

SUPERSYMMETRY AND NEUTRINO MASS

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(Received 31 January 2003; Accepted 30 June 2003)

The existence of neutrino mass and mixing is a strong pointer towards physics beyond the Standard Model. An overview of the possibility of having neutrino masses in supersymmetric theories is attempted here. Some of the recent works reviewed suggest Dirac masses, whereas others include Majorana masses as well. Side by side, it is shown how R-parity violating supersymmetry opens new avenues in the neutrino sector. Reference is also made to light sterile neutrinos, nearly degenerate neutrinos and neutrinos acquiring masses from hard supersymmetry breaking terms which are suppressed by the Planck scale. In several of the cases, it is pointed out how the models that give neutrino masses and mixing have independent motivations of their own, and can be tested in accelerator experiments.

Key Words: R-Parity Violation; Light Sterile Neutrinos; Quasidegenerate Neutrinos; Neutrino Masses Suppressed by Planck Scale

1 Introduction

As has been amply established in the other articles in this volume, there is a strong evidence nowadays in favour of neutrino masses. In addition, the solar¹ and atmospheric² neutrino data have their most obvious explanation in neutrino oscillation, requiring mixing among neutrinos, or, more generally speaking, in the leptonic sector, in analogy with quark mixing which is controlled by the Cabibbo-Kobayashi-Maskawa (CKM) matrix³. However, in contrast to quark mixing, the most favoured explanations of the solar and atmospheric neutrino deficits require very large—even close to maximal—mixing between the first two families and the last two. Side by side, the data indicate a hierarchy of mass splitting, the mass-squared difference being in the range 10^{-3} - 10^{-2} eV² between the second and the third families, and, most favourably, 10^{-5} - 10^{-4} eV² between the first and the second. Though such splittings are most often translated into a corresponding hierarchy in the masses themselves, the existence of near-degenerate neutrinos, too, cannot be ruled out.

According to many, all this is an indication of physics beyond the Standard Model. To see why, let us recall that, thanks to the electrically neutral character of neutrinos, they can have both Dirac and Majorana masses. While the second possibility which

entails lepton number violation clearly entails new physics, albeit at high scale, the first one can be *prima facie* dismissed as a 'trivial' extension in the form of a right-handed neutrino component for each family. However, the fact that such a right-handed neutrino has none of the strong, weak and electromagnetic interactions is curious, if not suggestive of some new interaction in which it takes part. The extreme suppression of neutrino Yukawa couplings necessitated by sub-eV Dirac masses is also puzzling. Side by side, if the LSND claim suggesting the disappearance of ν_μ 's is to be taken seriously⁴, we most likely need a fourth light neutrino, sterile in nature. Since the mass of a sterile vectorlike neutrino is not protected by any symmetry, and since we can hardly think of any new physics scale below a TeV or so, a light sterile neutrino, if it is there at all, warrants a drastically novel mechanism for its justification.

The new physics scale to which appeal has mostly been made to understand neutrino masses is that pertaining to Grand Unified Theories (GUT), restricted to be at least⁵ about 10^{16} GeV. However, there are other motivations for physics beyond the Standard Model within the TeV scale itself. One such is the so-called naturalness problem which reflects our lack of understanding why the Higgs mass (and consequently the electroweak scale M_{EW}) should be stable against quadratically divergent radiative corrections.

The most popular solution to this problem has been offered in terms of supersymmetry (SUSY), a symmetry between bosons and fermions, which can provide the necessary cancellations to control the large radiative corrections⁶. Most importantly, it is possible to keep the Higgs mass within acceptable limits even if SUSY is broken *in mass*, so long as the breaking scale (characterising the boson-fermion splitting) is approximately within the TeV scale. Side by side, the observation that the threshold effects arising from TeV scale SUSY breaking ensures better convergence of the three coupling constants at the GUT scale provides an added impetus to SUSY⁷.

In the minimal SUSY Standard Model (MSSM)⁸, the particle spectrum of the Standard Model (SM) gets doubled, there being a superpartner for each known particle, apart from the necessity of two Higgs doublets which lead to three neutral and a pair of mutually conjugate singly charged scalars. There is no experimental evidence yet for any of these superparticles; collider experiments have set lower bounds of about 100 GeV upwards on most of them. Further consequences of SUSY also depend on the details of the spectrum which in turn is crucially dependent on the SUSY breaking mechanism. We know that SUSY has to be broken at any rate if it is there, since we do not observe degenerate superpartners for the SM particles. No completely acceptable SUSY breaking scheme has been found so far, although most studies depend upon a scenario based on $N = 1$ supergravity (SUGRA)⁹ where gravitational interactions with a ‘hidden sector’ characterised by a high scale ($O(\sqrt{M_p M_{EW}})$) lead to soft SUSY breaking terms in the observable sector. In addition, schemes of SUSY breaking, for example, via gauge interactions of a messenger sector¹⁰ or via anomaly terms¹¹ have also been investigated.

