5 order turbulent kinetic energy closure
Cumulus parameterization	Kain-Fritsch and Modified Kuo schemes
Microphysics	Extensive bulk-water parameterization
Radiation	Shortwave absorption by water vapour and longwave emissivities of water vapour and carbon dioxide
Lower boundary	Based on Monin-Obukhov similarity theory
Upper boundary	Rigid, free-slip surface
Lateral boundaries	Radiative and large scale nudging boundary condition
Initialization	Based on 4D data assimilation
Dispersion	Eulerian and Lagrangian aerosol and gas dispersion

and carbon dioxide[28]. OMEGA uses an Optimum Interpolation analysis scheme[29] to create initial and boundary conditions and supports piecewise four dimensional data assimilation using a previous forecast as the first guess for a new analysis. In addition, an Incremental Analysis Update scheme[30] is in process for continuous data assimilation. Finally, OMEGA contains both Eulerian (grid based) and Lagrangian (grid free) dispersion models embedded into the model.

a. Grid Adaptation

Since the accurate solution of any complex computational problem depends on fine spatial discretization of the computational domain, the accurate representation of multiscale events in numerical models has long been a principal issue in computational fluid dynamics. For example, one typically desires to capture not only the development and evolution of small scale features but also their interaction with and influence upon the larger scale flow. This is a particularly important requirement in atmospheric models, because numerous events such as fronts, clouds, and plumes are not only relatively localized with respect to their environment, but are also forced on scales larger than their own. Because practical limitations in computer size and speed prohibit the use of uniformly high spatial resolution appropriate for the smallest scales of interest, numerous techniques have been developed to deal with multiscale flows.

Grid nesting techniques involve the sequential placement of multiple finer scale meshes in desired regions of the domain so as to provide increased spatial resolution locally. The decision to spawn one or more sub-meshes is typically subjective and manually directed. Some formulations allow the sub-meshes to move with particular features in the flow, such as hurricanes[31]. A principal limitation of grid nesting technique, however, is that one must know *a priori* and for the duration of the calculation the regions of the domain that will require high spatial resolution. In other words, the trajectory of the moving grid has to be pre-defined and therefore cannot be used for prediction. Another principal limitation of grid nesting technique is the interaction among multiple nested meshes, particularly the tendency for propagating dispersive waves to discontinuously change their speeds upon passing from one mesh to the next and to reflect off the boundaries of each nest due to an impedance mismatch across the mesh boundaries[32].

Grid refinement techniques are a relatively new and powerful class of methods and can be subdivided into two basic categories. The first includes methods in which grid points are added locally to the computational domain as the calculation proceeds, or finite elements are subdivided locally, to provide increased spatial resolution based on predetermined physical criteria. This grid refinement technique has been widely used in aerospace applications with successful results. For example, Lottati and Eidelman[33], Baum and Löhner[15], and Baum *et. al.*[13], have applied this dynamic grid generation for simulating flows over aircrafts, tracking shock waves, and in simulating fluid-structure interactions. The second category of refinement technique involves methods that redistribute, in some predetermined manner, a fixed number of grid points so as to provide locally increased resolution and thus an improved solution in certain regions of the domain. For example, Dietachmayer and

Droegemeier[34], and Fiedler *et al.*[35], have used this method for atmospheric flows using structured grids. However, their methodologies have been restricted to idealized problems so far and are not suitable for real atmospheric flow simulations, which can include complicated terrain features.

The unstructured grid technique is rather new to the atmospheric science community. As noted in the previous paragraph, the unstructured grid method has been used for more than a decade by the aerodynamics community because of its efficiency in the modelling of irregular domains. The flexibility of unstructured grids and their ability to adapt to transient physical phenomena are the features that give unstructured grid algorithms for partial differential equations their great power. Grid adaptivity improves the fidelity of all finite difference or finite element numerical schemes by increasing resolution in high gradient regions. The improvement comes from the ability to adapt the grid structure to the flow and from the local refinement of the grid in the vicinity of fixed high gradient surface features and rapidly changing horizontal spatial structures in the atmosphere.

Another advantage of unstructured grids is the ease with which dynamic grid adaptation can be implemented. There is no longer a need for involved user-expertise/interaction for creating topologies of complicated terrain features; the whole procedure can be fully automated, a feature that is not only highly desirable, but is required in operational settings. Also, since the unstructured grid is a single mesh with a smooth and continuous transition from coarse to fine regions within the whole domain, the model is naturally two-way scale interactive without the interpolation error caused by the transfer of information from one nest to another. This also eliminates the wave reflection problem common in nested grid models.

OMEGA is the only operational atmospheric flow model based on unstructured grid technique, which fully exploits the advantages and flexibility of unstructured grids. It can adapt its grid both statically and dynamically to different criteria. *Static* adaptation creates a numerical grid resolving fixed features (*e.g.*, land-water boundaries, terrain gradients, and/or any other feature that the user includes in the adaptation scheme) with a resolution that smoothly varies from the maximum to the minimum specified (Fig. 3a). (In addition, the OMEGA grids can also be further refined in one or more specific geographical areas, by the creation of sub-domains in which higher resolutions can be specified.) *Dynamic* adaptation (Fig. 3b) adds the periodic re-adaptation of the grid to regions that require high resolution during the course of a simulation (*e.g.*, frontal zones, hurricane circulation, pollutant plumes). The computational grid does not change during the course of a static adaptation simulation; however, dynamic adaptation consists of three major steps taken at pre-set time

