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NEUTRINO OSCILLATIONS: A SOLUTION TO ANOMALIES IN OBSERVATIONS OF SOLAR, ATMOSPHERIC AND LABORATORY NEUTRINOS

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Evidence in support of neutrino oscillations is now compelling. A pedagogic introduction to vacuum neutrino oscillations and resonant flavour conversion is presented in this article. The results from solar and atmospheric neutrinos are discussed and their combined implications on neutrino properties are outlined. Laboratory tests of neutrino oscillations are also summarized.

Key Words: Vacuum Oscillations; MSW Effect; Adiabatic and Non-Adiabatic Cases; Solar and Atmospheric Neutrinos

1 Introduction

Neutrino physics has come of age. The entities which sneaked into physics in the garb of a 'desperate measure' to solve the problems associated with beta decay – undetectable particles which elope with energy and angular momentum – have now matured into a useful probe for alternate models of particle physics itself. Twenty years from their postulation, neutrinos were experimentally detected and more than a decade later it was established that neutrinos associated with electrons were not the same as those that go with muons. They were incorporated in the Standard Model (SM) as massless, difficult to detect, particles. That is where the matter rested. Or did it?

Painstaking experiments of increasing size and number kept on looking for neutrinos from terrestrial and celestial sources. Decades of patient work have now borne fruit. It is now established beyond any doubt that neutrinos from the Sun, from cosmic rays, and possibly even from accelerators do not behave as theoretically predicted. It is intriguing that the observations all fall in line if neutrinos of one flavour (say, the electron-type) are assumed to oscillate to those of another flavour (say, the muon-type). This merits close examination because the oscillation mechanism is truly the tip of an iceberg: it is possible only if neutrinos are massive. In fact, through neutrino oscillations, quantum mechanics has permitted us to probe the smallest masses measured so far. More importantly, in the SM neutrinos must be massless. The results from neutrino experiments are thus setting the stage for an extension of the Standard Model itself. Massive neutrinos also have major ramifications on cosmology, the early universe, and astrophysics. These important matters are discussed elsewhere in this volume. Our brief is to give a broad introduction to the concept of neutrino oscillations and to relate it to the recent experimental data.

2 Vacuum Oscillations

We begin with a discussion of oscillation between two neutrino flavours. The idea of neutrino oscillations[1,2] is rooted in quantum mechanics. The time evolution of a stationary state $|\psi_k\rangle$ (in units such that $\hbar = c = 1$) is:

$$|\psi_k(t)\rangle = |\psi_k\rangle \ \exp\left(-i\,E_k t\right), \qquad \ldots(1)$$

where E_k is the energy eigenvalue corresponding to $|\psi_k\rangle$. Thus, the stationary state vectors at different times differ simply by an overall phase change. The time evolution of an arbitrary, *i.e.*, non-stationary, state, $|\psi\rangle$, is more complicated. For such a state we can write at $t = 0$:

$$|\psi(0)\rangle = \sum_k a_k\, |\psi_k\rangle,$$

where a_k are constants. Using eq.1 one finds:

$$|\psi(t)\rangle = \sum_k a_k\, |\psi_k\rangle\, \exp\,(-iE_k t).$$

For neutrinos, the basic assumption is that the familiar electron and muon neutrinos (ν_e and ν_μ) – the *flavour* eigenstates – are not the mass eigenstates (*i.e.,* the stationary states) ν_1 and ν_2, but their superpositions:

$$|\nu_e\rangle = |\nu_1\rangle\, c + |\nu_2\rangle\, s; \qquad |\nu_\mu\rangle = -|\nu_1\rangle\, s + |\nu_2\rangle\, c,$$

where $c = \cos\theta$ and $s = \sin\theta$. For two flavours a single angle, θ, suffices to completely specify one basis in terms of the other.

Consider now the state vector of a ν_e produced at $t = 0$. Thus, initially $|\psi(0)\rangle = |\psi_{\nu_e}\rangle = c|\psi_1\rangle + s|\psi_2\rangle$. If the stationary states $|\psi_1\rangle$ and $|\psi_2\rangle$ correspond to energies E_1 and E_2, respectively, then at a later time the state vector will be:

$$|\psi(t)\rangle = c|\psi_1\rangle\, \exp(-iE_1 t) \,+\, s|\psi_2\rangle\, \exp(-iE_2 t).$$