The question is: since the search for physics beyond the Standard Model has found a strong candidate in SUSY, could SUSY also be responsible for neutrino masses (and mixing), the clue that nature seems to dangle so tantalisingly in front of us? If that be so, then the mass patterns answering to the solar and atmospheric neutrino data should not only depend on certain specific aspects of the SUSY model, but also impose constraints on it. It may also be more convincing if models are built not just to answer questions on neutrinos but have independent motivations of their own from the viewpoint of SUSY as well. Side by side, since the the search for SUSY is already an im-

portant goal of accelerator experiments, it should be really interesting to look for the particular signatures of such theoretical schemes as are able explain the observations in the neutrino sector. In other words, the issue of neutrino masses could provide not only useful guidelines for theorisation, but might also end up predicting specific experimental signals in high-energy colliders. The present article is aimed at discussing some of these possibilities.

In very general terms, some of the ways in which SUSY can be of special significance to neutrino masses are as follows:

- The phenomenon of SUSY can provide new scales (in addition to that brought by GUT in which most SUSY theories are embedded). These scales open up additional possibilities in the neutrino sector and can be helpful in explaining mass hierarchies. Also, some features of the SUSY theory might help us in understanding ultra-small Yukawa couplings.
- The extended particle spectrum in SUSY can lead to mechanisms for mass generation, for example, through additional radiative effects.
- The possibility of low-energy lepton number violation inbuilt in certain types of SUSY theories might lead to the generation of Majorana masses.
- SUSY could explain a naturally light sterile neutrino, in case we need it to explain the observed data.

In section 2 we discuss Dirac masses in presence of SUSY. Section 3 is devoted to Majorana neutrinos in SUSY scenarios, where lepton number violation takes place at high-scale. Section 4 contains a summary of neutrino mass generation mechanisms in R-parity violating SUSY where the low-energy Lagrangian has lepton number violation. In section 5 we discuss respectively the issues of degenerate neutrinos in SUSY and neutrino masses from unusual SUSY breaking terms. We conclude in section 6.

2 When Lepton Number is Conserved—Dirac Masses in SUSY

If one takes the hierarchy in neutrino mass splitting to be an indication of the hierarchy in the masses them-

selves, then, assuming that the solar and atmospheric neutrino deficits are due to $\nu_\mu - \nu_\tau$ and $\nu_e - \nu_\mu$ oscillations respectively, the two heaviest neutrinos are about 10 to 11 orders of magnitude smaller in mass than the τ and the μ . The simplest extension of the Standard Model spectrum that explains the above masses is one right-handed neutrino per generation. However, the onus then falls on us to explain the wide disparity of Yukawa couplings that is responsible for the huge mass splitting within the same families, as indicated above. The question is: can SUSY provide some explanation of such disparity?

Normally, with right-handed neutrino superfield N , one would expect a term in the superpotential of the form

$$W_N = y_\nu \bar{N} L H_2 \quad \dots(1)$$

where H_2 is the Higgs doublet giving mass to fermions with $T_3 = 1/2$. Of course, here one would find it hard to justify the smallness of y_ν . On the other hand, one can forbid such a term with the help of some discrete symmetry Z_n , and assume instead a higher-dimensional term¹²

$$W_N = y_\nu \frac{k}{M_P} Z \bar{N} L H_2 \quad \dots(2)$$

where k is a coupling constant $O(1)$, M_P is the Planck mass, and Z is a superfield that is invariant under the Standard Model gauge group. Then the superpotential given in eqn (2) is allowed, as against the one in eqn (1), if the various superfield have the following charge assignments under Z_n :

$$Z_n(Z) = 1; Z_n(N) = 1; Z_n(f) = 0 \quad \dots(3)$$

f being any of the chiral superfields in MSSM. Note that (2) implies the existence of a non-renormalizable term in the superpotential, which, in the SUGRA context, can arise as an effective coupling, duly suppressed by M_P .

If A_z and F_z are respectively the vacuum expectation values (vev) of the scalar and auxiliary components of Z (the latter being the SUSY breaking vev), then the Dirac mass for the neutrino is given by

$$m_\nu = \frac{k A_z v_2}{M_P} \quad \dots(4)$$

If A_z is of the same order as the square root of the SUSY breaking vev F_z , then, in a SUGRA scenario, $A_z \simeq M_X = \sqrt{m_{3/2} M_P} \simeq 10^{11} \text{ GeV}$ (where $m_{3/2}$

is the gravitino mass), giving $m_\nu \simeq 10^3 \text{ eV}$. This is an unacceptably large value unless one has near-degenerate neutrinos. The solution, therefore, lies in having $A_z \ll \sqrt{F_z}$, i.e. in the SUSY conserving vev being much smaller than the SUSY breaking one. This can be realised, for example, in an O’Raifeartaigh-type model, where a hierarchy between the scalar and pseudoscalar components can be envisioned upon generating an effective low-energy scalar potential for Z through the condensation of some chiral superfield in the SUSY breaking sector through non-perturbative effects:

$$A_z = 16\pi^2 k x m_{3/2} \quad \dots(5)$$

with $x = O(1)$. This yields neutrino Dirac masses of the order of 10^{-2} eV —the right order of magnitude!

It may be relevant to comment here that A_z can directly lead to small neutrino masses even without the above mechanism in a gauge mediated SUSY breaking (GMSB) scenario, where the gravitino is a much lighter object. In such a case, however, mass splitting between the electron and muon neutrinos becomes considerably smaller than what has been reported above, and can at most place us in the solution space corresponding to vacuum oscillation. Since the current data strongly disfavour such a solution, the GMSB option is perhaps not of much value in this context.

