

tgeqtf kpi 'u{uygo u'ku'vj cv'vj g'o qv'kqp'ugpuqtu'j cxg'vq dg'y qtp d{ 'vj g'uwdlgev'cpf 'vj wu'k'ch'geu'vj g'f gz'vgtk'f qh'vj g'uwdlgev'0Hwt'j gt'vj g'equv'qh'tgo qt'tgeqtf kpi u{uygo 'ku'xgt{ 'j ki j 'cpf 'vj wu'o c{ 'pqv'dg'ch'qtf cdng d{ 'eqo o qp'o cp0Vj g'r t'gugpv'u{uygo 'f g'xgmr gf 'hqt j cpf "tgo qt"s wcp'vkh'ec'v'kqp'f qgu'pqv'ch'gev'vj g f gz'vgtk'f 'qh'vj g'uwdlgev'cpf 'ku'ch'qtf cdng"rguu'vj cp &422+'vq'c'eqo o qp'o cp'g'xgp'hqt'j qo g'wuci g0

Vtcf k'k'qpcn'o gy'qf u'qh'tgo qt'f kci p'kuku'wug ceegrqtgo g'vgtu"*o q'v'kqp'ugpuqtu+'vj cv'ctg'r r'ceg'f'cv f k'htg'gpv'r ct'w'qh'vj g'uwdlgev'v'dqf {0Cu'vj g'uwdlgev r gthqto u'c'lgv'q'hf c'k'f 'ce'v'k'k'g'u'vj g'ug'ugpuqtu'i cvj gt vj g'ceegrqt'cv'kqp'cpf 'qt'k'p'v'k'q'p'k'p'htqto c'v'k'p'0Vj ku'ku r'vgt'r t'q'egu'gf "qh'rk'p'g'vq'f g'v'ge'v'vj g'r t'gugpeg"qh vtgo qtu0Ceegrqtgo g'vgtu'b c{ 'dg'h'k'gf 'q'p'vj g'uwdlgev'v r cm "cpf 'vj g'uki p'cni'ctg'v'c'puo k'v'gf "vq'c"o letq eqp'v'q'ngt'hqt'ht'vj gt'r t'q'egu'kpi "J ctkuj g'v'crl'422; +0 O gcuwtgo gpw'j cxg'dggp'v'cngp'c'v'c'uco r r'kpi 't'cv'q'h 347472'J | 'htqo 'uwdlgev'y kj 'cpf 'y kj qw'vtgo qt0 Vj g'vtgo qt'f g'v'ge'v'k'p'c'ri qtk'j o "t'cp"q'p'g'xgt{ "5"uge y k'p'f'qy 'q'h'f'c'v'c' 'cpf 'vj g'ht'gs w'p'e{ 'cpf 'co r r'k'w'f'g'q'h vtgo qt'cm'p'i 'cm'vj t'gg'cz'gu'y g'g'f'g'v'g'to k'p'gf 0Vj g'ug v'y q'r ctco g'vgtu'y g'g'h'q'w'p'f 'vq'd'g'f'k'v'k'p'v'v'f 'f k'htg'gpv hqt'vj g'vtgo qt'cpf "p'q'p'vtgo qt'ec'ug'0J g'g'c'w'j qtu j cxg'w'ug'f "c"v'ko g'f'qo c'k'p'c'p'c'n'f'uku'vq"s wcp'v'kh{ "vj g vtgo qt'ht'gs w'p'e{ 'cpf 'co r r'k'w'f'g0

Kp"*Uc'rt'k'p g'v'crl'4229+.'b k'p'k'w't'g'i {t'que'qr gu cv'cej g'f "vq"uwdlgev'v'ht'g'cto u'j cxg'dggp"w'ug'f "vq o gcuwt'g'vj g'qt'k'p'v'k'q'p'q'h'o q'x'go gp'0Vj q'f k'htg'gpv o gy'qf u'q'h'o gcuwt'go gp'v'y g'g'f'g'ht'qto g'f 'k'p'q'p'g'vj g uwdlgev'v'ht'q'ny g'f 'c'h'z'gf 'v'g'u'v'f' t'q'v'q'eq'n'c'p'f 'k'p'c'p'q'vj gt vj g'uwdlgev'y g'g'ht'g'v'q'f'q'vj g'k'f'c{/vq/f c{ 'ce'v'k'k'g'u y j k'g'vj g'b gcuwt'go gpw'y g'g'd'g'k'p'i 'v'c'ng'p'0Vj g'f'r'v'gt k'p'x'q'k'gf "c'rk'p'i "o gcuwt'go gp'v'v'ko g'q'h'c'd'q'w'6"j qwtu cpf 'vj g'g'ht'g'c'v'gr ct'c'v'g u{uygo 'ec'ng'f 'C'w'q'p'qo q'w' U'g'p'k'p'i "W'p'k'T'g'eq't'f'gt "C'U'W'T+"y cu'w'ug'f 0'G'cej C'U'W'T'w'p'k'ku'c'uo cm'k'p'f'g'r'g'p'f'g'p'v'w'p'k'v'eq'p'uk'p'i 'q'h v'y q'i {t'que'qr gu.'c'f'c'v'c'v'q'i i g't.'c"dc'w'gt{ 'cpf 'c'ht'c'uj o go qt {0'Vj g'c'p'i w'rt'x'g'm'ek'v{ "y cu'q'd'v'k'p'gf "d{ h'ng't'k'p'i "vj g'c'v'w'k'gf "uki p'c'ri'w'uk'p'i "c"dc'p'f/r'cu'v'HT' h'ng't'y kj "ew'v'q'h'ht'gs w'p'ek'g'u'q'h'50'J | 'cpf '90'J | 0 Vj g'ht'gs w'p'e{ "ur g'v't'wo "q'h'g'cej "5"uge'q'p'f y k'p'f'qy uki p'c'ri'y cu'g'v'ko c'v'gf "w'uk'p'i "c'8'j q't'f'g't'c'w'q't'g'i t'g'u'k'x'g o q'f'g'r'0'k'i'vj g'f'qo k'p'c'p'v'ht'gs w'p'e{ "h'm'd'g'y g'gp'50'/ 90'J | 'cpf 'k'h'ku'co r r'k'w'f'g'y cu'w'ht'k'k'g'p'v'f'j ki j . 'vj gp

c'tgo qt'y cu'tgr q't'v'gf 'k'p'vj cv'r ct'v'ew'rt'y k'p'f'qy 0

Kp"*U'w'g'v'crl'4225+.'vj t'gg'f'ko g'p'uk'q'p'c'n'g'g'v'g'v'q/ o ci p'g'v'k'ug'p'q'v'k'p'v'gi t'c'v'gf "k'p'v'c"i'f'c'v'c'i' m'q'x'g'j cxg d'ggp'w'ug'f "vq'ec'r w't'g'v'j g'b q'v'k'q'p'q'h'vj g'j cpf 0Vtgo qt uki p'c'ni'y g'g'ec'r w't'g'f "f'w'k'p'i "f'k'ht'g'p'v'v'c'v'g'u'v'k'ng'< f'w'k'p'i "t'g'u'v'v'q'g'v'ko c'v'g't'g'u'v'v'go qt+.'y j k'g't'q'v'c'v'k'p'i y t'k'u'v'i t'c'd'd'k'p'i "q'p'g'v'j' c'p'f . "g'v'0'F'k'v'et'g'v'g'v'g'v'k'g't v'c'p'uh'q'to "y cu'w'ug'f "vq'q'd'v'k'p'vj g'ht'gs w'p'e{ "qh'vj g vtgo qt'cpf 'vj g'k'p'v'c'p'v'c'p'g'q'w'v'ur g'g'f' y cu'q'd'v'k'p'gf "d{ eqo r w'k'p'i "vj g'f'g't'k'x'c'v'k'x'g'q'h'vj g'v'ko g'f'qo c'k'p'uki p'c'ri'0 Hwt'j gt.'c'5'F "g'g'v'g't'q'o ci p'g'v'k'k'o ci k'p'i u{uygo 'y cu w'ug'f "vq't'g'eq't'f'x'c't'k'q'w'v'h'c'w'v't'g'u'q'h'v'c't'k'p'v'q'p'v'f'k'ug'c'ug r'k'ng'v'tgo qt.'t'k'i'k'f'k'f' 'c'p'f'd't'c'f {n'k'p'g'uk' "0'c't'ng'j g'v'crl' 4222+'f'g'x'g'm'r'g'f'c'v'tgo qt's wcp'v'kh'ec'v'k'q'p'f'g'x'k'g'y j g'g'c'p'g'g'v'g't'q'o ci p'g'v'k'v't'c'eng't'y cu'c'h'k'z'gf "vq'vj g'h'k'p'i g't' c'p'f "f'c'v'c'y cu'c'v'w'k'gf ""'y t'q'w'i j "c'j' q'v'v'eqo r w'g't'0 H'g's w'p'e{ 'c'p'f 'co r r'k'w'f'g'q'h'vtgo qt'y g'g'f'g'v'g'to k'p'gf 0 K'p'c'm'q'h'vj g'cd'q'x'g'o g'y q'f' u.'vj g'c'v'w'k'gf "f'c'v'c'ku c'p'c'n'f' g'f "k'p'vj g'ht'gs w'p'e{ "f'qo c'k'p'v'q'q'd'v'k'p'vj g ht'gs w'p'e{ "qh'vtgo qt."uk'p'eg'v'tgo qt'ku'c'r'ug'w'f'q/ t'j { vj o k'e'c'v'k'x'k'f' 'c'p'f 'v'f'r'k'ec'm'f' 'vj g't'g'ku'c'ht'gs w'p'e{ cu'q'ek'c'v'gf "y kj "g'c'ej "n'k'p'f'q'h'vtgo qt"*J ctkuj g'v'crl' 422; +."C'nd'g'tu'g'v'crl'3; 95+0Vj g'p'q'to c'ri'j c'p'f'vtgo qt j cu'c'ht'gs w'p'e{ 'q'h; /47'J | 'c'p'f'ku'q'h'v'ko c'm'c'o r r'k'w'f'g0 Vj g't'g'v'k'p'i "vtgo qt'k'p'v'c't'k'p'v'q'p'v'f'k'ug'c'ug'w'w'c'm'f' t'c'p'i'g'u'ht'qo 5/: 'J | *J ctkuj g'v'crl'422; +0

