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Research Paper

Hall Current Effect on Visco-Elastic MHD Oscillatory Convective Flow Through a Porous Medium in a Vertical Channel with Heat Radiation

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Hall current effect on visco-elastic MHD oscillatory convective flow through a porous medium in a vertical channel with heat radiation is investigated. An oscillatory MHD convection flow of visco-elastic, incompressible, electrically conducting fluid in a vertical channel filled with porous medium in the presence of Hall currents is studied analytically. A magnetic field of uniform strength is applied in the direction normal to the planes of the plates. The temperature of one of the plates varies periodically and the temperature difference of the plates is high enough to induce heat transfer due to radiation. A closed form solution of the problem is obtained. The effects of various parameters on the velocity profiles, the skin-friction in terms of the amplitude and the phase angle are shown graphically and discussed in detail.

Key Words: Viscoelastic Fluid; Convective Flow; Hall Current; Porous Medium; Thermal Radiation

Introduction

The flow problems of electrically conducting fluids are currently receiving considerable attention. The magnetohydrodynamic (MHD) flows has many practical applications such as electromagnetic flow meters, electromagnetic pumps and hydromagnetic generators etc. The interest in magnetohydrodynamic (MHD) convective flows with heat transfer is renewed due to its importance in the design of MHD generators and accelerators in geophysics, in systems like underground water and energy storage. Several scholars have shown their interest in studying MHD and heat transfer flows in porous and non-porous media. The effect of transversely applied magnetic field on convection flows of an electrically conducting fluid has been discussed by several authors notably (Nigam and Singh, 1960; Soundalgekar and Bhat, 1971; Vajravelu, 1988; Attia and Kotb, 1996; etc). The fluid flow through porous medium is another important aspect which has attracted the attention of scientists and engineers because of its usefulness in the fields of agricultural engineering to study the underground water resources, seepage of water in river beds, in chemical engineering for filtration and purification processes. Raptis *et al.*, 1982 studied hydromagnetic free convection flow through porous medium between two parallel plates. Raptis and Perdikis 1985 analyzed oscillatory flow through porous medium by the presence of free convection flow. Hossanien and Mansour 1990 investigated unsteady magnetic flow through a porous medium between two infinite parallel plates.

When the strength of the magnetic field is strong enough then one cannot neglect the effects of Hall currents. Even though it is of considerable importance

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to study how the results of the hydrodynamical problems get modified by the effects of Hall currents. A comprehensive discussion of Hall currents is given by Cowling 1957. Soundalgekar 1979 studied the Hall and Ion-slip effects in MHD Couette flow with heat transfer. Soundalgekar and Uplekar 1986 also analyzed Hall effects in MHD Couette flow with heat transfer. Hossain and Rashid 1987 investigated Hall effect on hydromagnetic free convection flow along a porous flat plate with mass transfer. Attia 1998 studied Hall current effects on the velocity and temperature fields on an unsteady Hartmann flow. Effects of Hall currents on free convective flow past an accelerated vertical porous plate in a rotating system with heat source/sink is analyzed by Singh and Garg 2010. Taking into account the heat radiation and the Hall currents. Singh *et al*., 2012 studied heat and mass transfer in an unsteady MHD free convective flow through a porous medium bounded by vertical porous channel.

The flows of viscoelastic fluids through porous medium are very important particularly in the fields of petroleum technology for the flow of oil through porous rocks, in chemical engineering and in the cases like drug permeation through human skin. Aldoss *et al*. 1995 studied MHD mixed convection flow from a vertical plate embedded in porous medium. Rajgopal *et al*., 2006 analyzed oscillatory flow of an electrically conducting viscoelastic fluid over a stretching sheet in a saturate porous medium. Attia and Ewis, 2010 investigated an unsteady MHD Couette flow with heat transfer of a viscoelastic fluid under exponential decaying pressure gradient. Singh, 2012 analyzed an oscillatory mixed convection flow of a viscoelastic electrically conducting fluid in an infinite vertical channel filled with porous medium. Khem Chand *et al*., 2013 studied oscillatory free convective flow of a viscoelastic fluid through a porous medium in a rotating vertical channel. Considering the Hall effects, Attia, 2004 discussed unsteady Hartmann flow of a viscoelastic fluid. Choudhary and Jha, 2008 analyzed heat and mass transfer in elastico-viscous fluid past an impulsively started infinite vertical plate with Hall current. Choudhury *et al*., 2013 studied visco-elastic flow with heat and mass transfer past a vertical porous plate in the presence of Hall current and radiation. Das, 2013 has also investigated visco-elastic effects on unsteady heat and mass transfer of a visco-elastic fluid in a porous channel with radiative heat transfer.