There is a very similar approach¹³ which puts the mechanism of neutrino mass generation in a somewhat bigger perspective. It is well-known that in MSSM, there is no natural way to keep the Higgsino mass parameter μ , a Supersymmetry-conserving mass, within the TeV scale. The parameter occurs in a term $\mu H_1 H_2$ in the superpotential, and it is not clear why it is not as big as any of the masses in the SUSY breaking sector. On the other hand, it is highly desirable to have it around the electroweak scale so that the minimisation condition for the scalar potential can be naturally satisfied.

With the μ -problem in view, one can think of a global symmetry group G protecting the Higgs masses, and forbidding the μ -term in the original superpotential. There can again be a gauge singlet superfield X associated with the SUSY breaking sector, transforming non-trivially under the group G , which finds its way into the superpotential via the term

$$W_X = \frac{1}{M_P} X H_1 H_2 \quad \dots(6)$$

Remember also that F_X , the vev auxiliary component of X , is of the order of M_X^2 ($\simeq 10^{22}$ GeV²) as defined above. One can immediately see that this ‘SUSY breaking’ vev gives rise to a μ -parameter in the range of $M_X^2/M_P \simeq m_{3/2}$. Thus a value of μ in the naturally expected range is ensured.

Suppose now that the same scenario includes a right-handed neutrino superfield N . If lepton number is conserved, then one can envision a scenario where a term in the superpotential of the form LNH_2 is disallowed by the charge assignments of the corresponding superfields under G . However, the term ENH_1 may still be allowed if N , also a gauge singlet, has a different charge compared to the Standard Model superfields. In this case, the source of the neutrino Dirac mass is

$$W_N = \frac{1}{M_P} X L N H_2 \quad \dots(7)$$

Again, a scalar vev for X on the order of M_X leads to inadmissibly large neutrino masses. The interesting contribution comes again from the auxiliary component which yields a Dirac mass, given by

$$m_D \simeq \frac{M_X^2 v_2}{M_P^2} \simeq \frac{v_2^2}{M_P} \quad \dots(8)$$

which turns out to be around 10^{-3} eV. Once more, one is left with the task of preventing neutrino masses from the SUSY-conserving vev of X . This has been done in the literature by introducing additional $U(1)$ symmetries in the SUSY breaking sector¹⁴, and preventing, in a style similar to the one mentioned earlier, the scalar potential from developing a vev in the lowest order.

Of course, the neutrino mixing pattern still needs to be explained, the particular problem being the possibility of large mixing both between the first and second generations and the second and third. The only reasonable explanation of this can come from a texture of the $XNLH_2$ coupling. However, there is no clear understanding of how a suitable texture can be naturally ensured.

An alternative explanation of small Dirac neutrino masses has been offered from the assumption that the gauge singlet superfield N is prevented by a global from having an NLH_2 term, but that the charges are such that a heavy superfield H' can replace H_2 in the superpotential¹⁵. Now, if there is mixing between H_2 and H' after SUSY breaking, the Yukawa coupling of N with the resultant physical Higgs can have

a suppression of $\frac{M_{EW}}{m_{H'}}$ compared to the unsuppressed Yukawa strengths of the corresponding charged lepton. This suppression can be used to account for the smallness of the Dirac neutrino masses compared to those of their charged partners, although a nearly bi-maximal texture remains unexplained.

Before we end this section, some remarks about radiatively generation Dirac masses in SUSY models are in order. Radiative generation is possible through diagrams mediated by neutral gauginos¹². However, the fact that the right-handed neutrino superfield is a Standard Model gauge singlet implies that such a diagram can contribute only when additional gauginos are present. An extension of the gauge group is therefore a necessity for such a mechanism to be operative.

3 $\Delta L = 2$ Terms in the Lagrangian: Majorana Masses

If there is lepton number violation at a high scale M , it is possible to have $\Delta L = 2$ neutrino mass terms via the dimension-5 operator¹⁶

$$\mathcal{L}_5 = \frac{\lambda}{M} LLHH \quad \dots(9)$$

which gives neutrino masses on the order of v^2/M . The most obvious model that gives rise to Majorana masses of this kind has heavy right-handed neutrinos in the scale M , with both Yukawa couplings with $SU(2)$ doublets and L -violating mass terms of its own:

$$\mathcal{L}_N = \frac{M}{2} NN + y_N \bar{N} LH \quad \dots(10)$$

so that it is possible to generate very small neutrino mass eigenstates without requiring inordinately small Yukawa coupling. This is the essence of the well-known seesaw mechanism^{16,17}. It is also seen that one obtains the light neutrino masses in the expected range when M is in the Grand Unification scale of about 10^{16} GeV. Thus it is customary to treat N as a right-handed neutrino belonging to the fundamental representation of a GUT group such as $SO(10)$. In addition, there can be a right-handed neutrino in each generation, so that M in general can be a matrix, real and symmetric. The prediction of two large mixing angles, however, is a dilemma that is yet to be satisfactorily addressed in GUT-inspired textures of M .