Vj g'w'ug'q'h'j cr v'k'v'g'ej p'q'q'ij { 'vq'f'g'v'ge'v'vtgo qtu q'h'w'r'g't'f'ko d'j cu'd'ggp'k'p'k'k'c'm'f'g'z'r'q't'g'f'k'p'v'c't'f'q'ht'g' g'v'crl'4223+0C'x'k't'w'c'n'g'p'x'k'q'p'o gp'v'y cu'v'et'g'c'v'gf 'y j k'ej eq'p'uk'v'gf "q'h'c'5'F'v'c'd'f'k'p'v'j "r'k'ng'v't'w'v'w't'g'0Vj g'v'c'm' i k'x'g'p'v'q'vj g'uwdlgev'y cu'v'q'o q'x'g'c'd'c'm'v'j t'q'w'i j "vj g v'c'd'f'k'p'v'j 'ht'qo 'c'v'c't'v'q'c'p'g'p'f'v'q'k'p'v'0X'c't'k'q'w'v'h'c'w'v't'g'u r'k'ng'vj g'v't'c'l'g'v'q't { "qh'vj g'r'c'v'j . "eq'm'k'uk'q'p'ko r'c'ev f'w'c'v'k'p'c'p'f'eq'm'k'uk'q'p'ko r'c'ev'v'q't'g'v'g'g'c'p'c'n'f' g'f "vq k'f'g'p'v'k'h'f'p'g'w't'q'q'ij k'ec'n'f'k'v'q't'f'g'tu'0'U'k'p'eg'vj g'v'c'm'v'y cu r'g't'q'to g'f "k'p'c'x'k't'w'c'n'g'p'x'k'q'p'o gp'v'k'v'q'h'g't'g'f'vj g k'p'j g't'g'p'v'c'f'x'c'p'v'c'i'g'q'h'g'r'g'c'v'd'k'k'f'q'h'vj g'g'z'r'g't'ko gp'v0 C'nu'q.'v'c'd'f'k'p'v'j u'q'h'x'c't'k'q'w'v'eqo r'ng'z'k'k'g'u'y g'g'f'g'v'k'i'p'gf "vq's wcp'v'k'c'v'k'x'g'v'c'cu'g'u'v'j g'v'g'x'g't'k'f'q'h'vj g f'k'ug'c'ug'0Vj g't'g'u'w'u'y g'g'q'd'l'g'v'k'x'g.'vj w'u't'g'f'w'el'p'i j wo c'p'h'c'v'q't'g't'q'tu'0'G'x'g'p'vj q'w'i j "vj g'c'f'x'c'p'v'c'i'g'u'v'et'g o c'p'f. "vj ku'o g'y q'f "ku'p'q'v'v'w'k'c'd'ng'v'ht'cp'ce'w't'c'v'g ur g'v't'c'n'c'p'c'n'f'uku'q'h'vj g""v'tgo qt'uki p'c'n'cu'vj g g'z'r'g't'ko gp'v'f'q'gu'p'q'v'r't'q'x'k'f'g""c'p'c'f'g's'w'c'v'v'ko g y k'p'f'qy "vq'q'd'ug't'x'g'v'tgo qt'0'Vj g't'g'ku'c'ej'c'p'eg'vj cv

eqpf wevgf "wvf gt 'vj g'uco g'r tqvqeqn'htq'cm'uwdlgeu. vj ku' lkgu'tkug'vq'c'wvcpf ctf 'b gvj qf 'hqt'f kci pquku'cpf s wcpv'k'ecv'kp'qh'tgo qt0Cf f k'k'qpcmf. 'y kj 'vj g'cf xgpv qh'xctk'qwu' i co kpi 'r m'v'qto u. "uqo g'qh'vj g'j cr v'k' k'p'v'g'hc'eg'f g'x'legu'ctg'c'x'c'k'c'd'ng'c'v'c'x'g' { "c'h'q'f'c'd'ng' r'k'leg' "c'd'q'w'&422+. 'y j gp'c'r'c'v'k'p'v'ecp'j' c'x'g'u'we'j' 'c' o' g'cu'w'k'pi' 'f'g'x'k'eg'c'v'j' k'uj' g't'q'y' p'j' qo' g'c'p'f' 'r'g't'q'to' u'g'r'h'v'g'u'u'cu'c'p'f' 'y' j' gp' 't'g's'w'k't'g'f' 'y' k'j' q'w'v'j' g'j' g'r' "q'h' c'p' { 'vj' g't'c'r' k'u'0

Vj g'f ghqto cvkqp 've'j pls wgu'wugf 'vq'b qf g'nl'q'lv qdl'geu'ctg'g'k'j' g't' i' g'qo' g't' { "d'cu'g'f' "q't' 'r' j' { 'u'k'eu'd'cu'g'f' 0 k'p' i' g'qo' g't' { "d'cu'g'f' 'v'g'ej' p'ls' w'gu' 'c'p' 'q'd'l'g'ev'k'u'f' g'h'q'to' g'f' d'cu'g'f' "q'p' "u't' w'e'w't' c'n' 'o' c'p'k' r'w'c'v'k'p'u'0'V'j' g' "w'ug't' o' c'p'k' r'w'c'v'g'u'x'g't' v'k'eg'u'q't' 'e'q'p't'q'n'r' q'l'p'u' 'y' j' k'ej' 'k'p' 'w't'p' f' k'ur' r'e'g'u' 'q'y' g't' 'x'g't' v'k'eg'u' 'h'q't' 'u'o' q'q'y' g't' 'f' g'h'q'to' c'v'k'q'p'u' *E'q's' w'k'n'c't'v' "3; ; 2+; *U'g'f' g't'd'g't'i' "c'p'f' "R'c't't' { "3; ; 8+0 J' q'y' g'x'g't' 'u'we'j' 'f' g'h'q'to' c'v'k'q'p'u'c't'g'b' q'u'w'f' 'h'q'ec'n'c'p'f' 'c't'g' p'q'v'r'j' { 'u'k'ec'm' { 'x'c'r'k'f' 0'V'j' g'l' "c't'g'c'n'u'q'p'q'v' 'u'w'k's'c'd'ng' 'h'q't' i' n'q'd'c'n'f' g'h'q'to' c'v'k'q'p'0'R'j' { 'u'k'eu'd'cu'g'f' "f' g'h'q'to' c'v'k'q'p' v'g'ej' p'ls' w'gu' "o' q'f' g'n' 'v'j' g' "f' { 'p'c'o' k'eu' "k'p'x'q'n'x'g'f' "k'p' k'p'v'g't'c'ev'k'p'i' 'y' k'j' "c'p' 'q'd'l'g'ev'c'p'f' "v'c'n'g' 'k'p'v'q' 'c'ee'q'w'p'v'v'j' g' g'h'g'ew'u'q'h'c'm'v'j' g' 'k'p'v'g't'p'c'n'c'p'f' "g'z'v'g't'p'c'n' 'h'q't'eg'u'q'p' 'v'j' g' u'j' c'r' g'q'h'v'j' g'q'd'l'g'ev' *Y' k'n'k'p' "3; ; 9+0D'w'v'j' g'ug'b' g'y' q'f' u' c't'g' 'e'q'o' r' w'c'v'k'p'c'm'f' "g'z'r' g'p'u'k'x'g'0'J' q'y' g'x'g't' "v'j' g'l' "f' q' r' t'q'x'k'f' g' "c'p' "q'r' v'k'q'p' "h'q't' "u'k'o' w'w'c'p'g'q'w'u' "i' n'q'd'c'n' f' g'h'q'to' c'v'k'q'p'c'p'f' "t'c'p'ur'c'v'k'q'p'q'h'v'j' g'q'd'l'g'ev'0'J' g'p'eg' 'y' g' w'ug'c'b' c'u'u' 'r' t'k'p'i' "d'cu'g'f' 'b' q'f' g'n'q'h'v'j' g'q'd'l'g'ev' 'b' q'x'l'p'i' w'p'f' g't' 'v'j' g' "k'p'h'w'g'p'eg'q'h'x'c't'k'q'w'u' 'k'p'v'g't'p'c'n'c'p'f' "g'z'v'g't'p'c'n' *w'ug't' 'c'r' r' n'k'g'f' +h'q't'eg'u'0