The probability, $P(\nu_e,0;\nu_\mu,t)$, of the state $|\psi(t)\rangle$ (originating as a ν_e at $t = 0$) appearing as a ν_μ is $|\langle\psi_{\nu_\mu}|\psi(t)\rangle|^2$ and can be expressed as:

$$P(\nu_e,0;\nu_\mu,t) = c^2 s^2 |-\exp(-iE_1 t) + \exp(-iE_2 t)|^2.$$

The neutrinos are expected to have small masses, m_i, and are in the ultrarelativistic regime ($E_i \simeq p + \frac{m_i^2}{2p}$) where p ($\gg m_i$) is the magnitude of the neutrino momentum.[a] In this situation:

$$\begin{aligned} P(\nu_e,0;\nu_\mu,t) &= 4c^2 s^2 \sin^2\left(\frac{\Delta}{4p}t\right) \\ &= \sin^2 2\theta \sin^2\left(\frac{\pi x}{\lambda}\right), \quad \ldots(2) \end{aligned}$$

where $\Delta = |m_2^2 - m_1^2|$ and

$$\lambda = 2.47\text{m}\left(\frac{E}{\text{MeV}}\right)\left(\frac{\text{eV}^2}{\Delta}\right), \qquad \ldots(3)$$

is the so-called *oscillation length* expressed here in terms of the neutrino energy $E = p$. We use x and t interchangably, since the neutrinos move with essentially the speed of light (c = 1). In the right hand side of eq.2 the first factor is a consequence of the 'mixing' while the second factor leads to the 'oscillatory' behaviour. For vacuum oscillations, the former, dependent on the mixing angle θ, is a constant but in the Mikheyev-Smirnov-Wolfenstein (MSW) matter effect, discussed later, it changes with the matter density.

From eq.2,

$$\begin{aligned} P(\nu_e,0;\nu_e,t) &= 1 - P(\nu_e,0;\nu_\mu,t) \\ &= 1 - \sin^2 2\theta \sin^2\left(\frac{\pi x}{\lambda}\right). \end{aligned}$$

It is seen from the above that $P(\nu_e,0;\nu_e,t)$ can be less than or equal to unity. The essential ingredients for this are twofold:

1. The neutrinos must be massive and *non-*degenerate.

2. The mass eigenstates of the neutrinos – ν_1, ν_2 – must be different from the flavour eigenstates – ν_e, ν_μ.

3 Resonant Flavour Conversion

Of the variants of this basic theme of neutrino flavour change the most prominent is the MSW[4] matter induced resonant effect. The mass square matrix for the two-neutrino system in the flavour basis is:

$$\begin{aligned} M_f^2 &= \left(\frac{m_1^2 + m_2^2}{2}\right) I \\ &\quad + \frac{1}{2}\begin{pmatrix} -\Delta\cos 2\theta_V & \Delta\sin 2\theta_V \\ \Delta\sin 2\theta_V & \Delta\cos 2\theta_V \end{pmatrix}, \quad \ldots(4) \end{aligned}$$

where I stands for the 2×2 identity matrix. It is easy to check that M_f^2 is diagonalised by the orthogonal matrix characterised by the neutrino mixing angle – denoted here by θ_V. The average mass squared, $(m_1^2 + m_2^2)/2$, plays no rôle in our discussion of neutrino oscillations; only the difference in (mass)^2 (Δ) and the mixing angle (θ_V) are of relevance here.

The MSW effect originates from the additional interactions of a neutrino in a medium resulting in a

[a] Here, we assume that all neutrinos share the same momentum p. To include the effect of momentum spread one must consider a neutrino wave-packet. In most situations, such an analysis makes no practical difference[3] from the above.

varying mass akin to the effective mass of an electron moving in a solid. The most well-known application of the MSW mechanism has been to the case of solar neutrinos where the ν_e, produced in the interior by fusion reactions, on their way out must pass through dense regions of the Sun. Both charged and neutral weak interactions with the material can contribute to the effective mass. However, since the solar neutrinos typically carry an energy of a few MeV while the muon's mass is $\sim$ 105 MeV, the charged current interaction for the ν_μ–*i.e.*, $\nu_\mu + e^- \to \nu_e + \mu^-$– is kinematically forbidden. The upshot of this is that through neutral current interactions ν_e and ν_μ receive the same contributions to their masses–adding to the irrelevant term proportional to the identity in eq.4– while the charged current contributes only to ν_e. As a consequence, in place of eq.4 we now have:

$$M^2_{MSW} = \left(\frac{m_1^2+m_2^2}{2}+\delta_{nc}\right) I + \frac{1}{2}\begin{pmatrix} -\Delta\cos 2\theta_V + 2\delta_{cc} & \Delta\sin 2\theta_V \\ \Delta\sin 2\theta_V & \Delta\cos 2\theta_V \end{pmatrix}, \quad \ldots(5)$$

with $\delta_{cc} = 2\sqrt{2}G_F n_e(r)E$, where $n_e(r)$, the number density of electrons, is a function of the distance from the solar centre, r. The mixing angle in matter obtained from eq.5 is:

$$\theta = \frac{1}{2}\arctan\left(\frac{\Delta\sin 2\theta_V}{\Delta\cos 2\theta_V - \delta_{cc}}\right). \quad \ldots(6)$$

It is important to bear in mind that both θ and the oscillation length λ are now functions of the energy and r. Two cases are distinguished.