In what way can SUSY contribute to the Majorana mass generation mechanism of the above type? Of

course, there are numerous versions of SUSY GUT's where various issues related to the requisite texture have been discussed. The recourse to SUSY GUT's has also its motivation in the observation that the convergence of the three gauge coupling constants at high scale is better achieved in a SUSY scenario where new threshold effects become important around the TeV scale. Particular SUSY breaking schemes such as GMSB have been also invoked to explain the large flavour mixing necessitated by the observed data. An important component, to which most existing studies of the subject owe their richness, is the question of compatibility of large mixing with the limits on lepton flavour violating processes such as $\mu \rightarrow e\gamma$ or $\tau \rightarrow e\gamma$. In the SUSY context, the mismatch between the neutrino and sneutrino mass matrices at low-scale is a source of potentially dangerous flavour violation, and thus the parameters of the theory must be subject to strong constraints. A large number of investigations in this direction can be found in the literature¹⁸.

Here we discuss the following question: in addition to the GUT scale, can the additional scale(s) made available to us from SUSY breaking be of any use in Majorana mass generation? In relation to Dirac masses, we have already found an answer in the affirmative. Now we include a brief discussion related to Majorana masses^{12, 13, 19}.

One can, for example, extend the picture outlined above by including a $\Delta L = 2$ mass term for the right-handed neutrino N through a term of the form $X^\dagger NN$ in the superpotential. In exact analogy with the situation where the μ -parameter is generated around the electroweak scale, this mass may also be generated only from the vev of the auxiliary component of the field X . The Majorana mass is thus given by

$$m_N \simeq \frac{MX^2}{M_P} \simeq v \quad \dots(11)$$

v being of the same order as the electroweak scale. For the Dirac mass, however, the scalar vev of X may be used in the term $XLNH_2$, yielding

$$m_D = \frac{m_X v_2}{M_P} \quad \dots(12)$$

so that the seesaw mass for the light neutrino(s) is given by $m_D^2/m_N \simeq v^2/M_P$, which is in the desired range. Note that unlike in the case of Dirac neutrinos, here one does not need to invoke a special mechanism such as a $U(1)$ symmetry to keep the vev of the scalar

component of X small. The only thing that needs justification is the Majorana masses for N as well as the μ -term only out of the auxiliary component of X . For the latter such a condition is essential if one has to have μ in the electroweak scale. It has been argued that for the former a similar fate is expected since the two terms are of the same form. A contribution from the scalar vev of X would make the Majorana mass much higher and the lighter eigenvalue much lower than is admissible, unless one can again think of a symmetry to restrict the scalar component to a vev within the TeV scale. It is with an argument of this kind that the contributions from the scalar component of X have sometimes been dropped from the low-energy effective theory, although this may not be totally above criticisms of arbitrariness.

Once more, the problem of generating two large mixing angles is not solved in a construction of the above type. For that, one has to assume specific textures in the XNLH couplings, which in turn requires appropriate modelling of the SUSY breaking sector.

It has also been shown in several works²⁰ that the above principle can be extended to include a light sterile neutrino, something that one might require if the claims from LSND are confirmed. An additional gauge singlet superfield S has to be added for this purpose. It is, however, necessary to suppress the Yukawa coupling for S , and allow S to develop a small mass via the scalar component of X , devised to be small by mechanisms mentioned earlier. This can be ensured through an appropriate assignment of charge for S under the group G .

Unlike the case of Dirac neutrinos, a Majorana neutrino can have loop-induced masses without any extension of the gaugino sector. The second reference in ref.[12] shows the representative diagrams from which such contributions can come. The contribution, for which explicit expressions can be found in the literature, depend on the effective $\tilde{\nu}\tilde{\nu}$ as well as left-right mixing in the sneutrino mass matrix. Such loop contributions, in regions where they are substantial, may be required to explain (a) the mixing pattern, and (b) the mass pattern itself where, for example, the right-handed neutrino develops a large Majorana mass from the scalar component of X , making the seesaw mechanism viable.

Before we conclude, two comments may be in order. First, the mechanisms discussed in this section and the last one are important, although they

might not be uniformly successful in explaining textures etc. The reason for this is the fact that in addition to the conventional GUT scale, here the scale m_X is made available to us by the SUSY breaking scheme. This enables one to explore newer avenues to address the yet unanswered questions, hopefully by combining inputs from the SUSY breaking scale with those from the GUT scale. Secondly, the kind of models outlined here favour, among other things, additional right-handed sneutrinos in the electroweak scale. In fact, since this sneutrino mass is not restricted by the Z -decay width, it can even become the lightest supersymmetric particle (LSP). This can have considerable implications in collider phenomenology as well as issues related to dark matter¹³.

4 $\Delta L = 1$ Terms in the Lagrangian–R-Parity Violating SUSY

Let us next consider the case where neutrinos can acquire masses through lepton number violating interactions at low-energy. This is realised in R-parity violating SUSY²¹, where R-parity is defined as $(-)^{3B+L+2S}$. It can be seen from this definition that all superparticles have $R = -1$ whereas R equals 1 for all the Standard Model particles. It is also clear from above that R-parity, a multiplicatively conserved quantity, can be violated when B or L is violated. This makes it possible for a superparticle to decay into two or more Standard Model particles, thus rendering the LSP unstable.