Vj g' "x'k't'w'c'n' "g'p'x'k't'q'p'o' g'p'v' "w'ug'f' "k'p' "q'w't' g'z'r' g't'k'o' g'p'u' 'e'q'p'u'k'u'u'q'h'c'f' g'h'q'to' c'd'ng' 'v'j' k'p' 'e' { 'n'k'p'f' t'k'ec'n' q'd'l'g'ev'c'u' 'u'j' q'y' p' 'k'p' "H'k'i' 0'3'0

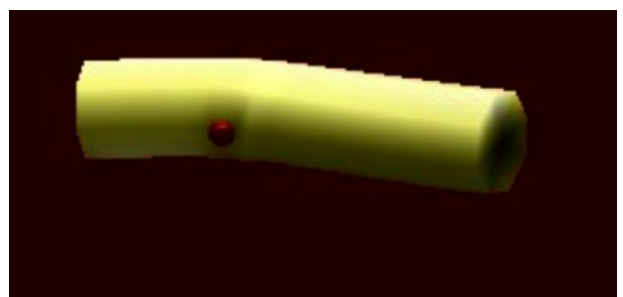


Fig. 1: 3D virtual environment consisting of a thin and soft object being deformed by the haptic device

Vj ku'b' c' { "d'g' 'e'q'p'u't'w'g'f' 'c'u'c' 'u'o' c'm' 'l'g'i' o' g'p'v'q'h'c' u'r' c'i' j' g'w'k'p'q'q'f' n'g'0'V'j' g' "i' g'q'o' g't' { "q'h' 'v'j' k'u' 'q'd'l'g'ev' 'y' c'u' f' g'uk'i' p'g'f' 'u'we'j' 'v'j' c'v'v'j' g't'g'c't'g' 'g'p'q'w'i' j' 'p'w'o' d'g't' 'q'h' 'u'w't'h'c'eg' p'q'f' g'u' 'h'q't' 'c' 'i' q'q'f' 'h'q'ec'n'f' g'h'q'to' c'v'k'q'p' *I' c't'i' . "4'2'3'3+0'C'v' v'j' g' 'u'c'o' g' 'v'k'o' g' 'v'j' g' 'v'q'w'n'p'w'o' d'g't' 'q'h' 'p'q'f' g'u'j' c'f' "v'q' 'd'g' n'g'u'u' 'v'q' 't'g'f' w'eg' 'v'j' g' 'q'x'g't'c'm' 'e'q'o' r' w'c'v'k'q'p' 'v'k'o' g' 'u'q' 'v'j' c'v' i' q'q'f' "i' n'q'd'c'n'f' g'h'q'to' c'v'k'q'p'u'c'p'f' "t'c'p'ur'c'v'k'q'p'u'c't'g'c'n'u'q' r' q'u'k'k'd'ng'0'V'j' g'ug' 'h'c'ev'q't'u' 't'g'u'w'w'g'f' "k'p' 'v'j' g' 'u'g'r'g'ev'k'q'p' 'q'h' v'j' g' "i' g'q'o' g't' { "c'u' 'u'j' q'y' p' 'k'p' "H'k'i' 0'4'0

Vj g'q'd'l'g'ev' i' g'q'o' g't' { "k'u'c' "f'c'o' r' g'f' "o' c'u'u' 'r' t'k'p'i' o' q'f' g'n'o' c'f' g'w'r' 'q'h'o' c'p'f' { "p'q'f' g'u'0'V'j' g' 'e't'q'u'u' "u'g'ev'k'q'p'c'n' x'k'g'y' . "c'u' 'u'g'g'p' "k'p' "H'k'i' 0'4' *c'+ "u'j' q'y' u'c' "u'k'p'i' n'g' "t'k'p'i' e'q'p'u'k'v'k'p'i' 'q'h' : "u'w't'h'c'eg'p'q'f' g'u'c'p'f' "q'p'g' 'k'p'v'g't'p'c'n'p'q'f' g'0' H'k'i' 0'4' *c'+ 'u'j' q'y' u'c' 'e't'q'u'u' 'u'g'ev'k'q'p'q'h'v'j' g' 't'c'p'u' /c'z'k'n'f'k'g'y' q'h'v'j' g' 'c't't'c'p'i' g'o' g'p'v'q'h' 'u'r' t'k'p'i' u'0'J' g't'g' 'Q' 't'g'r'g' 't'g'g'p'w'v'j' g' n'q'ec'v'k'q'p' "q'h' "v'j' g' "e'g'p'v'c'n' "c'z'k'u' "c'n'q'p'i' "y' j' k'ej' *r' g't'r' g'p'f' k'ew'r'c't' 'v'q' 'v'j' g' 'r' n'c'p'g'f' g'h'k'p'g'f' d' { "v'j' g' 't'k'p'i' w'r'c't' t'g'i' k'q'p' "C'Q'D'+ 'v'j' g' 'p'q'f' g'u' 'u'j' q'y' p' "k'p' 'v'j' g' 'e't'q'u'u' 'u'g'ev'k'q'p' t'g'r' g'c'v' { 'k'g'r'f' k'p'i' "c' "n'q'p'i' "e' { 'n'k'p'f' t'k'ec'n' 'u't' w'e'w't' g'0'V'j' g' t'g'r'c'v'k'q'p'j' k'r' 'y' k'j' "v'j' g' 'p'q'f' g'u'c'n'q'p'i' "v'j' g' 'r' g't'r' g'p'f' k'ew'r'c't' f' k't'g'ev'k'q'p' "k'u' 'u'j' q'y' p' "k'p' "H'k'i' 0'4' *d'+ "h'q't' "q'p'n'f' "v'j' g' 't'g'i' k'q'p' "C'Q'D'0'J' g'p'eg' 'H'k'i' 0'4' *d'+ 'u'j' q'y' u'c' 'u'g'ev'q't'c'n'f'k'g'y' 'f' g'r' k'ev'k'p'i' v'j' g' "r' q'u'k'k'q'p'u' 'q'h' 'v'j' g' "p'g'k'i' j' d'q'w't'k'p'i' "p'q'f' g'u'0'C'm' 'v'j' g'ug' p'q'f' g'u'j' c'x'g' "c' "o' c'u'u' 'c'u'u'q'ek'ev'g'f' "y' k'j' "v'j' g'o' "c'p'f' "c't'g' k'p'v'g't'eq'p'p'g'ev'g'f' d' { 'u'r' t'k'p'i' u'c'p'f' 'f'c'o' r' g't'u'0'V'j' g'f'c'o' r' g't'u' c't'g' 'x'g't' { "g'u'ug'p'v'k'n' 'v'q' 't'g'f' w'eg' 'r' q'u'k'k'd'ng' "q'ue'k'n'c'v'k'q'p'u'c'v' v'j' g' "e'q'p'u'k'w'g'p'v' 'u'r' t'k'p'i' u'0'G'c'ej' "u'w't'h'c'eg'p'q'f' g' "k'u' e'q'p'p'g'ev'g'f' 'v'q' : "q'v'j' g't' 'u'w't'h'c'eg'p'q'f' g'u'c'p'f' '5' 'k'p'k'f' g'p'q'f' g'u'0' G'c'ej' "e'g'p'v'c'n'p'q'f' g' "k'u' 'e'q'p'p'g'ev'g'f' "v'q' '4'6' 'u'w't'h'c'eg'p'q'f' g'u' c'p'f' '4' 'p'g'k'i' j' d'q'w't'k'p'i' 'e'g'p'v'c'n'p'q'f' g'u'0'V'j' g' 'u'w't'h'c'eg'p'q'f' g'u' c't'g' 't'g'u'r' q'p'u'k'd'ng' 'h'q't' 'h'q'ec'n'f' g'h'q'to' c'v'k'q'p'c'p'f' 'v'j' g' 'e'g'p'v'c'n' p'q'f' g'u'd't'k'p'i' 'c'd'q'w'f' n'q'd'c'n'f' g'h'q'to' c'v'k'q'p'c'p'f' "t'c'p'ur'c'v'k'q'p' q'h'v'j' g'q'd'l'g'ev'0'V'j' g' 'e'q't't'g'ev'g'u'k'o' c'v'k'q'p'q'h' 'u'r' t'k'p'i' 'u'k'h'p'g'u'u' e'q'g'h'k'ek'p'v'k'u'x'k'c'n' 'h'q't' 'v'j' g' 'u'w'c'd'k'k'v' { "c'p'f' "d'g'j' c'x'k'q'w't' 'q'h' v'j' g' 'b'o' c'u'u' 'r' t'k'p'i' "o' q'f' g'r'0'H'q't' 'v'j' g' 'g'u'k'o' c'v'k'q'p'q'h' 'u'r' t'k'p'i' 'u'k'h'p'g'u'u' 'e'q'g'h'k'ek'g'p'v' "o' c'p'f' { "c'n'i' q't'k'j' o' "u'j' c'x'g' "d'g'g'p' r' t'q'r' q'ug'f' *F' g'w'au'g'p' g'v'0'c'rl'0'3; ; 7= 'N'q' { f' g'v'c'rl'0'4'2'2'9+0' U'q'o' g'q'h'v'j' g'o' "w'r'f'c'v'g' 'v'j' g' 'u'r' t'k'p'i' 'u'k'h'p'g'u'u' 'e'q'g'h'k'ek'g'p'v' c'v'g'c'ej' "v'k'o' g' 'u'v'g'r' "c'p'f' "u'q'o' g' "w'r'f'c'v'g' 'v'j' g' 'u'k'h'p'g'u'u' e'q'p'u'c'p'v' 'c'h'v'g't' "c' "h'g'y' "v'k'o' g' 'u'v'g'r' "u' "w'uk'p'i' "k'v'g't'c'v'k'x'g' c'n'i' q't'k'j' o' u'0'V'j' g'ug' "o' g'y' q'f' u'c't'g' "w'ug'h'w'f'k'p' "i' t'c'r' j' k'e' f' k'ur' n'c' { "y' j' g't'g' "v'j' g' 'e'q'o' r' w'c'v'k'q'p'u' 'e'c'p' "d'g' "t'g'm'v'k'x'g'n'f' u'ny' g't' *c'v'c' 't'c'v'g'q'h' '5'2' 'h'c'o' g'u'r' g't' 'u'g'ev'q'p'f' +e'q'o' r' c't'g'f' v'q' "v'j' c'v'k'p' "j' c'r' v'k'eu' "j' g't'g' "v'j' g' "w'r'f'c'v'g' "j' c'u' 'v'q' "d'g'c'v'v'j' g' o' k'p'k'o' w'o' "t'c'v'g'q'h' "3' "n'j' | "h'q't' 'v'j' g' "t'g'c'v'q'p' "q'h' 'u'w'c'd'k'k'v' { 0' V'j' g't'g'h'q't'g' 'v'j' g' 'h'q'to' w'r'c't' r' t'q'r' q'ug'f' 'k'p' *I' g'r'f' g' "3; ; ; +y' c'u'