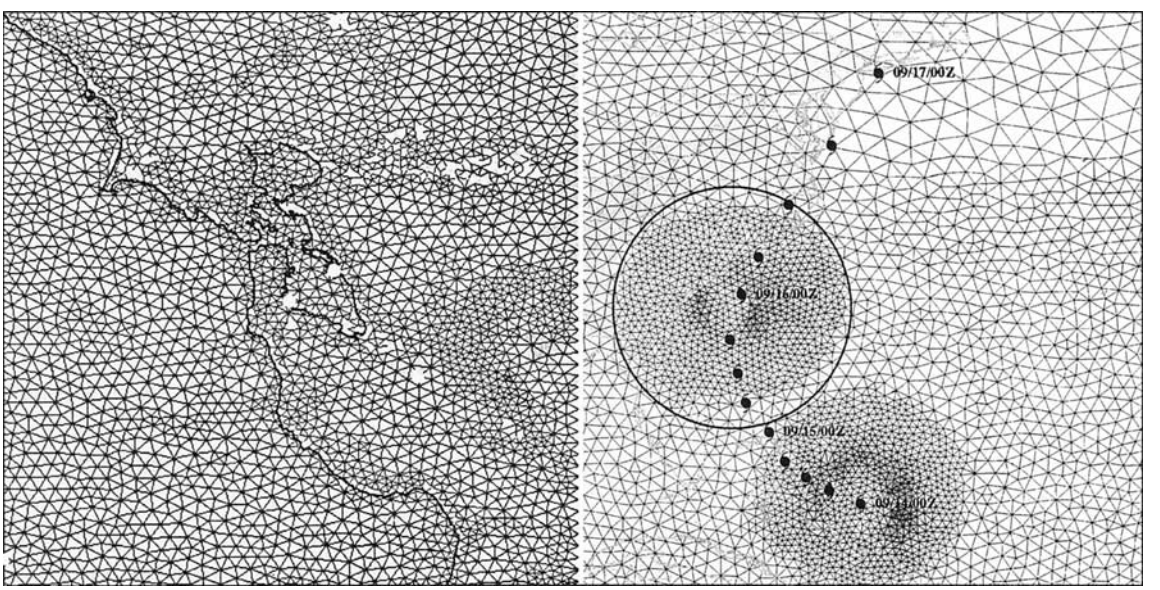

Fig. 3 (a) Static adaptation creates a grid that captures the complex terrain and land/water boundaries; (b) dynamic adaptation puts high resolution only where required leading to computational efficiency. The left panel shows a grid created for the San Francisco Bay area; the right panel shows the hurricane Floyd (1999) grid and wind speed (shading) at initialization and (inset) the high resolution portion 48 hours into the forecast. (The observed storm track is shown at six hour intervals by the symbols.)

intervals: (1) specific variables or their gradients are evaluated to see if they meet the adaptivity criteria; (2) the mesh is refined or coarsened depending on the pre-specified criteria; and (3) the physical variables are interpolated to the new cell centres.

(a) (i) Static Grid Adaptation

The total number of grid points necessary to perform a successful numerical computation that recovers the correct physics can be greatly reduced in an unstructured grid (*cf.*, Fig. 1). By this we mean that the recovery of the model physics at the smallest length scale resolved does not require the complete domain to have the same resolution. The resources of the numerical and computational machinery are focused on the regions of importance. This is especially significant for three-dimensional hydrodynamic problems, where our experience has shown the resulting economy can make the difference between tractability and intractability (*cf.*, Baum *et al.*[13]).

In OMEGA, the adaptation of the unstructured grid takes place through a variety of grid operations (Fig. 4-6). The first is vertex addition, which is followed by vertex reconnection. Fig. 4 illustrates these two steps, which are used for cells that require higher resolution. The vertex addition step is accomplished by adding a new vertex at the centroid of each affected cell and connecting it to the three original vertices of the cell. The reconnection step then involves the evaluation of each new cell to see if it is possible to create grid cells with lower aspect ratios by removing an edge and reconnecting the vertices.

Fig. 5 shows the reverse process, in which the grid is coarsened through the process of vertex deletion. Like vertex addition, vertex deletion is followed by a vertex reconnection step. It is important to note that the goal of grid adaptation is not to *move* the grid, but rather to refine the grid in advance of any important physical process that might require additional grid resolution, and to coarsen the grid behind the region. This differentiates the method from the adaptation techniques described by Dietachmeyer and Drogemeier[34], which used vertex movement to adapt to atmospheric features.