The object of the present paper is to analyze Hall current effect on the unsteady hydromagnetic convective flow of a viscoelastic fluid filled in a vertical channel. The transverse magnetic field is applied is strong enough so that the Hall currents are induced. The temperature difference between the plates of the channel is sufficiently high to radiate the heat.

Basic Equations

The equations governing the unsteady convective flow of an incompressible, visco-elastic and electrically conducting fluid in a vertical channel filled with porous medium in the presence of magnetic field are:

Equation of Continuity

$$\nabla \cdot \vec{V} = 0 \tag{1}$$

Momentum Equation

$$\rho\left[\frac{\partial \vec{V}}{\partial t^*} + (\vec{V}\cdot\nabla)V\right] = -\nabla p + J\times B$$
$$+\mu\nabla^2\vec{V} - \frac{\mu}{K^*}V + g\beta(T^* - T_0) + \nabla\cdot\exists \tag{2}$$

Energy Equation

$$\rho C_p\left[\frac{\partial T^*}{\partial t^*} + (\vec{V}\cdot\nabla)T^*\right] = k\nabla^2 T^* - \nabla q \tag{3}$$

Kirchhoff's First Law

$$div\vec{J} = 0 \tag{4}$$

General Ohm's Law

$$\vec{J} + \frac{\omega_e\tau_e}{B_0}(J\times B) = \sigma\left[E + V\times B + \frac{1}{e\eta_e}\nabla p_e\right] \tag{5}$$

Gauss's Law of Magnetism

$$div\ \vec{B} = 0 \quad (6)$$

where $\vec{V}$ is the velocity vector, $\vec{B}$ is the magnetic induction vector, $\vec{J}$ is the current density and $\vec{E}$ is the electric field. $\exists$ is the Cauchy stress tensor and the constitutive equation derived by Coleman and Noll, 1960 for an incompressible homogeneous fluid of second order is

$$\exists = -p_1 I + \mu_1 A_1 + \mu_2 A_2 + \mu_3 A_1^2. \quad (7)$$

Here $-p_1 I$ is the interdeterminate part of the stress due to constraint of incompressibility, μ_1, μ_2 and μ_3 are the material constants describing viscosity, elasticity and cross-viscosity respectively. The kinematics A_1 and A_2 are the Rivelen Ericson constants defined as

$$A_1 = (\nabla\bar{V}) + (\nabla\bar{V})^T,$$
$$A_2 = \frac{dA_1}{dt} + (\nabla\bar{V})^T A_1 + A_1(\nabla\bar{V}),$$

where ∇ denotes the gradient operator and d/dt the material time derivative. According to Markovitz and Coleman, 1964, the material constants μ_1, μ_3 are taken as positive and μ_2 as negative.

Formulation of the Problem

Consider an unsteady MHD free convective flow of an electrically conducting, viscoelastic, incompressible fluid through a porous medium bounded between two insulated infinite vertical plates in the presence of Hall current and thermal radiation. The plates are at a distance 'd' apart. We introduce a Cartesian coordinate system with x*-axis oriented vertically upward along the centreline of the channel. The z*-axis taken perpendicular to the planes of the plates is the axis along which a strong transverse magnetic field of uniform strength B_0 is applied. The schematic diagram of the physical problem is shown in Fig. 1. Since the plates of the channel are of infinite extent, all the physical quantities except the pressure, depend only on z* and t* only. Let (u^*, v^*, w^*) be the components of velocity in the directions (x^*, y^*, z^*) respectively. Since the plates are non-porous, therefore equation of continuity (1) on integration gives $w^* = 0$. Also the equation (6) for the magnetic field gives $\vec{B} = (B_x^*, B_y^*, B_z^*)$ $B_z^* = B_0$ (constant).

(j_x^*, j_y^*, j_z^*) are the components of electric current density $\vec{J}$ Equation (4) the conservation of electric charge gives j_z^* (constant).

For non-conducting plates

$$j_z^* = 0 \quad (8)$$

at the plates and hence zero everywhere in the fluid.

Under the usual assumptions that the electron pressure (for a weakly ionized gas), the thermoelectric pressure, ion slip and the external electric field arising due to polarization of charges is negligible.