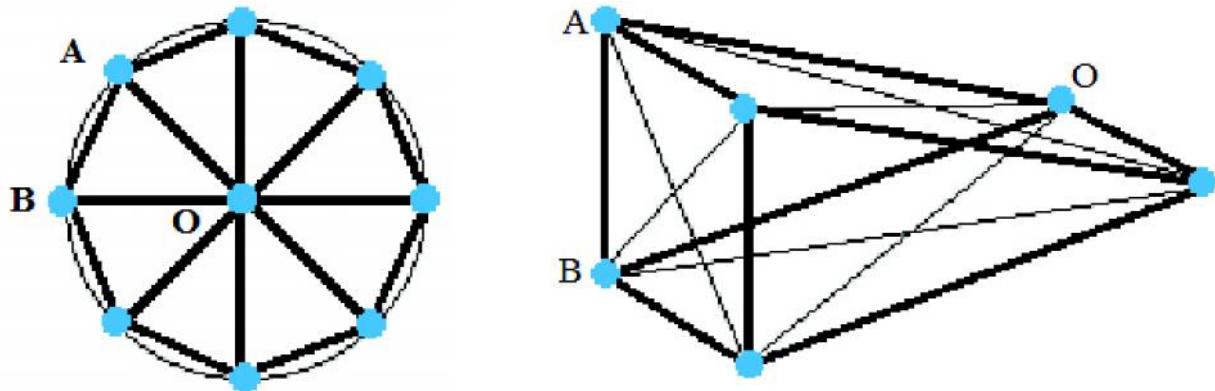


Fig. 2: (a) Cross-sectional view of the cylindrical object with dots indicating the nodes and the straight lines indicating the springs connecting them, (b) a finite volume tetrahedral element, for the wedge shape element ABO showing the diagonal springs [Adapted from: [Anish, 2011]]

wugf 0'k'ku'i kxgp'd{

$$m_e = G \sum_g \frac{Xqr^*V_g}{n_e^4} \tag{*3}$$

Y j gtg m_e tgr tgu p w u ' y j g ' l w k t h p g u u ' e q g h h e k g p v ' q h ur t k p i ' e . n_e k u ' y j g ' r g p i y j ' q h ' u r t k p i ' e ' c p f ' G k u ' y j g [q w p i a i ' o q f w w u ' q h ' y j g ' o c v g t k r i ' 0 v j g ' u w o o c v k a p ' k u q x g t ' c m ' v j g ' v g t c j g f t c n ' g r g o g p w u V_g ' c f l q k p i ' e 0 X q r ^ * V_g + f g p q v u ' y j g ' x q n w o g ' q h ' y j g ' v g t c j g f t c n ' g r g o g p v V_g u j q y p ' k p ' H k i ' 0 ' 4 ' d ' 0 v j g ' c f x c p c i g ' q h ' w u k p i ' y j k u h q t o w r ' k u ' y j c v ' y j g ' u r t k p i ' u w k t h p g u u ' e q g h h e k g p w ' c t g r t g / e q o r w u g f ' c p f ' x c n e d r g ' e q o r w c v k a p ' v k o g ' k u ' p q v y c u g f ' q x g t ' y j g k t ' e c r e w r c v k a p u ' c v ' g c e j ' v k o g ' u v g r 0 U g r g e v k a p ' q h ' u r t k p i ' f c o r k p i ' e q g h h e k g p w ' c t g ' g s w c m { k o r q t v e p v ' c u ' y j g { ' r t g x g p v ' y j g ' r c t v e r g u ' q h ' y j g ' q d l g e v h t q o ' q u e k r c v k a p u ' c u ' o g p v k a p g f ' k p ' * D w i k p g v ' c r i 0 3 ; ; : 0 v j g ' x c n w g ' q h ' u r t k p i ' f c o r k p i ' e q g h h e k g p v m_e^f k u ' c n g p ' r t q r q t v k a p c n i ' v ' y j g ' u r t k p i ' e q p u x p v ' * N n { q f g v c r i 0 ' 4 2 2 : + ' c u ' i k x g p d { .

$$m_e^f = \alpha m_e 0 \tag{*4}$$

Y j gtg \alpha ' k u ' c ' r q u k k x g ' e q p u x p v ' c p f ' 2 ' > \alpha ' > ' 3 0

F w t k p i ' j c r v k e ' k p v g t c e v k a p . " e q m k u k a p ' f g v g e v k a p c p f ' h q t e g ' e q o r w c v k a p ' c t g ' v j g ' y q ' k o r q t v e p v r t q d r g o u ' k p ' t g p f g t k p i ' 0 ' Y g ' h q n q y ' y j g ' v e j p l s w g r t q r q u g f ' k p ' * 1 c t i . ' 4 2 3 3 - h q t ' y j g ' c d q x g ' r w r q u g u 0 v j g u w t h c e g ' p q f g ' y j c v ' k u ' e n q u g u v ' v q ' y j g ' j c r v k e ' k p v g t h c e g r q k p v k u ' c n g p ' c u ' y j g ' e q m k u k a p ' r q k p v 0 C n i p q f g ' r q u k k a p u

ctg"wr f cvgf "cv'gxgt{ "vko g"uvgr 0'Vj g"co qwpv'qh f ghqto cvkqp'ku'f gr gpf gpv'qp'yj g'gz vgt pcm{ "cr r k g f h q t e g ' c p f ' u r t k p i ' c p f ' f c o r g t ' e q g h h e k g p w 0 v j g ' o q f g n c n u q ' c n g u ' k p v q ' c e e q w p v ' y j g ' h k e v k a p c n i ' h q t e g ' d g y g g p y j g ' q d l g e v ' c p f ' y j g ' r n e p g ' d { " c r r t q r t k c v n { " u g r g e v k p i y j g f { p c o k e ' h k e v k a p ' e q g h h e k g p v h q t ' y j g ' u w t h c e g 0 v j g j c r v k e ' y q t m i r c e g ' e q p u k u ' q h ' c 3 2 ' e o ' u k f g ' e w d g ' c p f y j g ' u w d l g e v ' e c p ' k p v g t c e v ' y k j ' y j g ' x k t w c n l q d l g e v ' y k j ' c h q t e g ' k p ' y j g ' t c p i g '] 2 . ' 7 _ P 0

Experimental Setup

Hki 0'5"uj qy u'yj g'r tqr qugf "gzr gtlo gpvcn'ugwr "hqt vtgo qt 'f gvge v k a p ' w u k p i ' c ' j c r v k e ' g p x k t q p o g p v 0

Y g ' c n u q ' u j q y ' y j g ' w u g ' q h ' c e e g r g t q o g v g t u ' k p ' y j g u c o g ' u g w r ' h q t ' e t q u u / x c r k f c v k a p ' r w r q u g u 0 C w e j g f v q ' y j g ' u w d l g e v u ' j c p f ' k u ' c ' o q v k a p ' v c e n g t ' y j k e j