A) The adiabatic case: If

$$\frac{d\theta}{dr} \ll \frac{1}{\lambda},$$

i.e., the change in the mixing angle is small over one oscillation length, then the usual adiabatic approximation of quantum mechanics is valid: an eigenstate adjusts as the Hamiltonian gradually changes. If an electron neutrino is produced in a region of the Sun where the mixing angle is θ and later detected on Earth (mixing angle θ_V) then

$$P(\nu_e,0;\nu_e,t) = \sum_{i=1}^{2} |\langle\nu_e|(\nu_i)_{Earth}\rangle\langle(\nu_i)_{Sun}|\nu_e\rangle|^2 = [1+\cos 2\theta_V \cos 2\theta]/2. \quad \ldots(7)$$

Interference terms average out to zero once integrations over the production region (the size of the Sun) and the detector size are performed.

B) The non-adiabatic case: Here

$$\frac{d\theta}{dr} \gg \frac{1}{\lambda}.$$

This is a situation where the change in the mixing angle is very rapid and there is a 'jumping probability' from one eigenstate to another ($\nu_1 \leftrightarrow \nu_2$). A resonance point is defined from eq.6 by $\delta_{cc} = \Delta\cos 2\theta_V$. In terms of the number density of electrons at resonance, $(n_e)_{res}$, the jumping probability, X, can be written as:

$$X = \exp\left[-\frac{\pi}{4}\frac{\Delta}{E}\frac{\sin^2 2\theta_V}{\cos 2\theta_V}\frac{1}{|\frac{d\,\ln n_e}{dr}|_{res}}\right].$$

Strictly speaking, the above expression is valid when the number density n_e varies linearly with r near the resonance point. In the non-adiabatic case the electron neutrino survival probability (see eq. (7)) is:

$$P(\nu_e,0;\nu_e,t) = [1+(1-2X)\cos 2\theta_V \cos 2\theta]/2.$$

The experiments on solar neutrinos see less ν_e than theoretically predicted. The atmospheric muon neutrinos show a depletion which is also dependent on the length of travel. These matters are discussed in the subsequent sections. The natural explanation of these observations is in terms of neutrino oscillations since the detectors are neutrino flavour sensitive. The common procedure is to confront the experimental data in terms of two-flavour neutrino oscillations, discussed in this and the previous section. It is found that the preferred solutions require two rather different mass splittings: $\Delta_{sol} \sim 10^{-5}$ eV^2 and $\Delta_{atm} \sim 10^{-3}$eV^2. The LSND accelerator experiment has seen evidence of $\nu_\mu - \nu_e$ and $\bar{\nu}_\mu - \bar{\nu}_e$ oscillation corrsponding to a mass splitting $\Delta_{LSND} \sim 10$ eV^2. The vastly different mass-splittings that are needed to explain these experimental results indicate that oscillations between more than two neutrino species is operative in Nature. It then becomes imperative to extend the discussion to more than two flavours.

4 Three Flavours

The LEP experiments established that there are three light neutrinos which is also supported by the requirements of nucleosynthesis in the early universe. The

ν_τ has been directly experimentally detected recently. A natural question then is how do experiments constrain three-neutrino mixing? The results of the previous sections are readily generalised to the case of three or more flavours. Here, for the purpose of illustration, we focus on three neutrinos. The 3×3 neutrino mass matrix M_ν can be diagonalized according to[b]

$$U^\dagger M_\nu U = \text{diag}(m_1, m_2, m_3), \qquad \ldots(8)$$

where U is the Maki-Nakagawa-Sakata[2] unitary matrix which relates flavour (α) and mass (i) eigenstates of neutrinos through $\nu_\alpha = U_{\alpha i}\nu_i$ and m_1, m_2, m_3 are the eigenvalues. For the discussion below, we assume U to be real and there is no CP-violation.

We let the flavour states be represented by greek indices with $\alpha = 1,2$, and 3 to correspond to e, μ, and τ respectively. The general expression for the probability that an initial ν_α of energy E gets converted to a ν_β after travelling a distance L in vacuum is

$$P_{\nu_\alpha \nu_\beta} = \delta_{\alpha\beta} - 4 \sum_{j>i} U_{\alpha i} U_{\beta i} U_{\alpha j} U_{\beta j} \sin^2(\frac{\pi L}{\lambda_{ij}}),$$

where, as in eq.3, $\lambda_{ij} = 2.47\text{m}(E/\text{MeV})(\text{eV}^2/\Delta_{ij})$, $\Delta_{ij} = |m_j{}^2 - m_i{}^2|$. The detailed expressions of the various survival and transition probabilities depend on the spectrum of Δ_{ij} and the three angles in the MNS[2] mixing matrix U relating the flavour states to the mass eigenstates.