In SUSY, squarks and sleptons, all spinless objects, carry lepton and baryon numbers. It is thus possible to violate one of these numbers by one unit while the other is conserved. This is not possible in the Standard Model due to the gauge structure and particle assignments. Such a provision in the SUSY scenario makes it free from the danger of destabilising the proton.

To see how R-parity violation actually takes place in SUSY, let us remember that the MSSM superpotential is given by

$$W_{MSSM} = \mu H_1 H_2 + h_{ij}^l L_i H_1 E_j^c + h_{ij}^d Q_i H_1 D_j^c + h_{ij}^u Q_i H_2 U_j^c \dots (13)$$

where the last three terms give the Yukawa interactions corresponding to the masses of the charged leptons and the down-and up-type quarks, and μ is the Higgsino mass parameter.

When R-parity is violated, the following additional terms can be added to the superpotential²²:

$$W_R = \lambda_{ijk} L_i L_j E_k^c + \lambda'_{ijk} L_i Q_j D_k^c + \lambda''_{ijk} U_i^c D_j^c D_k^i + \varepsilon_i L_i H_2 \dots (14)$$

with the λ'' -terms causing B-violation, and the remaining ones, L-violation. The need to avoid proton decay usually prompts one to have *only one* of the two types of nonconservation at a time. Since we are concerned with neutrino masses here, we will consider only lepton number violating effects.

The λ - and λ' -terms have been widely studied in connection with various phenomenological consequences, enabling one to impose various kinds of limits on them²³. Their contributions to neutrino masses can be only through loops, and their multitude (there are 36 such couplings altogether) makes the necessary adjustments possible for reproducing the requisite values of neutrino masses and mixing angles. We shall come back to these ‘trilinear’ effects later.

More interesting, however, are the three bilinear terms²⁴ $\varepsilon_i L_i H_2$. Since there are only three terms of this type, the model looks simpler and more predictive with them alone as sources of R-parity violation. This is particularly so because the physical effects of the trilinear terms can be generated from the bilinears by going to the appropriate bases²⁵. In addition, they have interesting consequences of their own²⁶, since terms of the type $\varepsilon_i L_i H_2$ imply mixing between the Higgsinos and the charged leptons and neutrinos. In this discussion, we shall assume, without any loss of generality, the existence of such terms involving only the second and third families of leptons.

The scalar potential in such a case contains the following terms which are bilinear in the scalar fields:

$$V_{\text{scal}} = m_{\tilde{L}_3}^2 |\tilde{L}_3|^2 + m_{\tilde{L}_2}^2 |\tilde{L}_2|^2 + m_1^2 |H_1|^2 + m_2^2 |H_2|^2 + B\mu H_1 H_2 + B_2 \varepsilon_2 \tilde{L}_2 H_2 + B_3 \varepsilon_3 \tilde{L}_3 H_2 + \mu \varepsilon_3 \tilde{L}_3 H_1 + \mu \varepsilon_2 \tilde{L}_2 H_1 + \dots \dots (15)$$

where m_{L_i} denotes the mass of the i th scalar doublet at the electroweak scale, and m_1 and m_2 are the mass parameters corresponding to the two Higgs doublets. B , B_2 and B_3 are soft SUSY-breaking parameters.

An immediate consequence of the additional (L-violating) soft terms in the potential is a set of non-vanishing vacuum expectation values (vev) for the sneutrinos. This gives rise to the mixing of electroweak gauginos with neutrinos (and charged leptons) through the sneutrino-neutrino-neutralino (and

sneutrino-charged lepton-chargino) interaction terms. The hitherto massless neutrino states enter into the neutralino mass matrix through such mixing and acquire see-saw masses, where the high scale is supplied by the massive states. The parameters controlling the neutrino sector in particular and R-parity violating effects in general are the bilinear coefficients ε_2 , ε_3 and the soft parameters B_2, B_3 .

For a better understanding, let us perform a basis rotation and remove the R-parity violating bilinear terms from the superpotential by suitably redefining the lepton and Higgs superfields. This, however, does not eliminate the effects of these terms, since they now take refuge in the scalar potential. The sneutrino vev's in this rotated basis (which are functions of both and the ε 's and the soft terms in the original basis) trigger neutrino-neutralino mixing. Consequently, the 6×6 neutralino mass matrix in this basis has the following form:

$$\mathcal{M} = \begin{pmatrix} 0 & -\mu & \frac{gv}{\sqrt{2}} & -\frac{g'v}{\sqrt{2}} & 0 & 0 \\ -\mu & 0 & -\frac{g'v'}{\sqrt{2}} & \frac{g'v'}{\sqrt{2}} & 0 & 0 \\ \frac{gv}{\sqrt{2}} & -\frac{g'v'}{\sqrt{2}} & M & 0 & -\frac{gv_2}{\sqrt{2}} & -\frac{gv_3}{\sqrt{2}} \\ -\frac{g'v}{\sqrt{2}} & \frac{g'v'}{\sqrt{2}} & 0 & M' & \frac{g'v_2}{\sqrt{2}} & \frac{g'v_3}{\sqrt{2}} \\ 0 & 0 & -\frac{gv_3}{\sqrt{2}} & \frac{g'v_3}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & -\frac{gv_2}{\sqrt{2}} & \frac{g'v_2}{\sqrt{2}} & 0 & 0 \end{pmatrix} \quad \dots(16)$$

where the successive rows and columns correspond to $(\tilde{H}_2, \tilde{H}_1, -i\tilde{W}_3, -i\tilde{B}, \nu_\tau, \nu_\mu)$, ν_τ and ν_μ being the neutrino flavour eigenstates in this basis. Also, with the sneutrino vev's denoted by ν_2 and ν_3 ,

$$\nu(\nu') = \sqrt{2} \left(\frac{m_Z^2}{\bar{g}^2} - \frac{\nu_2^2 + \nu_3^2}{2} \right)^{\frac{1}{2}} \sin\beta \quad (\cos\beta)$$

M and M' being the SU(2) and U(1) gaugino mass parameters respectively, and $\bar{g} = \sqrt{g^2 + g'^2}$.