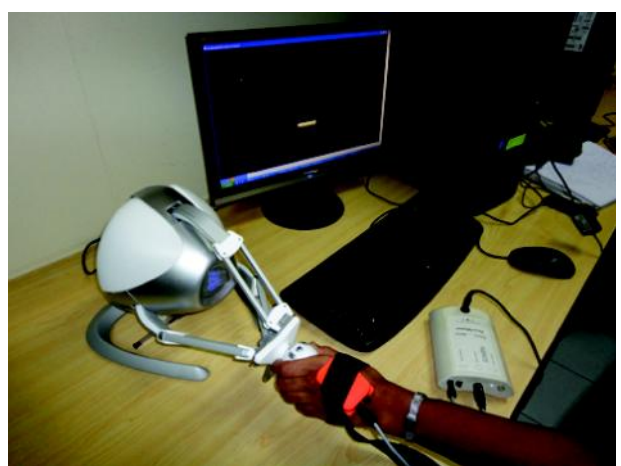


Fig. 3: Experimental set-up consisting of both haptic interface and motion sensors

kpvtpcm{ 'eqpvkpu'cp'beegrtqo gygt'cpf 'c'i { tqueqr g0 Vj g'uwdlgev'r gthqto u'c'vum'qh'o qxkpi "vj g'xktwcn qdlgev*f guetkdgf 'kp'vj g'r tgxkqwu'ugevqp+ 'r rcegf "qp c'j qtk qpvcn'uwthceg'y kj "c'j cr vke'f gxleg"cmppi "c ur gekhe'f kgevqp0Kp'vj g'ewtgp'gzr gtko gpv'vj g'vcum ku'vq'tqvcv'vj g'qdlgevd{"; 2² cdqw'vj g'xgtvceci'czku y kj "qpg'qh'vj g'gf i gu'cu'vj g'egpvg'qh'tqvcvqp0Hki 0 6*c+'cpf "Hki 0'6*d+'uj qy "vj g"kpvkcn'cpf "uqo g kpvtgo gf kv'g'r qukvkpu'qh'vj g'uqhv'qdlgev'f wkpi kpvtcevqp0Vj g'uwdlgev'o qxgu'vj ku'qdlgev'uj'vj cv'kv ku'hkpcmf "r ctmgn'vq'vj g'tghgtpeg'rkpg'cu'uj qy p'kp Hki 06*e+0Vj ku'r tqegu'vcngu'cdqw37/42'ugeqpf u'ht c'pqto cni'uwdlgev'cpf "o c{ 'tgs vkt'g'c'mpi gt'f wcvkqp hqt"cu'uwdlgev'uwhtkpi 'htqo "j cpf "tgo qt0Vj g'uco g gpxktqpo gpv'y cu'cuq'xkuwcn{ 'tgpf gtgf 'kp'5F "wukpi vj g'uko r ng'b gj qf 'qh'cpci n{ r j ke'uvgtgq0Vj ku'kpxkngu xkuwcn{ 'tgpf gtkpi 'vj g'gpxktqpo gpv'cu'uggp'htqo 'vy q f'khtgtpv'eco gtc'mqecvku. "kp"vy q'f kvkpev'eqmtu. uwr gtr qugf "qp'vj g'uco g'uetggp0Kp'dwkn/hwpevku'qh Qr gpI N'j cxg'dggp'wugf "v'ko r ngo gpv'vj ku0

Cpci n{ r j ke'i rnuugu'ctg'wugf "v'xkgy "vj g'5F gpxktqpo gpv'y j krg'r gthqto kpi "vj g'j cr vke'vcun0Vj g j cr vkeu'CRK*J CRK'gpcdrgu'wu'vq"i gv'vj g'xgmekv{ kphqto cvkqp'y kj "y j kej "vj g'j cr vke'f gxleg'ku'o qxgf 0 Uko wncpgqwu{. 'beegrtcvkqp'cpf 'qtkgpvcvqp'f cvc'ctg cuq'qdvkpgf 'htqo 'vj g'o qvkap'tcengt0Vj wa'y g'pqy j cxg'kphqto cvkqp'cdqw'uwdlgevai"j cpf "o qxgo gpv vj tqwi j "vy q'kpf gr gpf gpv'luqtegu* j cr vke'f gxleg'cpf o qvkap'tcengt+0J qy gxgt. "vj g'uactv'cpf "vj g'gpf o gcuwtgo gpv'ko gu'qh'vj g'vy q'luqtegu'f khtg'dgecvug vj g{ 'ctg'uactvgf 'cpf 'uvqr r gf 'kpf gr gpf gpv{0Kp'qtf gt vq'kf gpvkh{ "vj g'eqo o qp'f cvc'tgi kqp."cu'c'r tqvqeqn vj g'uwdlgev'uj cngu"j kulj gt"j cpf "c'rkvwg'xki qtqwu{

dghgt'uvctvki "cpf "chgt'eqo r rgvkpi "vj g'i kxgp'vcum vq"j gr "u{pej tqpk'g'j cr vke'cpf "o qvkap'tcengt'f cvc ceevctcvgn{0Vj ku'f grkdgtcvg"uj cng"o wuv'pqv'dg tgr gvkkxg0Dghgt'vj ku'gpvkt'g'gzr gtko gpv'dgi kpu."vj g uwdlgev'ctg'htuvcmjy gf 'vq'i gv'hc0 kktk'gf 'y kj "vj g y j qrg"u{ ugo "cpf "r gthqto "ctqwpf "32"vkn0Vj gp. dcugf "qp"y j gvj gt"vj g{ "r tghgt "4F "qt "5F "xkuwcn gpxktqpo gpv'cp'cr r tqr tkvg'f kur r{ 'ku'ugv'wr 'cpf 'vj g g'zr gtko gpv' ctg" r gthqto gf 0' Vj g" 5F " xkuwcn gpxktqpo gpv'r tqxkf gu'c"o wej "dgwgt"j cr vq/xkuwcn ko o gtukqp'qh'vj g'uwdlgev'kp'vj g'xktwcn'y qtrf 0'Dw vj g'wug'qh'cpci n{ r j ke'rgpugu'o c{ 'dg's vkg'vkt guqo g vq"vj g'uwdlgevai'xkuqpo 0 K'uj qwrf "dg'pqv'f "vj cv'vj g r tgupeg'qh'o qvkap'ugpuqt'qp'vj g'j cpf "o ki j v'chgev vj g'f gzvktv{ 'qh'vj g'uwdlgev0Dw.'o qvkap'ugpuqtu'ctg wugf "j g'gt'lwuv'ht "eqo r ctvkap"r wtr qugu'cpf "vq guvdrkuj "kp'vj ku'r cr gt"vj cv'j cr vke'gpxktqpo gpv'ku kpf g'gf "c'i q'qf 'ecpf kf cv'g'ht"o gcuwtkpi "j cpf "tgo qt0 Chgt'gucdrkuj kpi "vj g'gs vkcrgpeg'dgy ggp'vj g'tgo qt f cvc'eqmgev'f d{ "vj g'j cr vke'f gxleg"cpf "o qvkap ugpuqtu."vj g'tguv'qh'vj g'tgo qt'f cvc'cpcn{ uki'ecp'dg ecttkgf "qww'wukpi "vj g'f cvc'htqo "j cr vke'f gxleg'cmppg0 K'o c{ "cuq'dg"o gpv'kppgf "vj cv'cp{ "j cr vke'kpvthceg f gxleg'wuwcn{ "j cu'ku'qy p'kpgtvc"Ucrkudw{ 'gv'crl0 3; ; 7+'cpf "uwej "cp'kpgtvc"o c{ 'chgev'vj g'ur gev'cn tgur qpug'qh'vj g'eqmgev'f lki pcr0Y g'cuuwo g'vj g'dcpf/ y kf vj "qh'vj g'j cr vke'f gxleg'vq'dg"o wej "j ki j gt"vj cp vj cv'qh'vj g'tgo qt'htgs wpe{ "wuwcn{ 'w'pf gt"32"J | + uq"vj cv'vj g'ghgev'qh'o gej cplecn'kpgtvc"o c{ "dg pgi rgev'f 0

Pre-Processing

Y g'cko "vq"eqo r ctg"vj g'r gthqto cpegu'qh'vj g

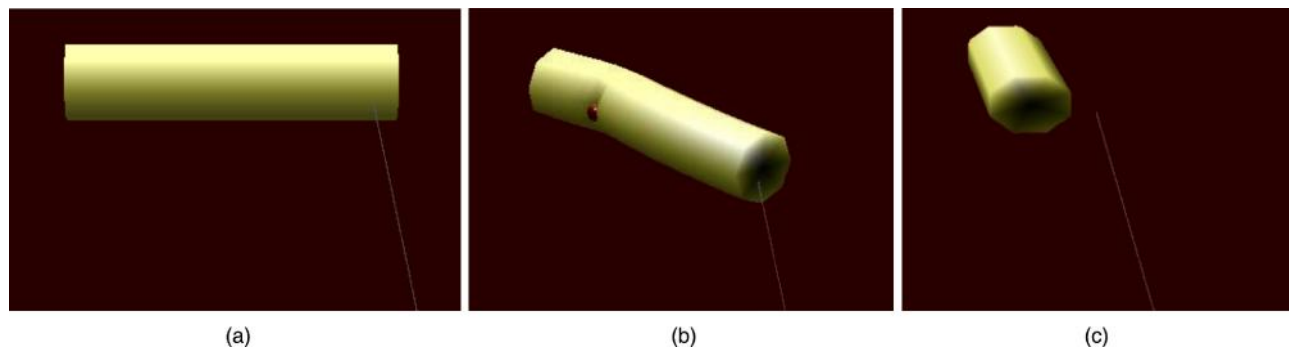


Fig. 4: (a) Initial, (b) some intermediate and (c) final positions of the soft object before, during and after performing the task. The thin line is a reference that helps the subject to align the object to the desired orientation

accelerometer based method with the haptic based method. But the sampling rates and work space axes of the two devices are different. Also, data capturing is asynchronous because the two devices are turned on independently. Further, the sampling rates for the haptic device and the motion sensors are different. Due to these reasons, the following pre-processing steps are required.