As in the quark sector, different parametrizations of U have appeared in the literature. A popular form[5] is $U = R_{23}R_{13}R_{12}$ where R_{ij} denotes the rotation matrix in the ij-plane. This results in

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12} & c_{23}c_{12} - s_{23}s_{13}s_{12} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12} & -s_{23}c_{12} - c_{23}s_{13}s_{12} & c_{23}c_{13} \end{pmatrix}$$

To accommodate the CHOOZ constraint one has to choose $s_{13} \simeq 0$ in the above mixing matrix. Further, in this parametrization, the mass spectrum constraint, $\Delta_{atm} \sim 10^{-3}$ eV^2 $\gg$ $\Delta_{sol} \sim 10^{-5}$ eV^2, then permits the identification θ_{12} (θ_{23}) $\equiv$ solar (atmospheric) mixing angle in a two-flavour picture.

Another form of the MNS matrix which has appeared in the literature[6] is obtained by defining $U = R_{13}R_{12}R_{23}$. This yields:

$$U = \begin{pmatrix} c_{12}c_{13} & c_{12}s_{13}s_{23} + s_{12}c_{23} & -c_{12}s_{13}c_{23} + s_{12}s_{23} \\ -s_{12}c_{13} & -s_{12}s_{13}s_{23} + c_{12}c_{23} & s_{12}s_{13}c_{23} + c_{12}s_{23} \\ s_{13} & -c_{13}s_{23} & c_{13}c_{23} \end{pmatrix}$$

A third parametrization which has been used is $U = R_{13}R_{12}R_{23}$. This yields:

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13}c_{23} + s_{13}s_{23} & c_{13}s_{12}s_{23} - s_{13}c_{23} \\ -s_{12} & c_{12}c_{23} & c_{12}s_{23} \\ s_{13}c_{12} & s_{13}s_{12}c_{23} - c_{13}s_{23} & s_{12}s_{13}s_{23} + c_{13}c_{23} \end{pmatrix}$$

This form was used for a combined analysis of the atmospheric, accelerator, and reactor neutrino data[7]. There the above choice of U has the merit that θ_{23} does not appear in the expressions for the probabilities for the laboratory experiments.

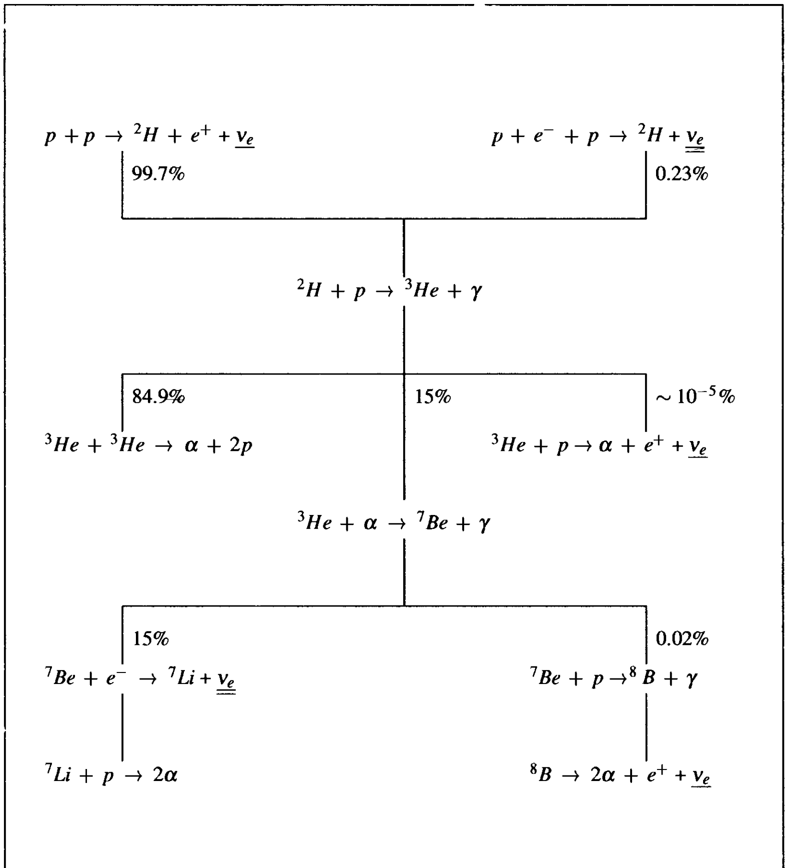
In a three-flavour framework it is possible that a combined analysis to find the allowed range of parameters would uncover regions in the parameter space sensitive to the presence of the third generation which cannot be probed in the two-flavour limit. For a combined analysis of the solar and atmospheric data this turns out not to be the case. The result of the CHOOZ experiment, which requires the U_{e3} element to be small, plays a key role here. It can be shown that this decouples the solar and atmospheric sectors and the three-flavour case effectively reduces to separate two-neutrino mixing-like scenarios[8]. On the other hand, a three-flavour analysis of the atmospheric neutrino data along with accelerator and reactor neutrino results was shown to permit new solutions beyond the ones in the two-generation approximation[7].