One can now define two states ν_3 and ν_2 , where

$$\nu_3 = \cos\theta \nu_\tau + \sin\theta \nu_\mu \quad \dots(17)$$

and ν_2 is the orthogonal combination, the neutrino mixing angle being given by

$$\cos\theta = \frac{\nu_3}{\sqrt{\nu_2^2 + \nu_3^2}} \quad \dots(18)$$

Clearly, the state ν_3 — which alone develops cross-terms with the massive gaugino states — develops a

see-saw type mass at the tree-level. The orthogonal combination ν_2 still remains massless. Interestingly, now we have a new seesaw mechanism, where the SUSY breaking scale in the observable sector takes the place of the GUT scale or the scale m_X discussed in the earlier sections.

The massive state ν_3 can be naturally used to account for atmospheric neutrino oscillations, with $\Delta m^2 = m_{\nu_3}^2$. Large angle mixing between the ν_μ and the ν_τ corresponds to the situation where $\nu_2 \simeq \nu_3$.

The tree-level mass here is clearly controlled by $\nu' = \sqrt{\nu_2^2 + \nu_3^2}$. This quantity, defined as the 'effective' sneutrino vev in the basis where the ε 's are rotated away, can be treated as a basis-independent measure of R-parity violation in such theories²⁷. The SK data on atmospheric neutrinos restrict ν' to be on the order of a few hundred keV's. However, it should be remembered that ν' is a function of ε_2 and ε_3 , both of which can still be as large as on the order of the electroweak scale. For example, in SUGRA-based models, it is possible to have a very small value of ν' starting from large ε 's, provided that one assumes the R-conserving and R-violating soft terms (as also the slepton and $Y = 1$ Higgs mass parameters) to be the same at the scale of dynamical SUSY breaking at a high energy²⁸.

Also, one has to address the question as to whether the treatment of ν_3 and ν_2 as mass eigenstates is proper, from the viewpoint of the charged lepton mass matrix being diagonal in the basis used above. In fact, it can be shown that this is strictly possible when ε_2 is much smaller than ε_3 , failing which one has to give a further basis rotation to define the neutrino mass eigenstates. However, the observable consequences described here are still valid, with the requirement of near-maximality shifted from the angle θ to the effective mixing angle.

Furthermore, a close examination of the scalar potential in such a scenario reveals the possibility of additional mixing among the charged sleptons, whereby flavour-changing neutral currents (FCNC) can be enhanced. It has been concluded after a detailed study that the suppression of FCNC requires one to have the ε -parameters to be small compared to the MSSM parameter μ (or, in other words, to the electroweak scale) unless there is a hierarchy²⁹ between ε_2 and ε_3 .

However, one still needs to find a mechanism for mass-splitting between the massless state ν_2 and the electron neutrino, and to explain the solar neutrino

puzzle. While there exist studies which attempt to explain both the puzzles in terms of bilinear terms only, the existence of the various λ and λ' -terms can also give rise to loop contributions to the neutrino mass matrix³⁰.

The generic expression for such loop-induced masses is

$$(m_\nu^{\text{loop}})_{ij} \simeq \frac{3}{8\pi^2} m_k^d m_p^d M_{SUSY} \frac{1}{m_{\tilde{q}}^2} \lambda'_{ikp} \lambda'_{jpk} + \frac{1}{8\pi^2} m_k^l m_p^l M_{SUSY} \frac{1}{m_{\tilde{l}}^2} \lambda_{ikp} \lambda_{jpk} \quad (19)$$

where $m^{d,(l)}$ denote the down-type quark (charged lepton) masses. $m_{\tilde{l}}^2, m_{\tilde{q}}^2$ are the slepton and squark mass squared. $M_{SUSY} (\sim \mu)$ is the effective scale of supersymmetry breaking. The mass eigenvalues can be obtained by including these loop contributions in the mass matrix.

Again, it should be noted that there may be other ways of looking at the problem. For example, it has been shown in³¹ that, if one assumes either purely bilinear or purely trilinear R-violating interactions at a high scale, running of the mass parameters can lead to significant sneutrino vev's at low energy, and at the same time generate loop-induced masses.

If we want the mass thus induced for the second generation neutrino to be the right one to solve the solar neutrino problem, then one obtains some constraint on the value of the λ' 's as well as λ 's. In order to generate a splitting between the two residual massless neutrinos, $\delta m^2 \simeq 5 \times 10^{-6} \text{ eV}^2$ (which is suggested for an MSW solution), a SUSY breaking mass of about 500 GeV implies $\lambda' (\lambda) \sim 10^{-4} - 10^{-5}$.