(a) Calibration

The data obtained from motion sensors is in the local sensor co-ordinate system. Whenever the subject changes its hand orientation (which definitely occurs while performing the task), the direction of the local co-ordinate system also changes. Thus all the data captured during the task corresponds to values along many different unknown directions. We need to convert all of them to a common co-ordinate system. With the help of the orientation information, this data is converted to the global co-ordinate system using the Following transformation.

$$X_{GS} = R_o^T X_s \quad (3)$$

Where X_s denotes the data from motion sensors in the local sensor co-ordinate system, R_o represents the orientation data available from motion sensors and X_{GS} represents the data in the global co-ordinates system. Since the motion sensor is firmly placed on the palm (outside) used to move the stylus on the haptic device, the translation component can be assumed to be zero. The directions of global co-ordinate system follow a right handed co-ordinate system and are as follows: X-North; Y-West; Z-Vertically up. The orientation calibrated motion sensor data is used for all subsequent processing. It may be noted that the 3D accelerometers measure all accelerations, including the acceleration due to gravity. This is inherent to all accelerometers. Therefore, we remove this component of acceleration due to gravity by subtracting the local mean from the Z-component of the accelerometer data (Pawar *et al.*, 2008). Next, the haptic workspace may not coincide with this co-ordinate system. For this, we adjust the haptic device so that the haptic workspace coincides with the global co-ordinate system using a compass.

The z-coordinate of the haptic space is aligned to the magnetic north of the compass with x-y plane aligned to the top surface of the table housing the haptic device. Having done that, the position of the haptic device on the table is kept unchanged during subsequent experiments.

(b) Temporal Synchronization

The useful data lies between the deliberate shakes done at the beginning and at the end of the task. These shakes correspond to peaks in the recorded data from both haptic device and accelerometer as shown in Fig. 5 and 6. These peaks are easily identified in both accelerometer and haptic data in all the three axes and the start and the end points from both these sources are matched so that an accurate resampling rate for the haptic data can be calculated to perform the next step. This can be mathematically described as:

$$h_e \triangleq \min(t_{lx}^h, t_{ly}^h, t_{lz}^h) \quad (4)$$

$$h_s \triangleq \max(t_{fx}^h, t_{fy}^h, t_{fz}^h) \quad (5)$$

$$m_e \triangleq \min(t_{lx}^a, t_{ly}^a, t_{lz}^a) \quad (6)$$

$$m_s \triangleq \max(t_{fx}^a, t_{fy}^a, t_{fz}^a) \quad (7)$$

Where $t_{fx}^a, t_{fy}^a, t_{fz}^a, t_{fx}^h, t_{fy}^h$ and t_{fz}^h denote the time instants of first peak detected in the accelerometer and haptic data along x, y and z axes, respectively, and $t_{lx}^a, t_{ly}^a, t_{lz}^a, t_{lx}^h, t_{ly}^h$ and t_{lz}^h denote the time instants of the last peak detected in the accelerometer and haptic data along x, y and z axes, respectively, h_e and h_s stand for end and start sample indices for the haptic signal, and m_e and m_s are the corresponding indices for the motion sensor data. Thus synchronization means setting $h_s = m_s$ and $h_e = m_e$. To identify the peaks accurately, a bilateral filter (Tomasi and Manduchi, 1998) is applied to both data. This will smoothen the signal without disturbing the peaks corresponding to the initial and final shakes. The peaks are then detected from this smoothed data. For all further processing, the data in-between these peaks are considered. It may be mentioned here that matching the start and the end points by cross correlation cannot be applied as the haptic and motion sensor data are sampled at different rates.

(c) Resampling

In our trials, the haptic sampling rate is about 140

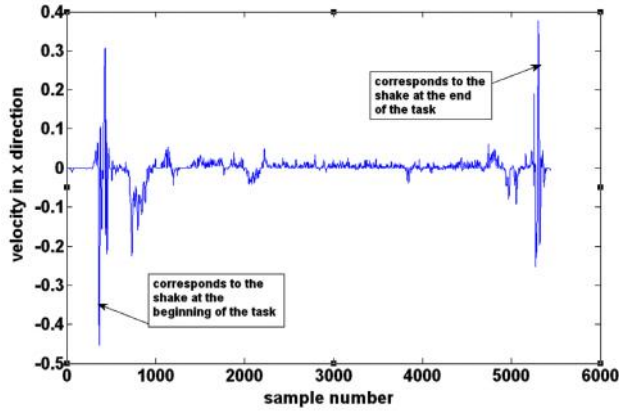


Fig. 5: Velocity signal in x-direction vs sample number as sensed by the haptic device

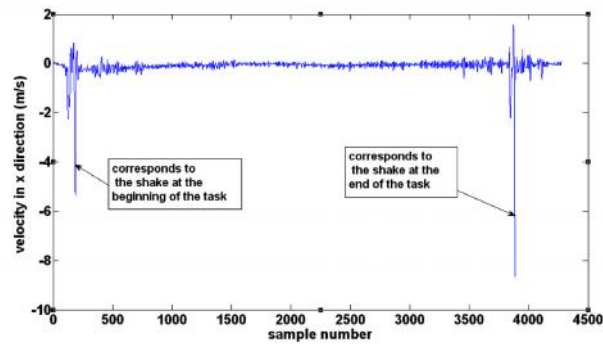


Fig. 6: Velocity signal along x-direction as sensed by the accelerometer

Hz, while the motion tracker used here samples data at the rate of 100 Hz. Therefore, the haptic data is downsampled to match the sampling rate of the motion tracker. The exact downsampling factor D for the haptic signal is given by

$$D = \frac{h_e - h_s}{m_e - m_s}, \quad (8)$$

The 'resample' function of MATLAB is used to down sample the data. Now, both the motion sensor and haptic data are temporally registered, geometrically aligned and correspond to the same sampling frequency of about 100 Hz.

Tremor Data Extraction

The haptic device gives the velocity of hand motion and motion sensor provides the acceleration. Thus the acceleration data from the motion sensor has to be integrated to obtain the velocity. The resultant signal obtained by performing all the steps mentioned above can be considered to be represented as the sum of two components as

$$r(n) = e(n) + f_{tr}(n) \quad (9)$$

with, $r(n)$ denoting baseline data signifying task related hand motion along a particular coordinate axis and $f_{tr}(n)$ denoting the corresponding high frequency tremor component. The base line data (task related hand motion) of the velocity of hand motion along x , y and z directions from the haptic device and motion sensor are computed using an averaging window filter. The estimated base line data at instant n is given by

$$\hat{e}(n) = \frac{\sum_{k=-w_d}^{w_d} r(n+k)}{2w_d + 1} \quad (10)$$

where, w_d denotes the half width of the averaging window.

The averaging filter smoothes the captured velocity of hand motion by removing the higher frequency components and allowing only low

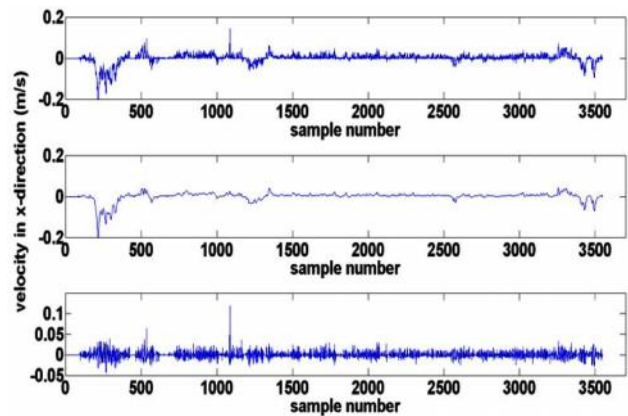


Fig. 7: Plots of haptic data after down sampling (top) the estimated motion trajectory (middle), and the identified hand tremor signal (bottom)

frequency components to pass through. This method also allows us to remove the gravity component along the z-component. Fig. 7 shows an example of the downsampled haptic velocity data and the extracted base line data (motion trajectory). This motion trajectory is then subtracted from the original data $r(n)$ to get only the high frequency components $\hat{f}_{tr}(n)$ as

$$\hat{f}_{tr}(n) = r(n) - \hat{e}(n) \quad (11)$$

Now $\hat{f}_{tr}(n)$ serves as the estimate of the tremor data. A typical signature of such a hand tremor is also shown in Fig. 7. This data is amenable to different types of signal analysis techniques as described in the next section. Just to illustrate one signal analysis technique, we have considered the power spectral density (PSD), computed using the the periodogram, of the high frequency component $\hat{f}_{tr}(n)$ to detect the presence of tremors. The PSDs of non-tremor and tremor data from both the haptic device and the motion sensor are shown in Figs. 8 and 9, respectively.