If the results of the LSND experiment receive independent confirmation then the third different Δ will make it imperative to consider mixing between four neutrinos. Since only three active neutrinos (ν_e, ν_μ, and ν_τ) are known, the fourth state would have to be a sterile neutrino without a charged lepton partner and immune to the weak interactions. Indeed, much work has already been done in this direction[9].

5 Solar Neutrinos

The Sun generates heat and light through fusion reactions. There are cycles of such reactions which take place in the Sun (*e.g.*, the *pp* chain, the CNO cycle, *etc.*) where neutrinos are produced at various stages, the effective process being $4p \to \alpha + 2e^+ + 2\nu_e$ + 25 MeV. The *pp* chain of reactions is shown in Fig.1.

[b] For Majorana neutrinos the l.h.s of eq.8 reads $U^T M_\nu U$.



The *pp*-chain

Fig. 1 The reactions of the *pp*-chain. The probability of a particular reaction is shown as a percentage. The neutrinos are shown underlined. Those with double underlines are monoenergetic.

Neutrinos produced in each reaction of the cycle have a characteristic spectrum whose shape is precisely fixed by weak interaction theory while the absolute normalization is determined by the solar model. Over the years, the solar models have undergone a series of refinements so that now there is a broad agreement within the community with regard to the spectrum of neutrinos expected from the Sun[10].

The largest flux of neutrinos–the *pp* neutrinos–also happen to be of low energy ($E < 0.411$ MeV) and have been detected so far only by the radiochemical GALLEX and SAGE experiments. Neutrinos from other reactions, *e.g.*, the *Be* and *B* neutrinos, are of higher energies and have been under scrutiny for several decades at the Homestake Chlorine experiment. The Super-Kamiokande (SK) detector uses a water Čerenkov technique which has the comparatively high threshold of 5 MeV and is sensitive only to *B* neutrinos. SNO (Sudbury Neutrino Observatory) is the most recent solar neutrino experiment. It uses heavy water. At SNO the neutrinos are detected by three processes, namely, (a) charge current (CC) break up of the deuteron, (b) electron scattering by the neutrino, and (c) neutral current (NC) break up of the deuteron.

$$\nu_e + d \to p + p + e^- \quad \text{(CC reaction)}, \ldots (9)$$

$$\nu + e^- \to \nu + e^- \quad \text{(scattering)}, \quad \ldots (10)$$

$$\nu + d \to \nu + p + n \quad \text{(NC reaction)} \ldots (11)$$

For the scattering (10) and NC (11) reactions, ν stands for any active neutrino; it has to be borne in mind that for the μ and τ flavours only the Z exchange contribution is present for the former reaction while for the ν_e there is an additional (dominant) piece from W-exchange. For the CC reaction and for scattering, the electrons are detected by the emitted Čerenkov

Table I

The mismatch between observed and predicted solar neutrino fluxes seen by the Homestake (Radiochemical, Chlorine), GNO, Gallex and SAGE (Radiochemical, Gallium), Super-Kamiokande (Water Čerenkov), and SNO (D_2O Čerenkov) experiments. E_{th} is the threshold energy for detection.

Experiment	Homestake	Ga	Super – K	SNO ES	SNO CC
$\left(\frac{\text{Observed rate}}{\text{Predicted rate}}\right)$	0.335 ± 0.029	0.584 ± 0.039	0.459 ± 0.017	0.473 ± 0.074	0.347 ± 0.027
E_{th} (MeV)	0.8	0.233	5.0	5.0	6.75

radiation and hence their energy spectrum is directly measured, as at SK.

Solar neutrino experiments observe a smaller flux than the theoretical predictions. The situation is summarised[11] in Table I. This is the *solar neutrino problem.* Notice that the SNO NC measurement does not appear in this Table.