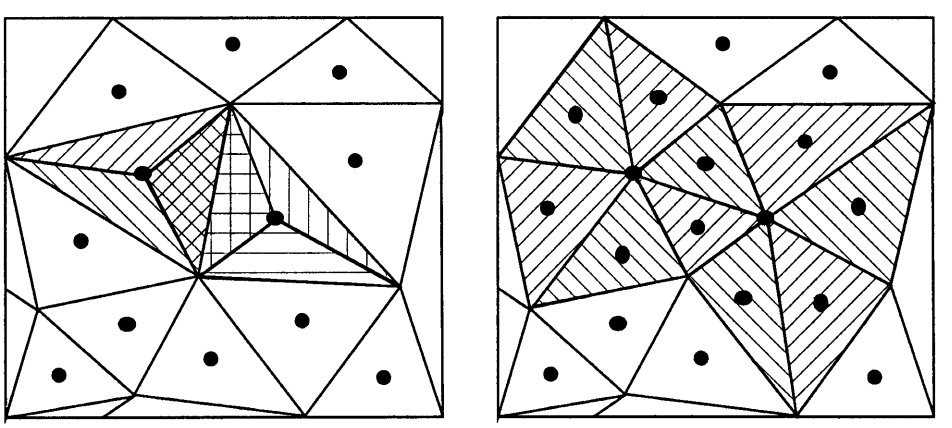

Fig. 4 Vertex addition and reconnection

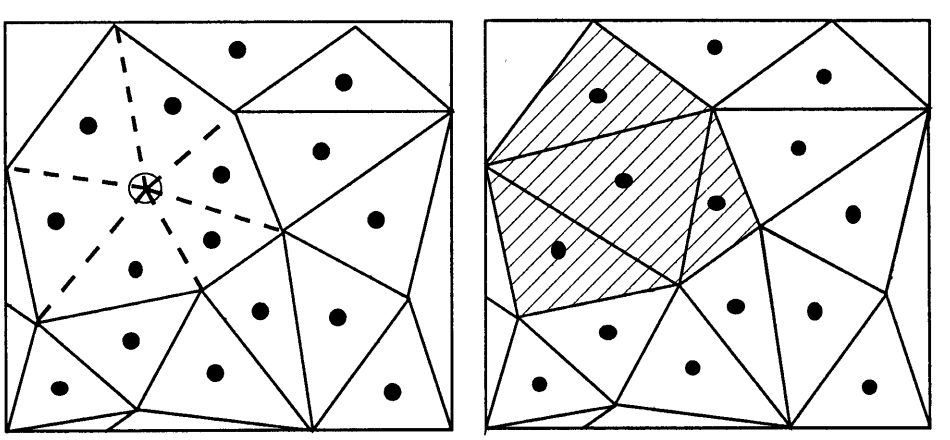

Fig. 5 Vertex deletion and reconnection

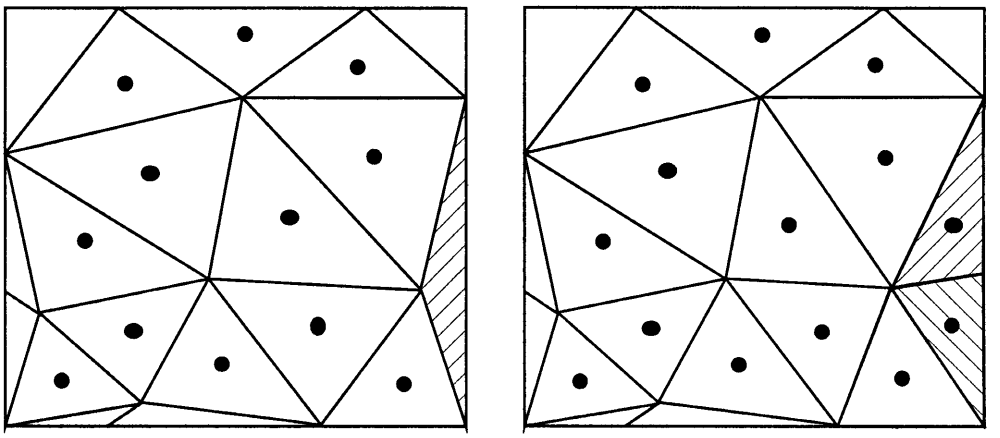

Fig. 6 Vertex relaxation and edge bifurcation

A different type of grid modification is shown in Fig. 6. In this figure we show vertex relaxation, in which the vertices are treated as masses and the connecting edges as springs in a mass-spring system that is allowed to relax to the minimum energy state. Also shown in Fig. 6 is edge bifurcation, which is equivalent to vertex addition in the special case of a cell at the edge of the computational domain. Vertex relaxation is used to keep the aspect ratio as near unity as possible to minimize numerical errors in computing gradients, while edge bifurcation is used to refine the grid along the computational boundary.

The OMEGA grid in the "static" adaptation case is adapted *a priori* to resolve static features such as terrain gradients, land-water boundaries, and/or any other feature that the user includes in the adaptation scheme. The grid does not change during the course of the OMEGA run. The OMEGA grid automatically adapts to static features of the underlying topography such as complex terrain and land/water boundaries. The grid can also be refined in one or more specified geographical areas, such as theaters of operation, by the creation of rectangular sub-domains in which higher resolutions are specified. Within each sub-domain, grid generation is governed by the user-specified minimum and maximum resolutions. Sub-domains can overlap to create a high-resolution region of almost any shape, or to produce a high-resolution area within an intermediate-resolution region. The user can alternatively specify a location in the domain and a radius of influence around it; the grid generator will then refine the grid within the region of influence. A smoothing process is performed on the resolution specifications to avoid high expansion ratios at the boundaries of sub-domains. A very important point, however, must be made: the result of the use of sub-domains and/or circular region refinement is a single, not nested, grid with variable resolution.

An important feature of the use of an unstructured grid is the ability to simulate mountains and coastal features without the "stair-step" geometry required by nested grid models (*cf.* Zhang *et al.*[36]). Triangular grids can naturally follow the coastline better, leading to improved land-water circulations, and can better capture the geometry of mountainous regions. This is especially important for near-surface simulations such as those affecting airport terminal operations (*cf.*, Fig. 3a).

(a)(ii) Dynamic Grid Adaptation

The grid adaptation discussed so far is statically adaptive, implemented in the grid generator prior to the beginning of an OMEGA run and the grid does not change during the simulation. OMEGA also has the ability to adapt its grid during a simulation to different criteria such as frontal activity, convection, hurricanes, and/or a pollutant plume. This enables atmospheric features that require additional grid points for adequate simulation to be resolved as they appear. Thus, through the combination of adaptation methods and criteria, the grid can be coarse where the circulation is regular and smooth, but greatly refined where there are sharp gradients, where there are discontinuities in the flow, where topographic features are important, or where model physics or dispersion source terms require fine resolution. The underlying philosophy of dynamic adaptation is to provide a refined grid to resolve important physical events and features as the simulation is in progress.

The dynamic grid adaptation in OMEGA consists of four major steps: (1) at a predetermined time step specific variables or their gradients are evaluated to

see if they meet the adaptivity criteria; (2) the mesh is refined where these criteria are satisfied; (3) the physical variables are interpolated to new cell centres; and finally (4) the mesh is coarsened where the criteria are not met. For example, if there are sharp discontinuities such as propagating fronts, the mesh is refined ahead of the front so that a refined mesh already exists when the front arrives at a specific location. As the front propagates, the refined mesh at the previous location of the front is coarsened to the resolution specific to the background mesh.

The cell-centered physical variables are interpolated to the vertices before each adaptation cycle using the pseudo-Laplacian weighted averaging scheme. As new cells are created or removed locally a simple linear interpolation is done to assign values to new vertices and cell centers. All the diagnostic variables are calculated for new cells, the particle location array and the underlying terrain are updated, and the mesh is re-masked to calculate new time steps for the time-splitting scheme. The entire procedure is completely automated and requires only a few user inputs at the beginning of the simulation.

Setting the right criteria for adaptation is very important. There is a significant cost associated with a grid adaptation, hence the ideal criteria are those that require minimum computational effort to evaluate yet indicate key regions requiring additional resolution. If the adaptation criteria are particle locations representing a pollutant plume or puff, then it becomes a simple step of locating the cells containing particles, and then tagging those cells for refinement. The alternative method is adapting to the flow itself, which is not as straightforward. Atmospheric flows are almost always turbulent and highly stratified in the vertical so the gradients are often weak and hard to detect.

To permit high resolution simulation of aerosol or gas plumes, OMEGA refines the grid around the particle locations and coarsens the grid where high resolution is not required (*e.g.*, areas from which the plume has already passed). The criteria for adaptation is set by defining a Gaussian function around each Lagrangian particle or puff. The region close to particles therefore gets higher weighting and is tagged for refinement; areas devoid of particles and consisting of relatively flat terrain are tagged for coarsening. The potential advantage of this type of dynamic adaptation was also demonstrated for a simple flow, but including the effects of sulfate chemistry[37]. This case demonstrated how the use of dynamic adaptation could capture 85% of the total sulfate production computed by a uniform high resolution simulation in 20% of the computational time.