It is assumed that no applied and polarization voltage exists. This corresponds to the case where no energy is being added or extracted from the fluid by electrical means (Meyer, 1958) i.e. electrical field $\bar{E} = 0$. Therefore, equation (5) takes the form:

$$\bar{j} + \frac{\omega_e \tau_e}{B_0}(j \times \vec{B}) = \sigma (V \times B). \quad (9)$$

After using equation (8), equation (9) in component form becomes

$$j_x^* + \omega_e \tau_e j_y^* = \sigma B_0 \vec{v}^* \quad (10)$$

$$j_y^* - \omega_e \tau_e j_x^* = \sigma B_0 \vec{u}^* \quad (11)$$

Solving (10) and (11) for $j_{\bar{x}^*}$ and $j_{\bar{y}^*}$, we get

$$j_x^* = \frac{\sigma B_0}{(1+H^2)}(H\vec{u}^* + v^*)$$

and

$$j_y^* = \frac{\sigma B_0}{(1+H^2)}(Hv^* + u^*)$$

where $H = \omega_e \tau_e$ is the Hall parameter.

Under the usual Boussinesq approximation momentum equation (2) in Cartesian components reduces to

$$\frac{\partial u^*}{\partial t^*} = -\frac{1}{\rho}\frac{\partial p^*}{\partial x^*} + v_1\frac{\partial^2 u^*}{\partial z^{*2}} + v_2\frac{\partial^3 u^*}{\partial z^{*2}\partial t^*} + \frac{\sigma B_0^2 (Hv^* - u^*)}{\rho(1+H^2)} - \frac{\vartheta_1 \bar{u}}{K^*} + g\beta(T^* - T_0), \qquad (12)$$

$$\frac{\partial v^*}{\partial \bar{t}} = -\frac{1}{\rho}\frac{\partial p^*}{\partial y} + v_1\frac{\partial^2 u^*}{\partial \bar{z}^{2}} + v_2\frac{\partial^3 u^*}{\partial z^{*2}\partial t^*} - \frac{\sigma B_0^2 (Hu^* - v^*)}{\rho(1+H^2)} - \frac{\vartheta_1 v^*}{K^*}, \qquad (13)$$

and energy equation (3) becomes

$$\rho C_p \frac{\partial T^*}{\partial t^*} = k\frac{\partial^2 T^*}{\partial z^{*2}} - \frac{\partial q}{\partial z^*}. \qquad (14)$$

The boundary conditions for the flow problem are

$$u^* = v^* = 0, T^* = T_0 \quad \text{at} \quad z^* = -\frac{d}{2}, \qquad (15)$$

$$u^* = v^* = 0,$$
$$T^* = T_0 + (T_w - T_0)\cos\omega^* t^* \quad \text{at} \quad z^* = -\frac{d}{2}, \qquad (16)$$

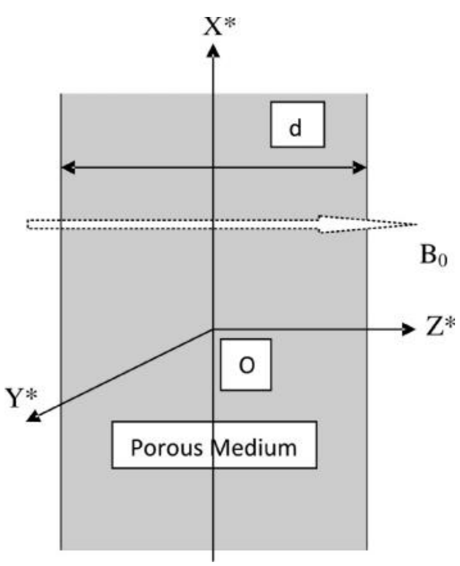

Fig. 1: Diagram of the problem

Following Cogley *et al*., 1968, it is assumed that the fluid is optically thin with a relatively low density and the last term in the energy equation (14)

$$\frac{\partial q}{\partial z^*} = 4\alpha^2 (T^* - T_0) \qquad (17)$$

stands for radiative heat flux, where

$$\alpha^2 = \int_0^\infty k_{\lambda n}\frac{\partial e_{b\lambda}}{\partial T} d\lambda \cdot$$

$k_{\lambda n}$ is the absorption coefficient at the walls and e_{λ_λ} is the Plank's function.