An interesting aspect of the scenario described above is that it can have distinctive signatures in collider experiments. The most striking ones among them pertain to decays of the lightest neutralino, produced either directly or via cascades. In presence of only the trilinear R-violating terms in the superpotential, the lightest neutralino can have various three-body decay modes which can be generically described by $\chi^0 \rightarrow \nu f \bar{f}$ and $\chi^0 \rightarrow l f_1 \bar{f}_2$, f, f_1 and f_2 being different quark and lepton flavours that are kinematically allowed in the final state.

Due to the mixing between neutrinos and neutralinos as also between charged leptons and charginos, the bilinears open up additional decay channels for the lightest neutralino, namely, $\chi^0 \rightarrow lW$ and $\chi^0 \rightarrow \nu Z$. When the neutralino is heavier than at least the W,

these two-body channels dominate over three-body ones over a large region of the parameter space, the effect of which can be observed in colliders such as the upgraded Tevatron, the LHC and a proposed high-energy electron-positron collider. In addition, superparticles such as the stop can sometimes decay dominantly via R-parity violating interactions, thereby altering the observed signals. Different observable quantities related to these decays have been studied in recent times^{32, 33, 34, 35}

Here we would like to stress upon one distinctive feature of the scenario that purportedly explains the SK results with the help of bilinear R-parity violating terms. It has been found that over almost the entire allowed range of the parameter space in this connection, the lightest neutralino is dominated by the Bino. A glance at the neutralino mass matrix reveals that decays of the neutralino (\simeq Bino) in such a case should be determined by the coupling of different candidate fermionic fields in the final state with the massive neutrino field ν_3 which has a cross-term with the Bino. Large angle neutrino mixing, on the other hand, implies that ν_3 should have comparable strengths of coupling with the muon and the tau. Thus, a necessary consequence of the above type of explanation of the SK results should be comparable numbers of muons and tau's emerging from decays of the lightest neutralino, together with a W -boson in each case^{32, 33}.

Of course, the event rates in the channel mentioned above will depend on whether the two-body decays mentioned above indeed dominate over the three-body decays. The latter are controlled by the size of the λ - and λ' -parameters. If these parameters have to be of the right size to explain the mass-splitting required by the solar neutrino deficit, then, for large angle MSW case, the decay widths driven by the trilinear term are smaller than those for the two-body decays by at least an order of magnitude.

The other important consequence of this picture is a large decay length for the lightest neutralino. We have already mentioned that the atmospheric neutrino results restrict the basis-independent R-violating parameter v' to the rather small value of a few hundred keV's. This value affects the mixing angle involved in calculating the decay width of the neutralino, which in turn is given by the formula

$$L = \frac{\hbar}{\Gamma} \times \frac{p}{M(\tilde{\chi}_1^0)} \quad \dots (20)$$

where Γ is the rest frame decay width of the lightest neutralino and p its momentum. The decay length decreases for higher neutrino masses, as a result of the enhanced flipping probability between the Bino and a neutrino, when the LSP is dominated by the Bino. Also, a relatively massive neutralino decays faster and hence has a smaller decay length. The interesting fact here is that even for a neutralino as massive as 250 GeV, the decay length is as large as about 0.1 to 10 millimetres, which should be observable in a detector³².

If the lightest neutralino can have two-body charged current decays, then the Majorana character of the latter also leads to the possibility of like-sign dimuons and ditaus from pair-produced neutralinos³⁵. Modulo the efficiency of simultaneous identification of W-pairs, these like-sign dileptons can also be quite useful in verifying the type of theory discussed here.

5 Some Other Possibilities

Nearly Degenerate Neutrinos

If the mass ranges to which the neutrino eigenstates belong are represented by mass-squared differences indicated by the solar and atmospheric neutrino deficits, then it is difficult to account for the hot dark matter content of the universe in terms of neutrinos. A way to surmount the difficulty is to postulate nearly degenerate neutrinos³⁶. Degeneracy also helps us understand large mixing in a somewhat ‘natural’ manner. At the same time, with a sterile neutrino with mass in the similar order, it may provide an explanation of the LSND results if they are substantiated.

However, there are problems with degenerate neutrinos. The limit on the electron neutrino mass from tritium beta decay provides the first restriction. More seriously, if neutrinos are of Majorana character, then degeneracy can come into serious conflicts with constraints imposed from the search for neutrinoless double-beta decay. There have been efforts to circumvent this difficulty by proposing neutrino mixing matrices which effect a cancellation between different eigenstates in such decay³⁷. Also, the literature contains proposals of a partial lifting of the degeneracy. On the whole, these scenarios cannot be completely ruled out, though some natural foundation for any of the models is yet to be found.