For Fig. 8, we observe that the spectra are quite flat in both cases, signifying that there is no pseudo rhythmic pattern in the data. On the other hand, the PSDs in both cases of Fig. 9 show a distinct peak and the locations are quite close in frequency. This corroborates the fact that the proposed haptics based system is indeed a viable option.

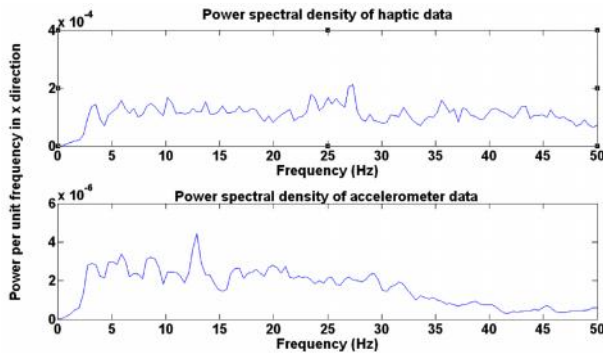


Fig. 8: Power spectral density of non-tremor data for a subject without any visible hand tremor collected from haptic device (top) and accelerometer (bottom) for the component along x-axis

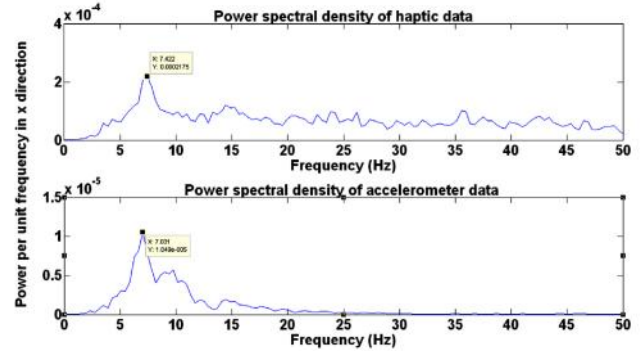


Fig. 9: Power spectral density of tremor data for a subject with visible hand tremor collected from haptic device (top) and accelerometer (bottom) for the component along x-axis

Signal Analysis

We use two different techniques for signal analysis. Since the equivalence between the haptic and the motion sensor data has already been established, we do not show any more results of analysis of the motion sensor data. For the analysis of haptic data, we first use the autoregressive (AR) model fitting to the time series to perform spectral estimation. This provides a good localization of the tremor frequency. Subsequently we perform a time frequency analysis of the data using the Choi-Williams method. The purpose of the study is to observe if there is a temporal variation in the tremor frequency.

(a) AR Modelling

A spectrum that is characterized by its peaks, as opposed to valleys, can be modeled well by an AR model, thus making AR modeling the most appropriate method for the analysis of tremor data. Two steps are involved in the modelling of tremor data using an AR model. First is the selection of a proper model order (p) and second is the estimation of model parameters from the data. Here the data is modelled by the following difference equation

$$f_{tr}(n) = \sum_{k=1}^p a_k f_{tr}(n-k) + w(n) \quad (12)$$

where, $w(n)$ is a white noise, a_k are the AR parameters and $f_{tr}(n)$ is the tremor data. The power spectral

density of the tremor data is given by

$$P_{ftr}(f) = \sigma_w^2 |H(f)|^2 \quad (13)$$

where, $P_{ftr}(f)$ is the power spectral density of the tremor data, σ_w^2 is the noise power and $H(f)$ is the frequency response of the model. The AR parameter estimation is done by using Burgs algorithm (Burg, 1968). The AR model order p is estimated by using the Akaike information criterion (AIC) (Akaike, 1974) and is based on selecting the order that minimizes

$$AIC(p) = \ln \hat{\sigma}_{wp}^2 + \frac{2p}{N} \quad (14)$$

Where, $\hat{\sigma}_{wp}^2$ is the estimated noise variance and N is the length of the data sequence. In the present work we identified that a 6th order AR model can fit the tremor data received from different subjects very well.

(b) Choi-Williams Spectrum

Time frequency signal representations characterize signals over a time frequency plane (Cohen, 1989). These kinds of analysis tools give a temporal localization of a signal's spectral components. Time frequency representations map an input signal $f_{tr}(n)$ into a two dimensional function of time and frequency, $T_{ftr}(n, f)$. This technique has been used to analyze non-stationary or time varying signals. Wigner-Ville distribution (WVD) is a member of Cohen class time-frequency distributions (Cohen, 1989). The well known Cohen class distribution is given by

$$C_{ftr}(i, k) = \sum_{m=-L/2}^{+L/2} \sum_n \phi(n, m) R(i-n, m) e^{-j2\pi km/L} \quad (15)$$

where $R(i, m)$ is the auto correlation function of the tremor signal and $\phi(n, m)$ is the kernel function. Here L is the window length and the variable n is also varied across an appropriate window. Compared to short-time Fourier transform based method of time

frequency analysis, this method can yield a better time frequency resolution. The Wigner-Ville distribution of a signal is obtained by correlating the signal with a time shifted version of the signal itself and the WVD of a signal $f_{tr}(n)$ is defined as

$$DWVD_{ftr}(n, f) = 2 \sum_{m=-\infty}^{+\infty} f_{tr}\left(n + \frac{m}{2}\right) f_{tr}^*\left(n - \frac{m}{2}\right) e^{-j2\pi fm} \quad (16)$$

WVD can be computed efficiently using a fast Fourier transform (FFT) (Swami and Mendel). One major drawback of WVD is the presence of cross term or interference term as WVD is quadratic in nature. The amount and the shape of this interference are directly related to the Particular kernel function $\phi(n, m)$ which is unity for WVD. Choi and Williams (Choi and Williams, 1989) proposed the use of an exponential kernel

$$\phi(n, m) = \exp\left(\frac{-n^2 m^2}{\sigma^2}\right) \quad (17)$$

to reduce the cross terms, but at the same time preserving most of the properties of WVD. The parameter σ controls the attenuation of the cross terms. However, increasing σ leads to a loss of spectral resolution. As a compromise we choose a value of σ equal to 0.5.

Experimental Results

The virtual environment was created using Visual Studio 2005 and the HAPI libraries. HAPI is an open source high level haptic API developed by Sense Graphics. HAPI is written in the C++ programming language and works on all major operating systems. HAPI provides a number of haptic force effects, surface effects, collision detection, primitive shape creation and thread handling in addition to the basic device handling. We use Novint FALCON, a commercially available haptic gaming platform in the prize range of \$200, as the haptic device to interact with the virtual world. For motion data, we use Xsens

sensors. Data from the motion tracker was transmitted to a computer via an USB cable. The captured data was processed offline in MATLAB on a Intel Core-2-Quad processor system with 8 GB RAM and a processing speed of 2.6 GHz working under Windows XP Operating system.

A normal subject is first asked to interact with the virtual soft object through the FALCON and the experience of the user is noted down. As the number of nodes in the object is increased, the haptic experience improves, signifying a smoother interaction. However, this increases the computation time for haptic rendering and one may not be able to complete the rendering within 1ms. Therefore, a GPU based rendering would be more useful to the user. Since all our computations are performed in the CPU, we had to restrict the number of nodes to 180. If the gap increases between two successive nodes along the length of the object, there could be a possibility of piercing through the object during interaction if a large amount of force is applied. This is a short-coming of the developed system which hopefully can be solved by introducing more number of nodes to model the object and by implementing it on a GPU. In order to have a better haptic-visual immersion into the 3D environment, the visual rendering was performed on a stereoscopic display by superimposing the images for two different viewpoints. An anaglyphic lens is used by the subject to perceive the depth during haptic interaction. A number of tremor affected volunteers are asked to interact with the created virtual environment and the velocity of hand motion is captured using the haptic device. The captured velocity data is subjected to tremor extraction routine discussed in section V. The power spectrum estimated using the AR model of two representative subjects are shown in Fig. 10 and Fig. 11.