Another demonstration of dynamic adaptation, this time applied to hurricane forecasting, was documented in Gopalakrishnan *et al.*[20] This study explored several issues including the "cross-over" between the resolved microphysics and the Kuo cumulus parameterization (achieved by weighting the cumulus parameterization by the cell area) and dynamic adaptation. An adaptation criterion was defined that maintained resolution around the low pressure centre (*e.g.*, Fig. 3b). This figure shows the initial OMEGA grid for a simulation of hurricane Floyd and (inset) the high resolution portion of the grid 48 hours after initialization. A significant feature is that outside the region of the adaptation, little or no change in the grid occurs. The grid maintains resolution over the core of the storm, but when the storm makes landfall and decays, the region of high grid resolution vanishes as there is no longer a need (according to the criteria) for high resolution.

b. Analytic Validation

Numerical weather prediction is a mathematically ill-posed problem. It is data sparse with poorly defined top, lateral, and bottom boundary conditions. As a result, it is impossible to mathematically *validate* an NWP model. However, some routines, such as the advection and diffusion operators, can be evaluated explicitly. This type of evaluation typically uses a rotating cone test (such as that presented in Bacon[11]); another common test problem is the counter-rotating vortex problem presented by Staniforth[38]. In this problem, six counter-rotating vortices are used to shear apart a conic distribution of a passive scalar. The initial state of this case is shown in Fig. 7a the state of the system after 1000 timesteps is shown in Fig. 7b with the shading representing the OMEGA solution and the black contours presenting the analytic solution.

c. Multiscale Simulations

OMEGA has been used at scales ranging from a few kilometers to global. The first application of OMEGA was to local scale dispersion of a tracer in complex terrain. A comprehensive model evaluation was conducted for the ETEX I experiment[17] (*cf.*, Fig. 8) that was continental in scale. This analysis demonstrated that OMEGA, as

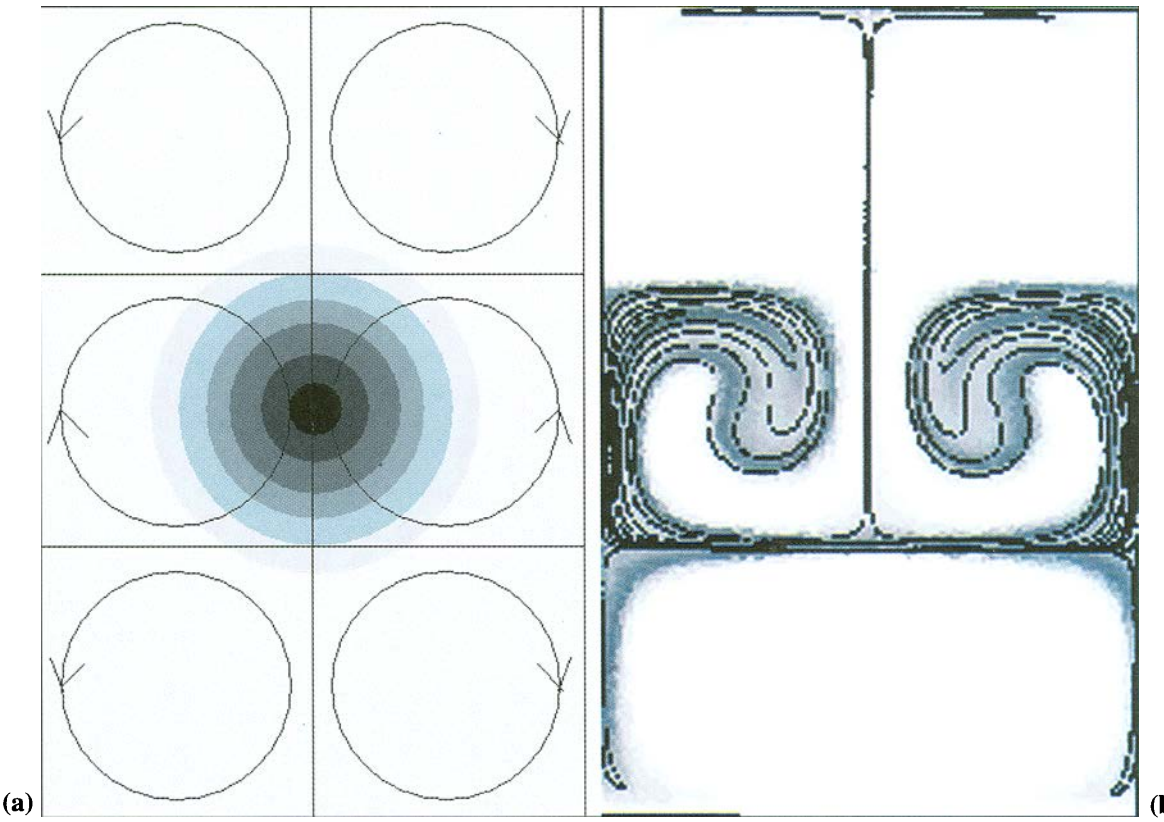

Fig. 7 The initial tracer density configuration for the counter-rotating advection test of Staniforth (left) and the OMEGA (shading) and analytic (black contours) at 1000 timesteps

a *forecast* system, could predict the dispersion better than most models operating in *analysis* mode.

Gopalakrishnan *et al.*[20] documents a series of hurricane forecasts, exploring the use of static and dynamic grid adaptation, involved grid resolution ranging from 100 km down to 10 km. The average track error (Table II) of the entire suite of eight storms was 56, 101, 140, 298, and 189 km for the analysis and the 24, 48, 72, and 96 hour forecasts, respectively. Given the small sample set, these numbers must be considered in context. For example, only three forecasts (two for Georges and one for Floyd) went out to 96 hours, and four storms (Beryl, Chris, Helene, and Xangsane) did not remain organized long enough