In the context of SUSY, too, efforts have been on

to justify degenerate neutrino scenarios, and we shall mention only one approach here³⁸. In this work, the close degeneracy of the neutrino masses can be *a priori* postulated to come from the form of the neutrino mass matrix at the Planck scale. Following works, for example of Georgi and Glashow, the matrix can be taken to correspond exactly to bimaximal mixing at the Planck scale. The evolution of the mass parameters should provide the requisite splittings at low energy. The evolution is crucially controlled by Yukawa couplings, and this is where the dependence on $\tan \beta$, the ratio of the vev’s of the two Higgs doublets becomes most important. However, it has been shown³⁸ that the solution space corresponding to the large mixing angle (LMA) MSW mechanism yields an inadmissible mass splitting unless $\tan \beta$ is very small, which is again incompatible with accelerator data. On the other hand a seesaw approach, with a high-scale Majorana mass in the range of 10^{10} GeV, leads to acceptable MSW solutions in the LMA regions. This, however, gives the best fit for $\tan \beta \simeq 2$ which is at the very edge of the phenomenologically viable MSSM parameter space.

Neutrino Mass from Unusual SUSY Breaking Terms

We normally agree to have ‘soft’ SUSY breaking terms only, the main reason being the need to control quadratic divergence of scalar masses. However, since the SUSY breaking interaction is usually an effective theory, one may expect higher order terms also to creep into the picture. Though such ‘hard’ terms are potential threats to the stability of scalar masses, they are suppressed by some power(s) of the cut-off scale for the effective theory, which in this case turns out to be the Planck mass m_P . Thus the quadratic corrections effectively shift the scalar masses by very small amounts, and the hard terms are usually ignored as phenomenologically insignificant. Such a possibility is conceivable also in the schemes suggested in ref.[38], with an enlarged SUSY breaking sector. Also, such terms have sometimes been exploited to stabilise flat directions of the scalar potential and generate intermediate scale vev’s.

It has been suggested³⁹ that some of these suppressed higher-dimensional terms may be responsible for neutrino masses. This is true in particular if lepton number is violated. Under such circumstances, one may, for example, have a gauge invariant term in the

Lagrangian, of the form

$$\mathcal{L}_{hard} = h(\varepsilon_{ij}\tilde{L}_i H_{2j})^2 \quad \dots(21)$$

where ε_{ij} is the completely antisymmetric rank-2 tensor. The dimensionless coupling h in this case depends on $(m_{\tilde{X}}^2/M_{\tilde{P}}^2)^n$ where n depends on the specific SUSY breaking mechanism. Note that this term is L -violating but R -parity conserving.

Such a term generates Majorana neutrino masses at one-loop level, involving virtual sneutrinos and $SU(2)$ gauginos. The induced mass has been shown to be of the form

$$m_\nu \simeq \frac{hg^2 v_2^2}{32\pi^2 m_{\tilde{\nu}}} F(M_2^2/m_{\tilde{\nu}}^2) \quad \dots(22)$$

where M_2 and $m_{\tilde{\nu}}$ are the $SU(2)$ gaugino and sneutrino masses respectively. The function F ranges between 0.5 and 0.1 for phenomenologically allowed values of the mass ratio in the argument.

Using such an expression, it can be seen that for a sneutrino mass in the range of 100 GeV and phenomenologically allowed values of the ratio of the Higgs vev's, the induced neutrino mass turns out to be too small to be consistent with observed results if $n = 1$, while for $n = 1/2$ it stays a little above the acceptable range.^a A mechanism of the above kind therefore favours SUSY breaking schemes where the dimension-4 terms shown in eq.4 are suppressed by some *fractional power* of the ratio $(m_{\tilde{X}}^2/M_{\tilde{P}}^2)$. An additional problem, of course, is to explain neutrino mixing in this scheme, for which the evolution of the term shown in eq.10 to low energies has to play a role.

Concluding Remarks

I have reviewed some of the various ways in which a SUSY scenario can be responsible for the generation of neutrino masses. I must admit that there are

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^a In ref.[39], the net induced mass has been claimed to be on the order of $M_{\tilde{X}}^2/M_{\tilde{P}}^3$ assuming that $n = 1$, which is misleading, for, as has been subsequently admitted in the same paper, the factor of $32\pi^2$ makes the contribution 'somewhat smaller'

many interesting approaches left out in this review. The point which has been emphasised here is the fact that SUSY notionally brings in additional mass scales into low-energy physics, which can have a role to play in the domain of neutrinos. Also, some special status of the right-handed neutrino superfield with respect to the governing symmetry in the SUSY breaking sector might well be responsible for the different nature of neutrino masses with respect to those of the other fermions. Such a point of view can be applied to both Dirac and Majorana masses, and also to cases which give rise to light sterile neutrinos. Side by side, the $\Delta L = 1$ terms in the superpotential of an R -parity breaking SUSY theory can use the electroweak symmetry breaking scale itself in a spectacular manner to explain not only neutrino masses but also their mixing pattern. Several of the theories discussed above have implications in other aspects of electroweak phenomenology including high-energy collider phenomena, which, quite desirably, integrates neutrino-related model-building into a much bigger canvas. Scenarios with degenerate neutrinos can also be encompassed by SUSY models. And finally, there exists the interesting conjecture that the otherwise undesirable hard SUSY breaking terms, suppressed by some power(s) of the Planck mass, can after all have a role to play in neutrino physics.

It should be admitted finally that flavour mixing, especially that of the bimaximal type, still requires special model assumptions. A better understanding of SUSY breaking schemes is necessary for further insight into the matter.

Acknowledgements

This work was partially supported by the Board of Research in Nuclear Sciences, Government of India.

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