Attempted solutions of this problem based on modifications of the Standard Solar Model have been unsuccessful and the general consensus today is that these results are a consequence of neutrino oscillations. The radiochemical detectors are sensitive only to ν_e while the Čerenkov detectors have a lower cross-section for ν_μ or ν_τ than for ν_e. Therefore, if neutrino oscillations are operative and some of the ν_e from the Sun appear on Earth as a neutrino of a different flavour then the detectors will register a lower count. The parameters which best fit the data are usually found by minimizing a χ^2 appropriately defined in terms of the data[c] in Table I. At the present time, there are three viable solutions which fit the results of all the experiments[13]: one depends on MSW flavour conversion in the Sun with $\Delta = 4.45 \times 10^{-5}$ eV^2 and $\tan^2\theta = 0.36$ (MSW LMA), while the two others are variants of the vacuum oscillation scenario with $\Delta = 1.53 \times 10^{-7}$ eV^2 and $\tan^2\theta = 0.68$ (LOW) or $\Delta = 4.53 \times 10^{-10}$ eV^2 and $\tan^2\theta = 2.94$ (VAC).

A major progress in the understanding of the solar neutrino problem has come from the recent publication of the SNO NC measurements[14]. Here the neutron from reaction (11) is detected via the gamma ray emitted when it is absorbed by a proton to produce a deuteron. They find a neutrino flux consistent with the prediction from the Standard Solar Model[d], an indication that oscillations are taking place to an active flavour rather than to a sterile neutrino, since all active neutrinos participate in the NC reaction with the same strength while the sterile neutrino does not at all. When this result is included in the analysis there is a clear preference for the MSW LMA solution with $\Delta = 6.07 \times 10^{-5}$ eV^2 and $\tan^2\theta = 0.41$, while the VAC and LOW solutions are included only at 3σ or higher[15].

6 Atmospheric Neutrinos

Atmospheric neutrinos originate from cosmic rays and were first detected by the KGF experiment in India[16]. Electron and muon neutrinos (in this section we refer to both neutrinos and antineutrinos as simply 'neutrinos') are produced through pion and kaon decay sequences (*e.g.*, $\pi^+ \to \mu^+ + \nu_\mu \to (e^+\bar{\nu}_\mu\nu_e) + \nu_\mu$) in the atmosphere and their energies can be in the GeV range. As this decay chain indicates, one expects neutrinos of the muon type and electron type in the ratio of 2:1. A proper analysis must include the different energy spectra of the two flavours of neutrinos (the ν_e are softer than the ν_μ), their detection efficiencies, *etc.* which are modelled by Monte Carlo simulations. Several experiments, including Kamiokande and IMB, which monitored atmospheric neutrino fluxes had reported an anomaly. The results are usually presented in terms of

$$R = \frac{\left[(\nu_\mu + \overline{\nu}_\mu)/(\nu_e + \overline{\nu}_e)\right]_{\text{obsvd}}}{\left[(\nu_\mu + \overline{\nu}_\mu)/(\nu_e + \overline{\nu}_e)\right]_{\text{MC}}}$$

where MC denotes the Monte-Carlo simulated value. Consistency with theoretical expectations would require $R = 1$, but both Kamiokande and IMB measured it to be more than 3σ smaller. The area received a new fillip with the presentation of high statistics data from the huge Super-Kamiokande experiment. They found[17] $R = 0.61 \pm 0.03 \pm 0.05$ for sub-GeV neutrinos ($E <$

[c] SK has also measured the energy spectrum of the scattered electon. This is often incorporated in the χ^2 analysis. In addition, moments of the spectrum have been suggested as variables sensitive to neutrino oscillation[12]

[d] For the NC reaction, SNO finds $\frac{\text{Observed rate}}{\text{Predicted rate}} = 1.008 \pm 0.123$.

1.33 GeV), [e] in agreement with other experiments[18].

What could be the reason behind this anomaly? How reliable are the theoretical calculations of the atmospheric ν fluxes? There are many uncertainties in the flux calculations (*e.g.*, the primary cosmic ray flux, the K/π ratio, geomagnetic effects, *etc.*) and, indeed, absolute fluxes calculated by different groups do differ somewhat. However, the ratio ν_μ/ν_e is a rather stable prediction and it is unlikely that the observed data can be attributed to an incorrect flux estimation.

Another possibility is the mis-identification of the electrons and muons. This issue has been addressed by the SK collaboration by verifying that the number of $\mu \to e$ decay events observed in the data agrees with expectation.