Introducing the following non-dimensional quantities

$$\eta = \frac{z^*}{d}, x\frac{x^*}{d}, y\frac{y^*}{d}, u\frac{u^*}{U}, v\frac{v^*}{U},$$

$$T = \frac{T^* - T_0}{T_w - T_0},\ t = \frac{t^* U}{d},\ \ \omega = \frac{\omega^* d}{U}, p = \frac{p^*}{\rho U^2}$$

into equations (11) to (13) and using equations (16), we get

$$\text{Re}\frac{\partial u}{\partial t} = -\text{Re}\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial \eta^2} + \gamma\frac{\partial^2 u}{\partial \eta^2 \partial t} + \frac{M^2 (Hv - u)}{(1+H^2)} - \frac{1}{K}u + GrT \qquad (18)$$

$$\text{Re}\frac{\partial v}{\partial t} = -\text{Re}\frac{\partial p}{\partial y} + \frac{\partial^2 v}{\partial \eta^2} + \gamma\frac{\partial^3 v}{\partial \eta^2 \partial t} + \frac{M^2 (Hv - u)}{(1+H^2)} - \frac{1}{K}v \qquad (19)$$

$$Pe\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial \eta^2} - N^2 T, \qquad (20)$$

where $\text{Re}\frac{Ud}{\upsilon_1}$ (Reynolds number),

$K = \frac{K^*}{d^2}$ (The permeability of the porous medium),

$H = \omega_e \tau_e$ (Hall parameter),

$M = B_0 d \sqrt{\frac{\sigma}{\rho \upsilon_1}}$ (Hartmann number),

$\gamma = \frac{\upsilon_2 \, \mathrm{Re}}{d^2}$ (Visco-elastic parameter),

$Gr = \frac{g\beta d^2 (T_w - T_0)}{\upsilon_1 U}$ (Grashof number),

$Pe = \frac{\rho C_p dU}{k}$ (Peclet number),

$N = \frac{2\alpha d}{\sqrt{k}}$ (Radiation parameter).

The corresponding transformed boundary conditions are:

$$u = v = 0, T = 0 \ \text{ at } \ \eta = -\frac{1}{2}, \tag{21}$$

$$u = v = 0, T = \cos\omega t \ \text{ at } \ \eta = \frac{1}{2}. \tag{22}$$

Following Singh and Pathak, 2013, for the oscillatory internal flow considered we shall assume that the fluid flows only under the influence of a non-dimensional pressure gradient oscillating only in the direction of x-axis which is of the form

$$-\frac{\partial p}{\partial x} = A\cos\omega t \quad \text{and} \quad -\frac{\partial p}{\partial y} = 0 \tag{23}$$

where A is a constant.

Solution of the Problem

In order to combine equations (18) and (19) into single equation, we introduce a complex function $F = u + iv$ and using (23), we get

$$\gamma = \frac{\partial^3 F}{\partial \eta^2 \partial t} + \frac{\partial^2 F}{\partial \eta^2} - \mathrm{Re}\frac{\partial F}{\partial t} - \left(\frac{M^2(1+tH)}{1+H^2} + K^{-1}\right) F = \mathrm{Re}\frac{\partial p}{\partial x} - GrT. \tag{24}$$

The boundary conditions (21) and (22) in complex form can be written as:

$$F = 0, \ T = 0 \quad \text{at} \quad \eta = -\frac{1}{2}, \tag{25}$$

$$F = 0, \ T = \cos\omega t \quad \text{at} \quad \eta = -\frac{1}{2}. \tag{26}$$

In order to solve equations (20) and (24) under the boundary conditions (25) and (26), we assume in complex form the solution of the problem as:

$$F(\eta, t) = F_0(\eta)e^{i\omega t}, T(\eta, t) = \theta_0(\eta)e^{i\omega t},$$

and

$$-\frac{\partial p}{\partial x} = Ae^{i\omega t}. \tag{27}$$

The real part of the solution will have physical significance.

The boundary conditions (25) and (26) become:

$$F = 0, \ \theta_0 = 0 \quad \text{at} \quad \eta = -\frac{1}{2}, \tag{28}$$

$$F = 0, \ \theta_0 = 1 \quad \text{at} \quad \eta = -\frac{1}{2}. \tag{29}$$

Substituting equation (27) in equations (20) and (24), we get

$$l^2 \frac{d^2 F_0}{d\eta^2} - m^2 F_0 = -A\mathrm{Re} - Fr\theta_0 \tag{30}$$

and

$$\frac{d^2\theta_0}{d\eta^2} - n^2\theta_0 = 0, \tag{31}$$

where, $l = \sqrt{1 + i\omega\gamma}$,

$$m = \sqrt{\frac{M^2(1+\iota H)}{(1+H^2)} + K^{-1} + \iota\omega \, \mathrm{Re}}$$