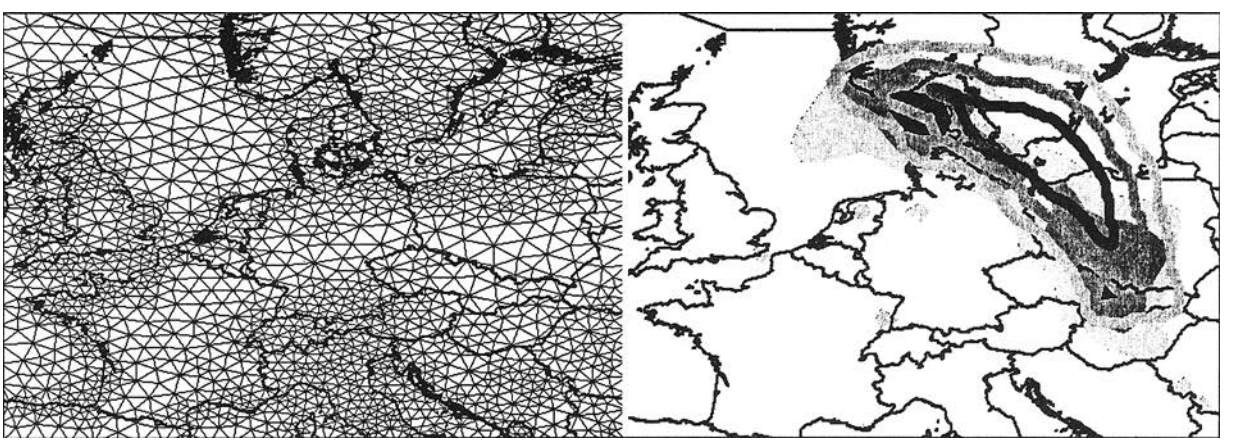

Fig. 8 OMEGA 100-40 km resolution grid (left) and dispersion results (right) for the ETEX I experiment. The observed and 60-hour forecasted plume concentrations are virtually on top of each other. (For clarity, slightly different concentration shading was used for the observations (solid) and the 60-hour forecast (contours).)

Table II

Summary of Track Error (km) as a function of forecast hour for 20 OMEGA forecasts

Forecast Hour:	0	12	24	36	48	60	72	84	96
GEORGES (2)	81	69	57	59	112	124	138	189	242
FLOYD (6)	25	49	50	46	65	139	224	338	83
BERYL (1)	48	101	94	192					
CHRIS (2)	68	160	77	93					
DEBBY (2)	124	151	206	372	378	294	305		
FLORENCE (3)	55	118	128	89	183	206	549		
HELENE (2)	54	115	150	180	114	70			
XANGSANE (2)	59	53	122	115	68	59			
Average Error	56	92	101	120	140	165	298	264	189

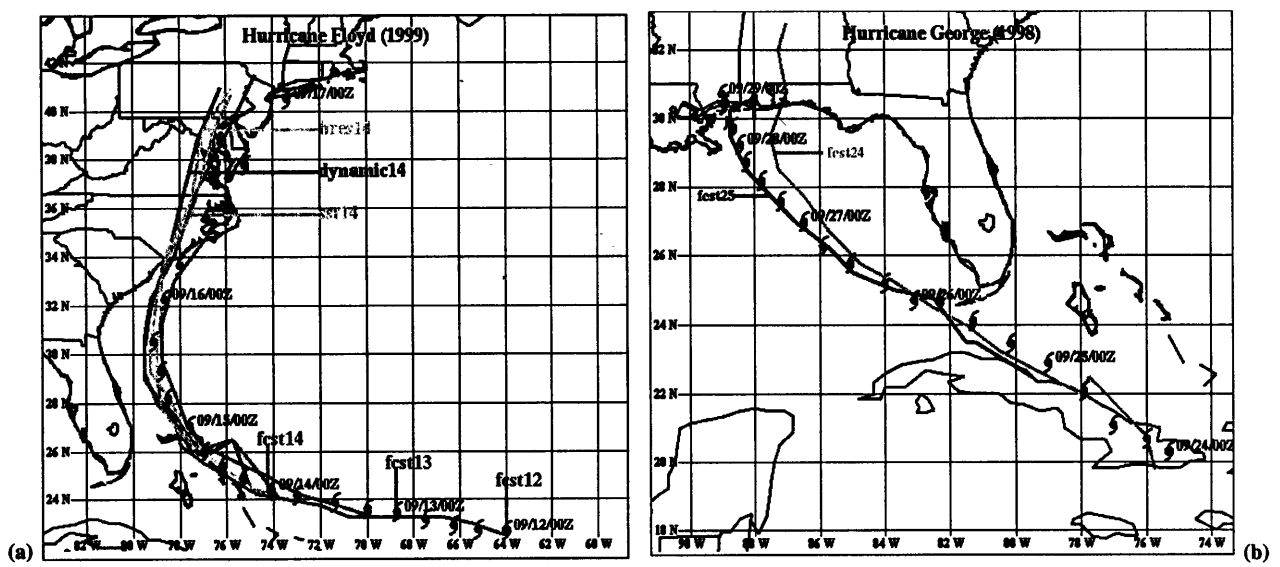

Fig. 9 OMEGA hurricane track forecasts for hurricanes (a) Floyd and (b) Georges.

to forecast for 72 hours. Nevertheless, the 24, 48, and 72 hour errors compare favourably with the 1998 NHC average errors[39] of 156, 268, and 374 km, respectively. Fig. 9 presents the track forecasts for the historic cases of this set: Floyd (Fig. 9a) and Georges (Fig. 9b). Fig. 10 presents the OMEGA track forecasts for the six storms (Beryl, Chris, Debby, Florence, Helene, and Xangsane) forecasted operationally in 2000. Recently, a 5-day simulation of hurricane Floyd was completed that included the interaction of the remnant of hurricane Floyd and the following storm, hurricane Gert (Fig. 11).

The ultimate in dynamically adapting multiscale simulations, however, is represented in a recent test problem with approximately 100 km resolution globally, but with 10 km resolution maintained dynamically over the core of hurricane Floyd. A figure showing the entire grid in this simulation would be unenlightening, but Fig. 12 shows the low level wind speed 24 hours after initialization; the reader can imagine the high-resolution that exists over the hurricane (circled) in the upper right quadrant of this figure.

The Earth is not the only place with strong scale-interactions. The atmosphere of Mars is also strongly affected by the interaction of the global tidal circulation with very local scale topographic features. A Martian version of OMEGA was created to permit the simulation of these types of interactions. Initial global and mesoscale simulations have been performed using this experimental tool. Fig. 13 shows the results from a mesoscale simulation in which a constant uniform low-level wind was applied and allowed to interact with the terrain around the Mars Pathfinder landing site. Shown are the terrain as constructed using data from the Mars Orbiter Laser Altimeter (MOLA) instrument[40], the OMEGA grid created using this data, the complex wind field that evolves when the specified uniform low-level wind interacts with the terrain, and the transport of dust injected using a simple model for the lifting of dust off the Martian surface.