As a further bolster supporting the neutrino oscillation hypothesis, Super-Kamiokande reported a strong dependence of the number of observed events on the ν_μ zenith angle. The ratio of the number of upward to downward ν_μ events was found to be $0.52^{+0.07}_{-0.06} \pm 0.01$ to be compared with the theoretical expectation of $0.98 \pm 0.03 \pm 0.03$ (see refs.[17] and [19]). No such up-down asymmetry was seen for the ν_e events. The neutrinos produced in the atmosphere pass through vastly different distances to reach the detector depending on the zenith angle of the event (tens of km for downward events to several thousand km for upward events). Thus the upward going events are generated by neutrinos which have a much larger time (or travel distance) for oscillating. The measured asymmetry can be used to set bounds on the allowed mixing angles and mass splitting of neutrinos. Taken together, the atmospheric neutrino data prefer 1×10^{-3} eV$^2 \le \Delta \le 4 \times 10^{-3}$ eV^2 and $\sin^2 2\theta \simeq 1$.

7 Accelerator Neutrinos

Neutrino beams are produced in a typical hadron accelerator by allowing a high energy proton beam to hit a target producing a beam of pions and kaons. The $\pi^\pm$ and $K^\pm$ are allowed to decay downstream producing a beam of ν_μ and $\bar{\nu}_\mu$. Accelerator neutrino experiments utilise these beams.

Over the years, several accelerator neutrino experiments have searched for oscillations[19]. CDHSW, a disappearance experiment measuring $P_{\overline{\nu}_\mu\overline{\nu}_\mu}$, obtained the 90% C.L. bound $P_{\nu_\mu\nu_\mu} \gtrsim 0.95$. E776 at BNL looked for $\overline{\nu}_\mu \to \overline{\nu}_e$ oscillations using the appearance technique. They found $P_{\overline{\nu}_\mu\overline{\nu}_e} \le 1.5 \times 10^{-3}$ (90% C.L.). E531 at Fermilab looked for $\nu_\mu - \nu_\tau$ oscillation, by ν_τ appearance. Their results can be translated into a 90% C.L. upper limit on the oscillation probability: $P_{\nu_\mu\nu_\tau} \lesssim 2 \times 10^{-3}$. All these are consistent with no neutrino oscillation.

More recent accelerator experiments, CHORUS and NOMAD[20], have used the neutrino beam produced at the 450 GeV CERN SPS. Using a baseline of about 1 km, they look for $\nu_\mu \to \nu_\tau$ oscillations. None of them see a positive evidence.

Another category of accelerator neutrino experiments use primary proton beams of lower energy. LSND, a typical experiment of this type, uses an 800 MeV proton beam to produce π^+ which are allowed to decay at rest producing a beam of $\bar{\nu}_\mu$ from the subsequent μ^+ decay. The LSND group searches for $\overline{\nu}_\mu \to \overline{\nu}_e$ oscillations using $\overline{\nu}_e$ appearance[21]. The $\overline{\nu}_e$s produce neutrons *via* the reaction $\overline{\nu}_e p \to e^+ n$ which in turn are captured by protons producing a 2.2 MeV γ. An excess of beam-on events with a γ of the above energy in time and space coincidence with an electron in the energy range 36 MeV $< E_e <$ 60 MeV is considered as a signal for $\overline{\nu}_e$. The mean source-detector distance is 30 metres. They report an excess of $51.0^{+20.2}_{-19.5} \pm 8.0$ events over the estimated background which, if interpreted in terms of neutrino oscillations, corresponds to a probability $P_{\overline{\nu}_\mu\overline{\nu}_e}$ of $(0.31 \pm 0.12 \pm 0.05)\%$. The LSND group have also looked for $\nu_\mu \to \nu_e$ oscillations and find results consistent with the above[21]. The energy and distance scales involved translate to a neutrino mass splitting $\Delta \sim \mathcal{O}(10)$ eV^2 for small mixing.

The other on-going accelerator based appearance experiment searching for $\overline{\nu}_\mu \to \overline{\nu}_e$ oscillations is KARMEN at the ISIS spallation neutron facility. Their KARMEN2 result[22] is consistent with no oscillation and they exclude large parts of the parameter region allowed by LSND.

MiniBooNE[24] is a new experiment that will search for neutrino oscillations in a set-up very similar to that of the LSND experiment. It will utilise muon-type neutrinos produced using Fermilab's 8 GeV proton Booster. ν_μ or $\bar{\nu}_\mu$ beams can be obtained by selecting the charge of the decaying pions by choosing the polarity of the focussing magnetic horn. The average

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[e] For multi-GeV neutrinos SK reported $R = 0.66 \pm 0.06 \pm 0.08$[17].

energy of the neutrinos is 1 GeV and the detector is located at a distance of about 500m downstream. Data taking will begin soon and it is expected that enough statistics will be accummulated in a year (5×10^{20} protons on target) to scan the LSND allowed region at a 5σ level. A larger experiment on similar lines–BooNE–is planned if the LSND results are verified.