$$n = \sqrt{N^2 + \iota\omega Pe}.$$

The ordinary differential equations (30) and (31) are solved under the boundary conditions (28) and (29) for the velocity and temperature fields. The solution of the problem is obtained as:

$$F(\eta,t)=\left[\frac{A\mathrm{Re}}{m^2}\left(1-\frac{\cosh\frac{m}{l}\eta}{\cosh\frac{m}{2l}}\right)+\frac{Gr}{(l^2n^2-m^2)}\right]$$

$$\left\{\frac{\sinh\frac{m}{l\left(\eta+\frac{1}{2}\right)}}{\sinh\frac{m}{l}}-\frac{\sinh \mathrm{n}\left(\eta+\frac{1}{2}\right)}{\sinh \mathrm{n}}\right\}e^{\iota\omega t} \tag{32}$$

$$T(\eta,t)=\left\{\frac{\sinh \mathrm{n}\left(\eta+\frac{1}{2}\right)}{\sin hn}\right\}e^{\iota\omega t} \tag{33}$$

Now from the velocity field we can obtain the skin-friction τ_L at the left plate in terms of its amplitude and phase angle as:

$$\tau_L=\left(\frac{\partial F}{\partial\eta}\right)_{\eta=-\frac{1}{2}}=\left(\frac{\partial F_0}{\partial\eta}\right)_{\eta=-\frac{1}{2}}e^{\iota\omega t}=|F|\cos(\omega t+\varphi)$$

with $|F|=\sqrt{F_r^2+F_i^2}$ and $\varphi=\tan^{-1}\left(\frac{F_i}{F_r}\right)$ (34)

where

$$F_r+\iota F_i=\frac{A\mathrm{Re}}{lm}\tanh\frac{m}{2l}+\frac{Gr}{(l^2n^2-m^2)}\left(\frac{\frac{m}{l}}{\sinh\frac{m}{l}}-\frac{n}{\sinh n}\right) \tag{35}$$

From the temperature field the rate of heat transfer Nu (Nusselt number) at the left plate in terms of its amplitude and phase angle is obtained as:

$$Nu=\left(\frac{\partial T}{\partial\eta}\right)_{\eta=-\frac{1}{2}}=\left(\frac{\partial\theta_0}{\partial\eta}\right)_{\eta=-\frac{1}{2}}e^{\iota\omega t}=|H|\cos(\omega t+\psi)$$

where

$$H_r=\iota H_i=\frac{r}{\sin hr} \tag{36}$$

with $|H|=\sqrt{H_r^2+H_i^2}$ and $\psi=\tan^{-1}\left(\frac{H_i}{H_r}\right)$ (37)

The temperature field, the amplitude and the phase angle Ψ of Nusselt number need no further discussion because Singh, 2013 has already discussed these in detail.

Discussion

The effect of Hall current on MHD convection flow of a viscoelastic fluid through a porous medium filled in a vertical channel with heat radiation is analyzed. In order to study the effects of different parameters appearing in the flow problem, we have carried out numerical calculations for the velocity field, skin-friction, in terms of its amplitude and phase.

The variations of velocity profiles under the influence of different parameters are exhibited in Figs. 2 to 11. Velocity variations with the visco-elastic parameter γ are shown in Fig. 2. It is observed from this figure that the velocity decreases with the increase of γ i.e. the flow retards as the visco-elasticity of the fluid increases. Fig. 3 depicts that the velocity increases tremendously with the increase of Grashof number Gr. Physically it means that the buoyancy force enhances the flow velocity. The variations of the velocity with Reynolds number Re are presented in Fig. 4. This figure reveals that the velocity increases with the increase of Reynolds number Re. The increasing Reynolds number means (being the ratio of inertial to the viscous forces) that inertial forces are predominant and strengthen the velocity field further.

It is evident from the curves of Fig. 5 that the velocity decreases with the increase of Hartmann number M. This is because of the reason that effects of a transverse magnetic field on an electrically conducting fluid gives rise to a resistive type force (called Lorentz force) similar to drag force and upon increasing the values of M increases the drag force which has tendency to slow down the motion of the fluid. Hall current effects on the flow velocity are shown by the curves shown in Fig. 6. We find from this figure that the velocity increases with the increase of Hall current parameter H.