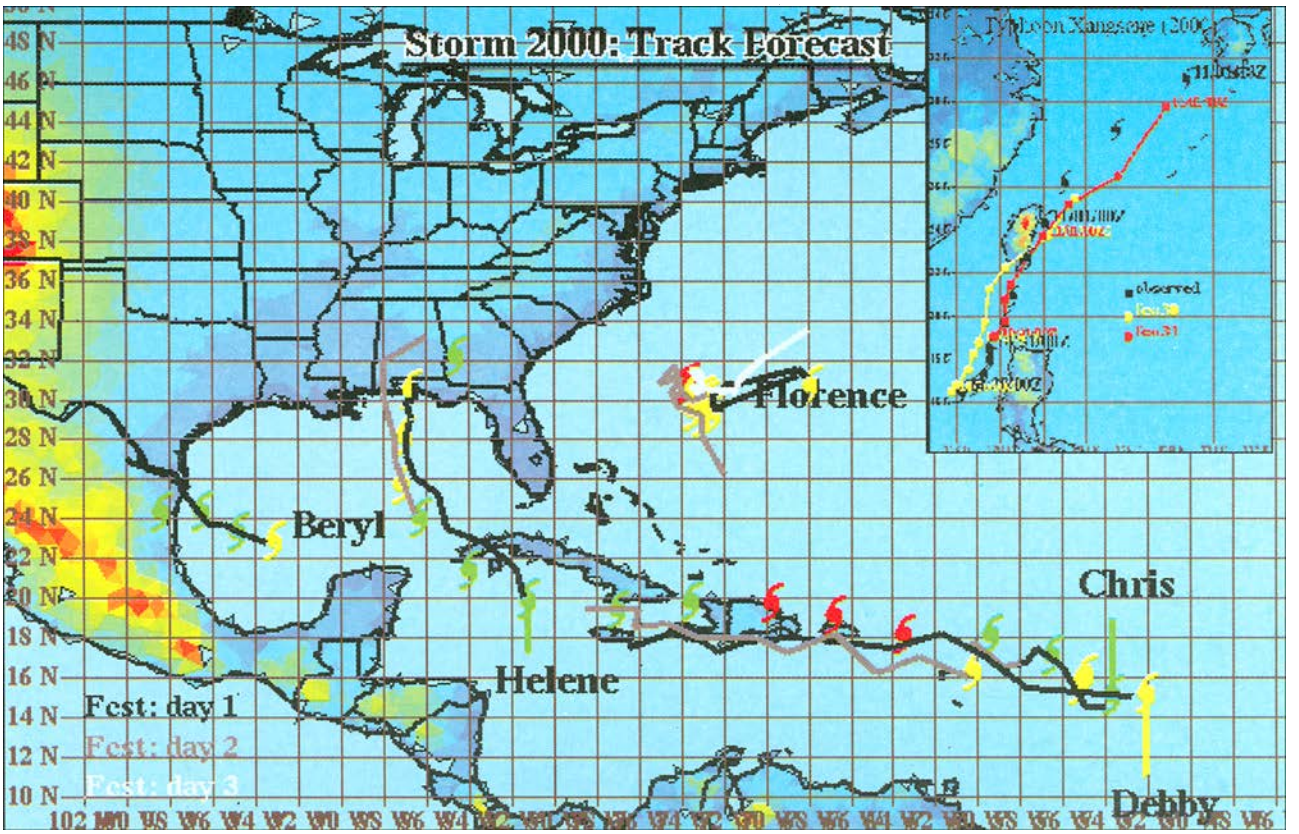

Fig. 10 Operational hurricane track forecasts produced by OMEGA for six storms in 2000

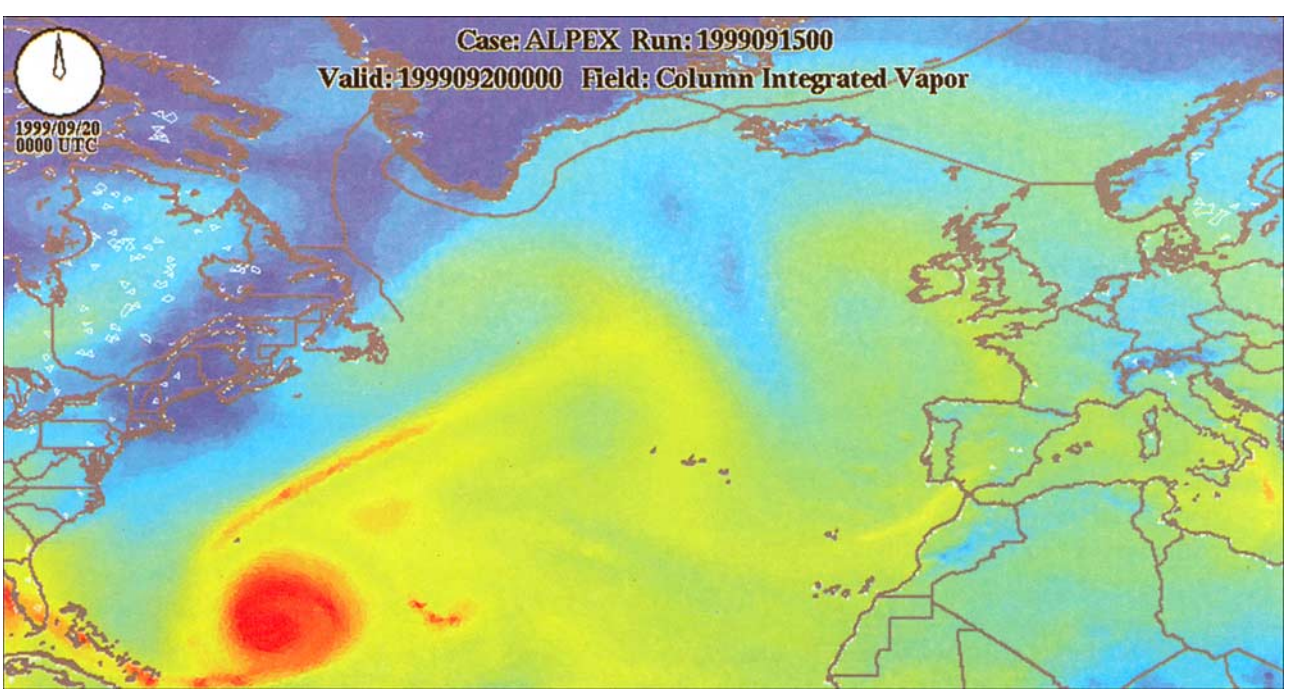

Fig. 11 This OMEGA simulation of hurricane Floyd captured the weak interaction between the remnant of hurricane Floyd (centre) and the following storm, hurricane Gert (lower left). This figure shows the integrated column water vapour 5 days after initialization. The static grid used for this simulation had a resolution ranging from 75 km down to 15 km