The frequency corresponding to the peak value is 4.6 Hz in subject 1 and 7.0 Hz in subject 2. These frequencies are in accordance with (Harish *et al.*, 2009). From both the plots, one can see a distinct peak in the spectrum. This indicates that the subject indeed has hand tremor, which is verified from the known history of the subject. These results illustrate

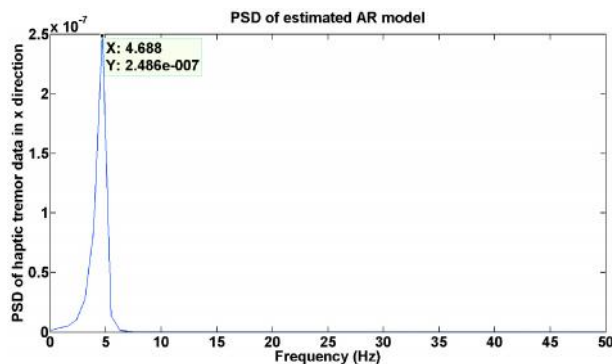


Fig. 10: PSD of haptic tremor data from subject 1 along x-direction

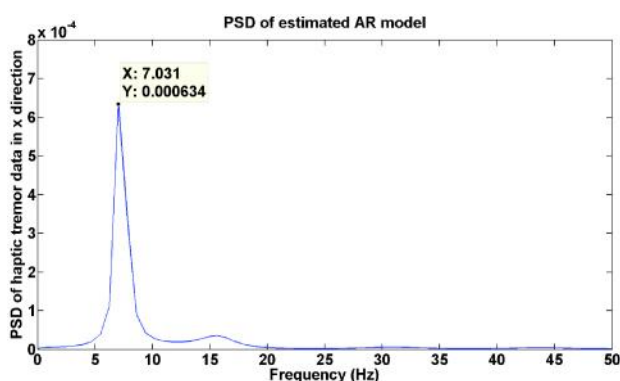


Fig. 11: PSD of haptic tremor data from subject 2 along x-direction

the usefulness of the proposed method.

The Choi-Williams spectrum is used to see if there is any temporal variation in tremor frequency. The Choi-Williams spectra from two representative subjects are plotted in Figs. 12 and 13.

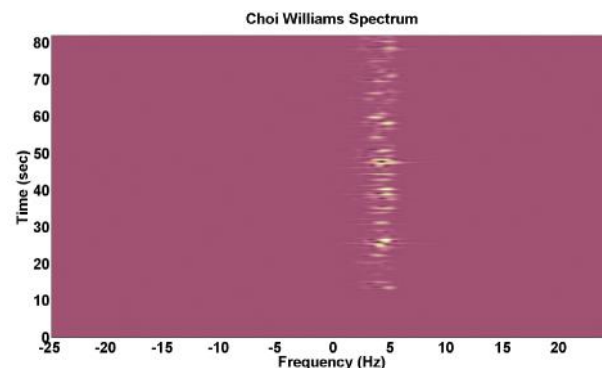


Fig. 12: Choi-Williams spectrum of tremor data for subject 1 along x-direction

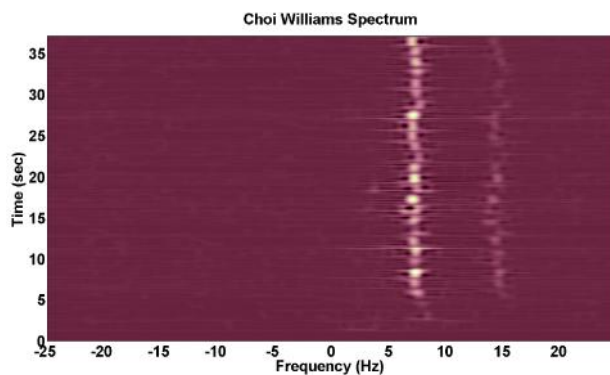


Fig. 13: Choi-Williams spectrum of tremor data for subject 2 along x-direction

From these two plots, we do not observe any noticeable change in the spectral content temporally. This suggests that both subjects had a fairly constant tremor frequency and it did not change during different phases of performing the haptic task. This further demonstrates that the proposed technique is quite reliable as there is no drift in frequency while performing the haptic task. In all these illustrations, we have given the spectral plots of tremor data along x-axis. Since the motion of the object is restricted to x-y plane only (as the object rests on the horizontal plane $z = 0$), we have also analyzed the tremor along the y-axis, and the corresponding results are found to be very similar. Hence they are not reproduced here.

Conclusions

We have proposed the use of a deformable object manipulation in a virtual 3D haptic environment as a

References

- Akaike H (1974) A new look at the statistical model identification, *IEEE Transactions on Automatic Control* AC-19, 716-723
- Albers J W, Potvin A R, Tourtellotte W W, Pew R W and Stribley R F (1973) Quantification of Hand Tremor in the Clinical Neurological Examination, *IEEE Transactions on Biomedical Engineering* BME-20, 27-37
- Bardorfer A, Munih M, Zupan A and Primozic A (2001) Upper limb motion analysis using haptic interface, *IEEE/ASME Transactions on Mechatronics* 6 253-260

method of tremor data acquisition. We collected data from several subjects, in the age group of 60-65, having tremor. We have been able to identify the presence of tremors based on the power spectral density of tremor signal in the velocity data, obtained from the haptic device. We have compared the proposed haptic based method with the most commonly used accelerometer based method and found the consistency of results from both the methods, thus experimentally validating our idea. Further, we are able to fit an AR model to the tremor data and estimate the PSD using the AR model. The haptic based method does not require any recalibration as required by accelerometer based method. Further by designing different types of deformable objects and interaction plan, any kind of tremor could be measured and quantified. In this paper our emphasis has been purely on developing a general purpose haptic system for measurement of hand tremor. The task of identifying various types of hand tremors and the progression analysis of tremor condition after appropriate rehabilitation remains to be investigated. Further the effect of loading during the task performance needs to be studied for an appropriate subject base.

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- Burg J P (1968) A new analysis technique for time series data, NATO advanced study institute on signal processing with emphasis on underwater acoustics 83 12-23
- Burgin J, Stephens B, Vahora F, Temkin B, Marcy W, Gorman P J and Krummel T M (1998) Haptic rendering of volumetric soft- bodies objects, Proceedings of Third PHANTOM Users Group Workshop, Massachusetts Institute of Technology, Cambridge, 9-13
- Choi H I and Williams W J (1989) Improved time-frequency representation of multicomponent signals using exponential kernels, *IEEE Transactions on Acoustics,*

- Speech and Signal Processing* **37** 862-871
- Cohen L (1989) Time-frequency distributions-a review, *Proceedings of the IEEE* **77** 941-981
- Coquillart S (1990) Extended Free-Form Deformation: A sculpturing Tool for 3D Geometric Modeling, *Proceedings of SIGGRAPH Computer Graphics* **24** 187-196
- Deussen O, Kobbelt L and Tucke P (1995) Using simulated annealing to obtain good nodal approximations of deformable bodies *Proceedings of Eurographics Workshop on Computer Animation and Simulation* 30-43
- Garg A (2011) Haptic Rendering of 3D objects - M.Tech Thesis, Indian Institute of Technology Bombay
- Gelde A V (1998) Approximate simulation of elastic membranes by triangulated spring meshes *Journal of Graphics Tools* **3** 21-42
- Harish K, Rao M V, Borgohain R, Sairam A and Abhilash P (2009) Tremor quantification and its measurements on Parkinsonian patients, International Conference on Biomedical and Pharmaceutical Engineering *ICBPE '09* 1-3
- Kay S M and Maple S L (1981) Spectral analysis - a modern perspective *Proceedings of IEEE* **69** 1380-1419
- Lloyd B A, Szekely G and Harders M (2007) Identification of Spring Parameters for Deformable Object Simulation, *IEEE Transactions on Visualization and Computer Graphics* **13** 1081-1094
- Lloyd B A, Kirac S, Szekely G and Harders M (2008) Identification of dynamic mass spring parameters for deformable body simulation, *Eurographics - Short Papers*, (Eds: Mania K and Reinhard E) 131-134.
- Markez S (2000) Development of a Hand Tremor Quantification Device for the Measurement of Pathological Tremor, A thesis in Master of Health Science, Institute of Biomaterials and Biomedical Engineering, University of Toronto
- Pawar T, Anantakrishnan N, Chaudhuri S and Duttagupta S (2008) Impact of Ambulation in Wearable ECG *Annals of Biomedical Engineering* **36** 1547-1557
- Rocon E, Belda-Lois J M, Sanchez-Lacuesta J J and Pons J L (2004) Pathological Tremor management: Modelling, compensatory technology and evaluation *Technology and Disability* **16** 3-18
- Salarian A, Russmann H, Wider C, Burkhard P R, Vingerhoets F J G and Aminian K (2007) Quantification of Tremor and Bradykinesia in Parkinson's Disease Using a Novel Ambulatory Monitoring System *IEEE Transactions on Biomedical Engineering* **54** 313-322
- Salisbury K, Brock D, Massie T, Swarup N and Zilles C (1995) Haptic Rendering: Programming Touch Interactions with Virtual Objects, *Proceedings of the ACM Symposium on Interactive 3D Graphics*.
- Sederberg T W and Parry S R (1986) Free-form deformation of solid geometric models *ACM SIGGRAPH Computer Graphics* **20** 151-160
- Swami A, Mendel J M and Nikias C L, Higher order spectral analysis toolbox with Matlab, <http://www.mathworks.com/matlabcentral/fileexchange/3013>
- Su Y, Allen C R, Geng D, Burn D, Brechany U, Bell G D and Rowland R (2003) 3-D motion system (data-gloves): application for Parkinson's disease *IEEE Transactions on Instrumentation and Measurement* **52** 662-674
- Tomasi C and Manduchi R (1998) Bilateral Filtering for Gray and Color Images *Proceedings of the IEEE International Conference on Computer Vision* 839-846
- Witkin A (1997) An Introduction to Physically Based Modeling: Particle System Dynamics, SIGGRAPH course notes.