The velocity variations due to the increase of permeability of the porous medium K are displayed in Fig. 7. For visco-elastic fluids the velocity goes on decreasing as the permeability of the porous matrix increases. Fig. 8 shows that the velocity decreases with the increase of Peclet number Pe. The thermal radiations effect on the velocity profiles are shown by the curves in Fig. 9. The velocity decreases with the increase of radiation parameter N. The effects of pressure gradient on the velocity profiles are shown in Fig. 10. It is obvious from these curves in this figure that the velocity increases with increasing favourable pressure gradient A. It is because of the fact that more is the drop in pressure gradient faster is the flow. As is evident from curves of Fig. 11 the velocity decreases with increasing frequency of oscillations ω.

The amplitude $|F|$ of the skin-friction τ_L on the left plate ($\eta = -0.5$) is plotted in Fig. 12 against w the frequency of oscillations. The values of various parameters listed in Table 1 represent different curves

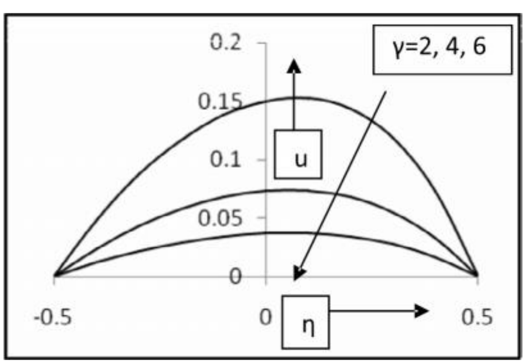

Fig. 2: Variations of velocity with γ

Table 1: The values of various parameters show different curves of Figs. 12 and 13

γ	Gr	Re	M	H	K	Pe	N	A	Curve
0.1	5	0.5	2	1	0.2	1	1	5	I
0.2	5	0.5	2	1	0.2	1	1	5	II
0.2	1	0.5	2	1	0.2	1	1	5	III
0.2	5	1.0	2	1	0.2	1	1	5	IV
0.2	5	0.5	4	1	0.2	1	1	5	V
0.2	5	0.5	2	5	0.2	1	1	5	VI
0.2	5	0.5	2	1	1.0	1	1	5	VII
0.2	5	0.5	2	1	0.2	7	1	5	VIII
0.2	5	0.5	2	1	0.2	1	5	5	IX
0.2	5	0.5	2	1	0.2	1	1	7	X

in Fig. 1. In order to study the effect of each of the parameter every curve is compared with the dashed curve II for viscoelastic parameter $\gamma = 0.2$. Slightly