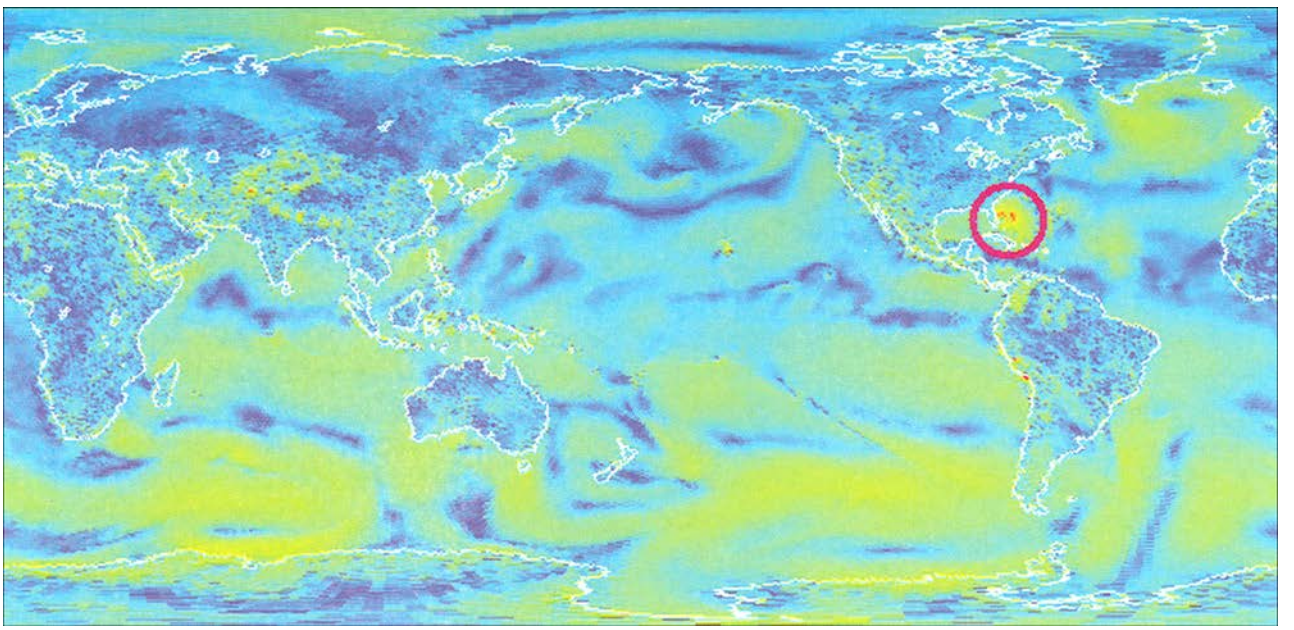

Fig. 12 This simulation of hurricane Floyd used a global grid with resolution of 100 km with dynamic adaptation maintaining 10 km resolution over the storm. This image shows the near-surface wind speed 24 hours after initialization

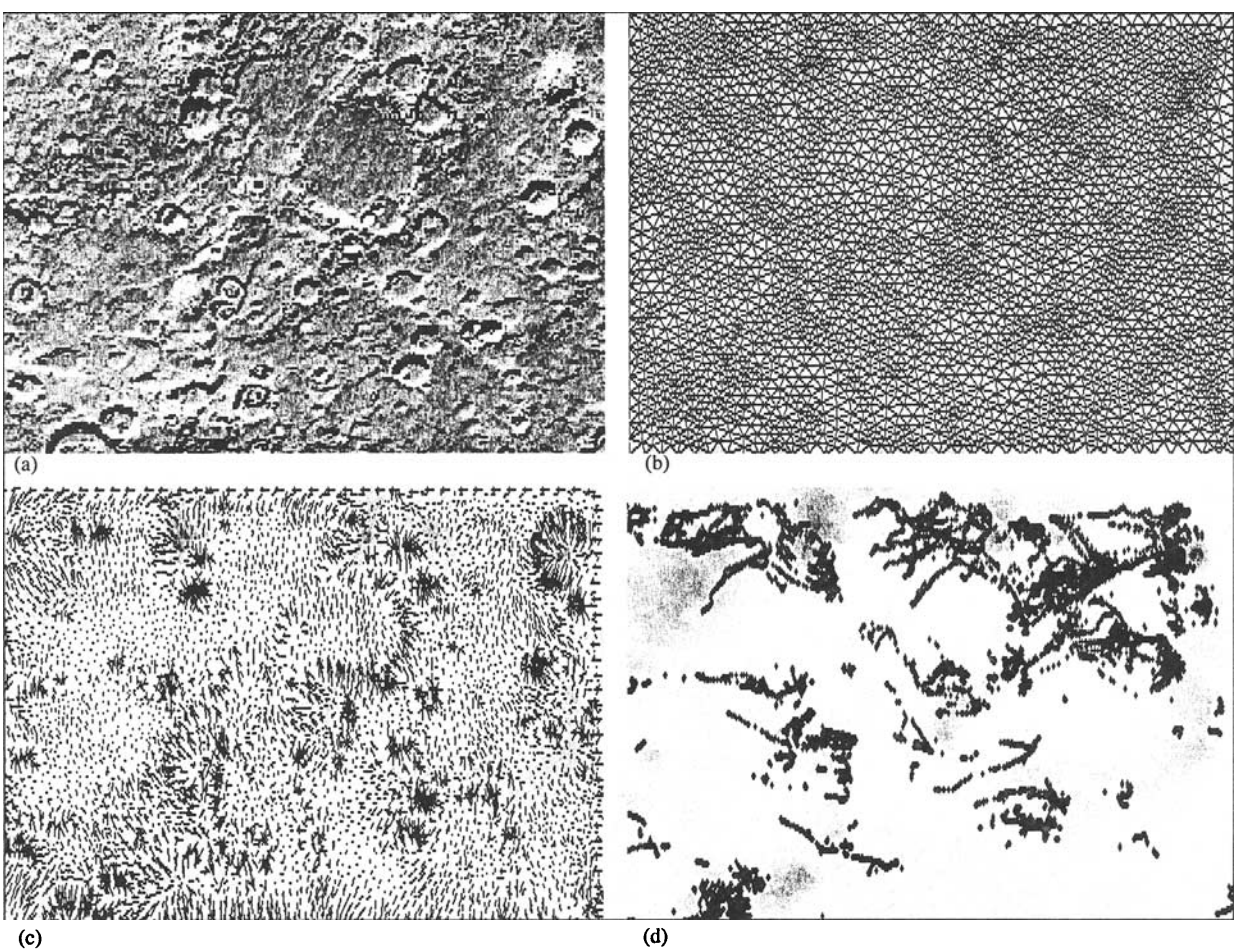

Fig. 13 A close-up view of (a) the Martian terrain derived from MOLA, (b) the OMEGA grid, (c) the 24 hr OMEGA simulated flow field, and (d) the 24 hr OMEGA simulated dust injection into the Martian atmosphere. The domain ranges from 60-70° S and 150-180°E