The first and only long baseline experiment currently in operation is K2K[25] in Japan. It uses a neutrino beam produced at KEK, a near detector 300 m downstream, and SuperKamiokande as its far detector at a distance of 250 km. The wideband ν_μ beam is produced using 12 GeV protons from KEK and have an average energy of 1.3 GeV. For this experiment L/E is in the right range to probe the atmospheric mass splitting $\sim 10^{-3}$ eV^2. The beam at source, according to Monte Carlo estimates, consists of 98% ν_μ, 1% $\bar{\nu}_\mu$, and 1% ν_e. K2K primarily intends to look for $\nu_\mu - \nu_\tau$ oscillations in the disappearance mode but is also sensitive to ν_μ - ν_e oscillations. The near detector consists of a 1kt water Cerenkov detector based on the same principles as SK. This cancels many systematic errors in event rate estimates for the far detector. There is also a fine grained near detector for a precise measurement of the ν_μ energy spectrum and flux, ν_e contamination, and the beam profile. The direction of the beam is fixed to a precision of < 0.1 mrad. Data taking for K2K has started and the preliminary results favour oscillations with parameters preferred by the atmospheric neutrino data.

8 Reactor Experiments

Nuclear reactors provide a copious source of $\bar{\nu}_e$ from fission reactions and, in fact, the first experimental detection of neutrinos was using a reactor. These anti-neutrinos have energies in the few MeV range. The absence of a collimated beam limits the distance at which the detector can be placed–the most recent reactor experiment, KAMLAND, has a previously unimaginable baseline of a few hundred km. Several reactor experiments searched for neutrino oscillations and found no positive evidence. Among the earlier experiments, the maximum exclusion was from Bugey which measured the spectrum of $\overline{\nu}_e$, coming from the pressurized water reactors running at the Bugey nuclear power plant, at 15, 40, and 95 metres using neutron detection techniques. Their 90% C.L. exclusion contour implied[26] $1 - P_{\overline{\nu}_e\overline{\nu}_e} \leq 0.05$.

A strong constraint on neutrino oscillations came recently from the CHOOZ nuclear reactor experiment[27]. Here the detector is located 1.0 km from the source and looks for oscillations in the disappearance mode. The experiment finds $\sin^2 2\theta < 0.10$ if Δ is in the atmospheric range. As discussed earlier, this has a strong bearing on three-neutrino solutions which address the solar and atmospheric neutrino results.

KAMLAND, the Kamioka Liquid Scintilator Anti-neutrino Detection facility[28] is designed to look for anti-neutrinos from nuclear reactors at different locations in Japan within a distance of a few hundred kilometres. The baseline for KAMLAND is about a hundred times more than earlier reactor experiments which enables it to probe neutrino oscilation parameters in the solar neutrino MSW LMA solution range. Results from this experiment are eagerly awaited.[f]

9 Future Directions

A major emphasis of future research will be in using super-beams or neutrino factories as the neutrino source[29]. Typically, for a neutrino factory, a beam of muons in the 20 – 50 GeV range is allowed to decay in the straight section of a muon storage ring. The resultant neutrino beam consists of an equal admixture of ν_μ and $\bar{\nu}_e$ ($\bar{\nu}_\mu$ and ν_e) for a storage ring containing μ^- (μ^+). Super-beams are high luminosity beams of ν_μ with a less than *per cent* level admixture of ν_e obtained from a hadron machine through the conventional route of pion and kaon decays. Typically beams with an average energy of 1 - 2 GeV are envisioned. These facilities will permit not only precise studies of neutrino oscillations but also of possible CP-violation in the neutrino sector.

Much effort is currently devoted in evaluating the possibility of setting up an India-based Neutrino Observatory[30]. Oscillation of atmospheric neutrinos and those from neutrino factories will be major thrust areas for this experiment.

Conclusions

In this article, the basic idea of neutrino oscillations and its application to explain recent experimental re-

[f] See the articles by S Goswami, C V K Baba and S Pakvasa & J W F Valle for discussions on the first KamLAND data.–Ed.

sults have been discussed. Neutrino oscillations pin down the mass and mixing in the neutrino sector–a pointer to new physics.

There are other areas of physics, *e.g.*, in supernovae and in the early universe, where neutrino oscillations may play an important role.

We have restricted ouselves exclusively to mass-mixing generated neutrino oscillations. It is possible for oscillations to be driven by other means, notably, violation of the equivalence principle[31, 32], neutrino magnetic moment[33, 34], *etc.*

An apology is due for the paucity of references to the original papers but this is largely ameliorated by the availability of the latest as well as the earlier references on the internet[35].

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