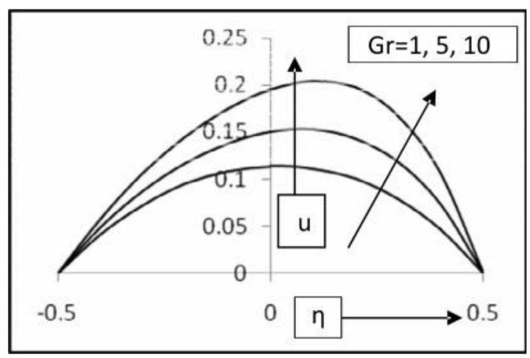

Fig. 3: Variations of velocity with Gr

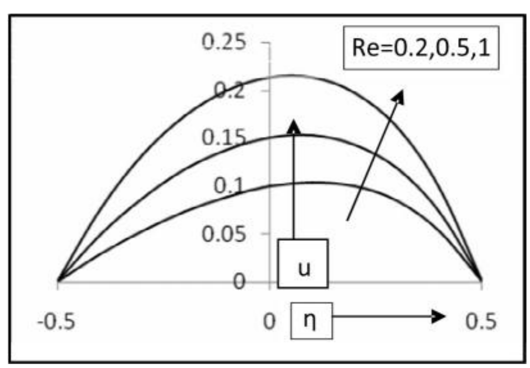

Fig. 4: Variations of velocity with Re

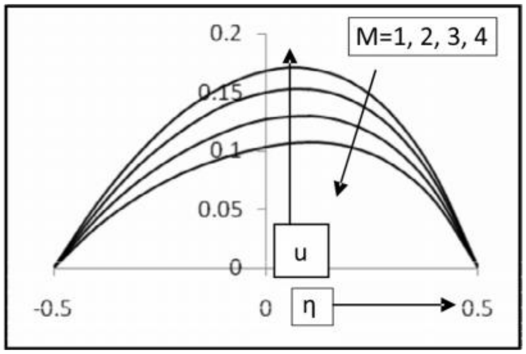

Fig. 5: Variations of velocity with M

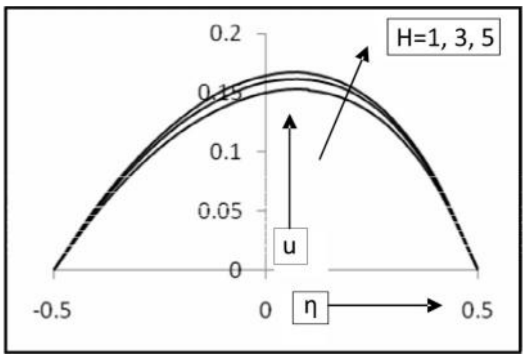

Fig. 6: Variations of velocity with H

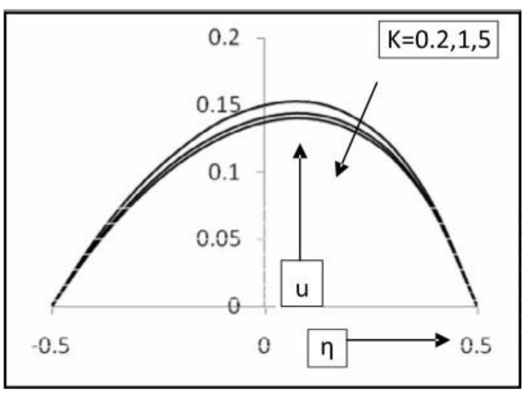

Fig. 7: Variations of velocity with K

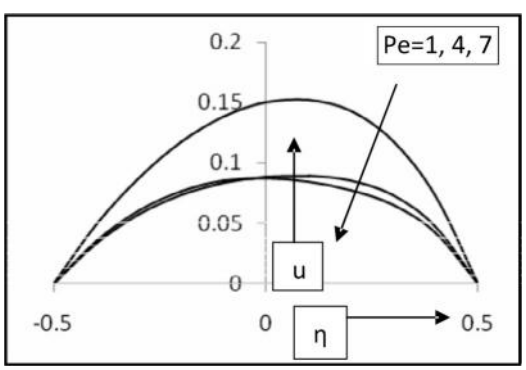

Fig. 8: Variations of velocity with Pe

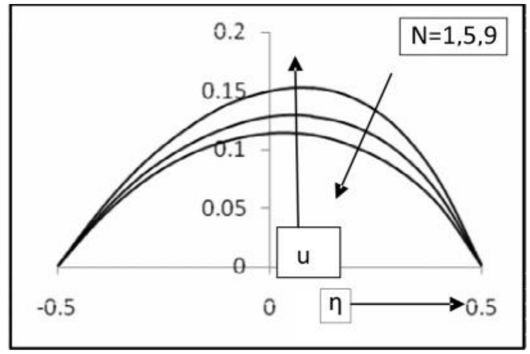

Fig. 9: Variations of velocity with N

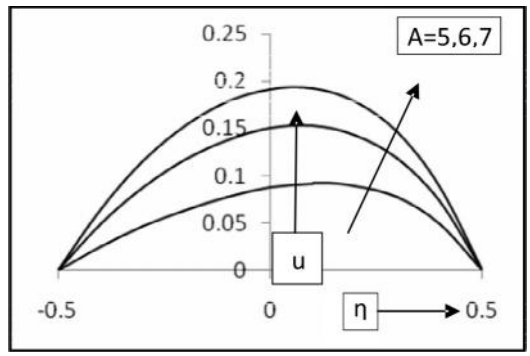

Fig. 10: Variations of velocity with A

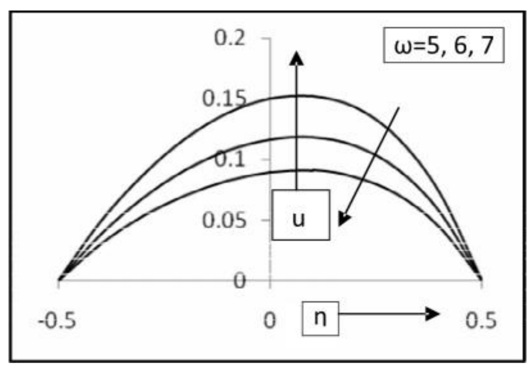

Fig. 11: Variations of velocity with ω

careful observation of the figure reveals that the skin-friction amplitude decreases quite significantly for smaller values of the frequency ω (varying from 0 to 10) in comparison to the larger values (varying from 15 to 25). The skin-friction goes on reducing further with the increase of ω although the rate of reduction declines. From the respective comparison of curves I, V, VIII and IX with the dashed curve II it is gathered that the skin-friction amplitude $|F|$ decreases with the increase of viscoelastic parameter γ, Hartmann