4 Multiscale Ocean Simulation System

Another natural implementation of the unstructured grid paradigm is in the construction of ocean circulation models. To explore the utility of this paradigm to ocean circulation, the Multiscale Ocean Simulation System (MOSS) was constructed. MOSS used a modified version of the OMEGA grid generator, utilizing bathymetric data instead of elevation data as the basis for the grid generation. A modification to the OMEGA data structures enabled the implementation of a varying number of cells in each vertical column. This allowed the creation of three-dimensional grids with the vertical index starting at the ocean surface and working downwards with a different number of cells in each column, permitting the creation of ocean grids with high vertical resolution near both the surface and the bottom of the ocean. Fig. 14 demonstrates this for a grid of the Mediterranean Sea (planview and cross-section); it shows the ability to provide high resolution not only horizontally, but also in critical vertical regions: the air-sea interface and the ocean floor.

5 Conclusions

A new paradigm has emerged for geophysical fluid dynamic simulation. The use of unstructured adaptive grids provides higher resolution of critical surface features (*e.g.*, terrain elevation, land/water fraction, land use, vegetation) in atmospheric and oceanic simulation. In addition, the coupling of this technique with dynamic adaptation provides the ability to automatically provide high resolution over those regions where the physics requires it. The use of dynamically adapting unstructured grids has been demonstrated in the OMEGA modelling system; a nascent capability demonstrating the implementation of this paradigm in oceanic simulation is existent in MOSS.

This new paradigm opens the way for simulation systems where the physics determines the actual resolution requirements, rather than having to maintain the highest resolution *potentially* required *anywhere* in the domain over the *entire* domain. The OMEGA modelling system can adapt its grid both statically and dynamically to different criteria such as fronts, clouds, hurricanes, and plumes. For real-time forecasting, the capability of grid adaptivity, given

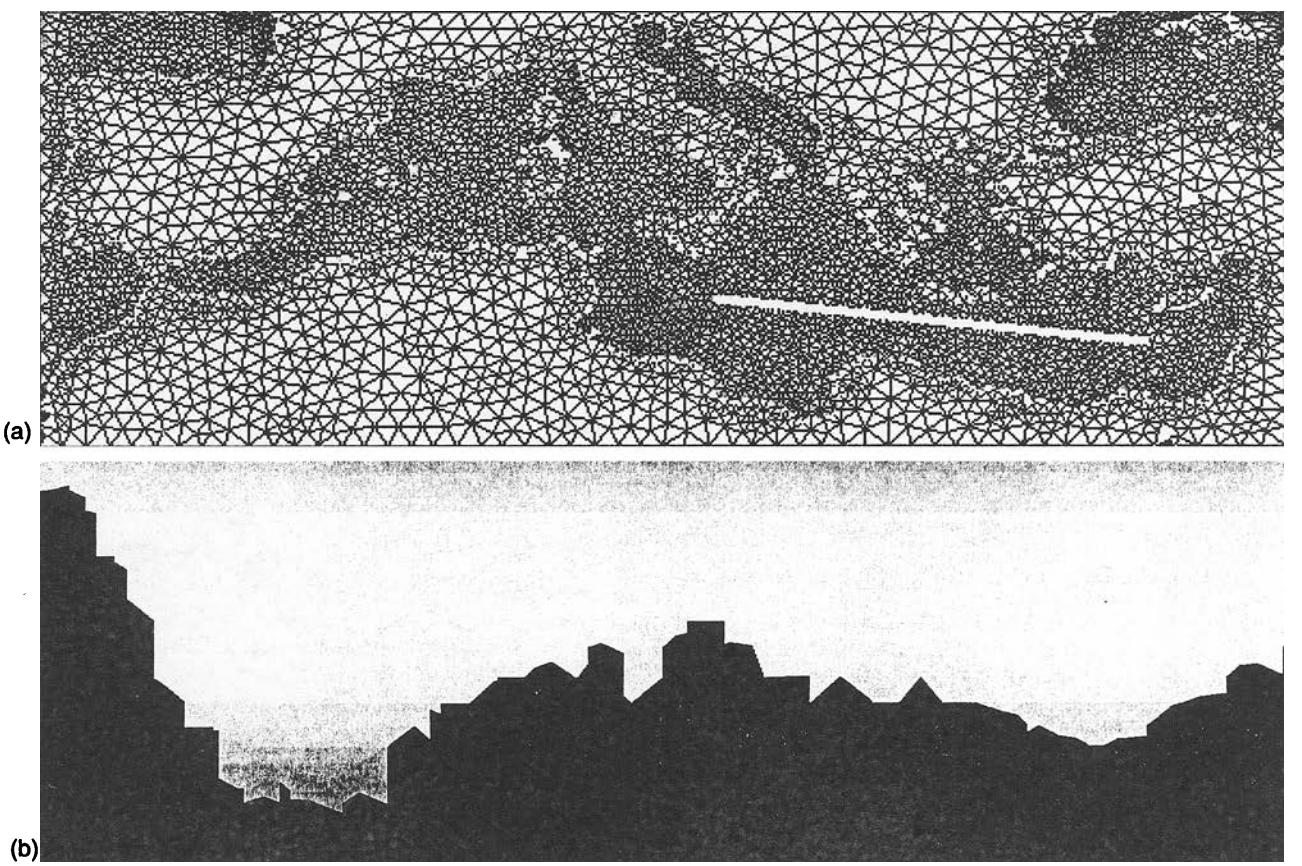

Fig. 14 (a) A MOSS grid of the Mediterranean Sea showing (white line) the location of (b) a West-East cross-section of this grid showing the variable number of layers in each column

the omnipresent CPU constraint may be the decisive factor in bringing the next significant jump in model resolution to operational reality. The flexibility of this paradigm is also important. With relative ease, the modelling system created for the terrestrial atmosphere has been adapted to the Martian atmosphere. In a different direction, the data structures and grid generation was readily modified to support oceanic simulation.

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