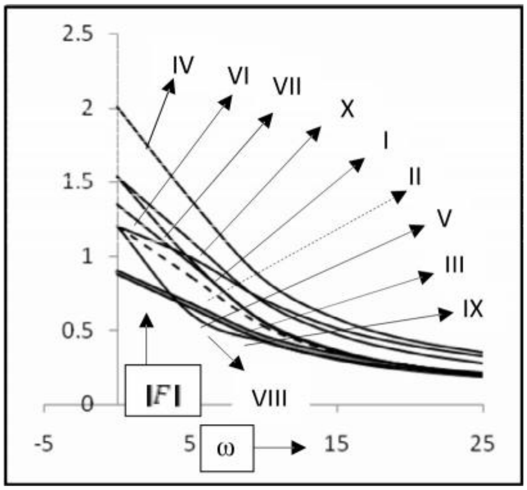

Fig. 12: Amplitude of the skin-friction

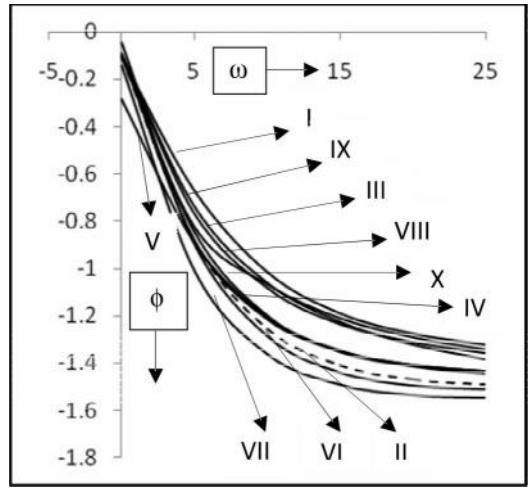

Fig. 13: Phase angle of the skin-friction

number M, Peclet number Pe and radiation parameter N. However, it is noticed from the respective comparison of curves III, IV, VI, VII and X with dashed curve I that the skin-friction amplitude $|F|$ increases with the increase of Grashof number Gr, Renolds number Re, Hall parameter H, permeability of the porous medium K and pressure gradient parameter A.

The effects of the variations of different flow parameters on the phase angle φ of the skin-friction τ are illustrated in Fig. 4. It is obvious from this figure that there is always a phase lag because the values of φ plotted against ω are negative throughout. This lag in the phase goes on increasing sharply as the frequency of oscillations ω increases from 0 to 10 and the increase is phase lag is marginal for further increase in ω. Various curves are compared with the dashed curve II. By the comparison of curves III, VI and VII with the dashed curve II we find that the lag in phase angle increases with the increase of Grashof number Gr, Hall parameter H and the permeability of the porous medium K respectively. Comparison of curves II and V reveals that as the Hartmann number M increases the phase lag increases initially for smaller values of the frequency ω and then decreases for larger values of the frequency. Comparing curves I, IV, VIII, IX and X with curve II shows that the phase lag decrease with the increase of viscoelastic parameter γ, Reynolds number Re, Peclet number Pe, Radiation parameter N and the pressure gradient A.

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List of Symbols

A	a constant
B_0	magnetic field applied
c_p	specific heat at constant pressure
e	electric charge
$\|F\|$	Amplitude of skin friction
Gr	Grashof number
g	gravitational force
H	Hall current
k	thermal conductivity
K	permeability of the porous medium
M	Hartmann number
N	heat radiation parameter
n_e	number density of electron

p	pressure
p_e	electron pressure
Pe	Peclet number
q	heat due to radiation
Re	Reynolds number
t	time variable
T	fluid temperature
T_0	temperature of $z^* = -\frac{d}{2}$ plate
T_w	mean temperature of $z^* = -\frac{d}{2}$ plate
U	mean flow velocity
u, v, w	velocity components
x, y, z	axial variables

Greek Symbols

α	radiation absorption coefficient
β	coefficient of volume expansion
γ	viscoelastic parameter
μ	viscosity
ρ	fluid density
σ	electric conductivity
ω	frequency of oscillations
ω_e	electron frequency
τ_e	electron collision time
τ_L	skin-friction at the left wall
φ	phase angle of the skin-friction
θ_0	non-dimensional temperature
*	superscript

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