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Medium Effects on the Transport Coefficients of a Hot Pion Gas

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The transport coefficients of a hot pion gas is evaluated in relativistic kinetic theory approach. The shear and bulk viscosities as well as the thermal conductivity of a pion gas are obtained by solving the relativistic transport equation in the well known Chapman-Enskog approximation. The in-medium propagator of the ρ and σ mesons at finite temperature is used to evaluate the $\pi\pi$ scattering amplitude in the medium. The real and imaginary parts of the self-energy calculated from one-loop diagrams using the tools from thermal field theory, are seen to have noticeable effects on the scattering cross section. The effect of early chemical freeze out in heavy ion collisions is implemented through a temperature-dependent pion chemical potential. These are found to affect the temperature dependence of the bulk and shear viscosities and as well as the thermal conductivity in a significant way.

Key Words : QGP; Hadron Gas; Heavy Ion Collision; Shear Viscosity; Thermal Conductivity; Finite Temperature

Introduction

The study of transport coefficients of quark gluon plasma and hot hadronic matters has been attracting much interest and attention in the recent years. The experimentally measured elliptic flow v_2 of hadrons in Au+Au collision at Relativistic Heavy Ion Collider (RHIC),can be interpreted in terms of viscous hydrodynamics with a small value of η/s, which is close to the quantum bound $1/4\pi$ (Kovtun *et al*., 2005) η and s being the coefficient of shear viscosity and entropy density respectively. These results indicate the strongly interacting nature of the matter created in heavy ion collisions. This interpretation is based on the measured elliptic flow v_2 of hadrons in terms of viscous hydrodynamics which is sensitive to the value of η/s used in the calculations. The behaviour of ζ and η as a function of temperature is particularly relevant in the context of

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non-ideal hydrodynamic simulations of heavy ion collisions. A lot of interest has been generated, leading to quite a few estimates of the transport coefficients of both partonic (Arnold *et al*., 2000, 2006) as well as hadronic (Prakash *et al*., 1993; Itakura *et al*., 2008) constituents of strongly interacting matter. The effects of heat flow in heavy ion collisions has received however much less attention. This is presumably on account of the fact that the net baryon number in the central rapidity region at the RHIC and LHC is very small. However, at FAIR energies or in the low energy runs at RHIC the baryon chemical potential is expected to be significant and heat conduction by baryons may play a more important role. Based on such a scenario a few studies of heat conduction by pions have been carried out. Using the experimental $\pi\pi$ cross-section the thermal conductivity of a pion gas was estimated in (Prakash *et al*., 1993; Davesne, 1996) whereas in (Dobado *et al*., 2007) a unitarized scattering amplitude was employed.

In the kinetic theory approach the dynamics of interaction resides in the differential scattering cross-section which goes as an input. In almost all estimations of the transport coefficients a vacuum cross-section was employed. In this work we study the temperature dependence of the transport coefficients of a hot pion gas. Medium effects are then incorporated by introducing in-medium propagators dressed by one loop self energies calculated in the framework of thermal field theory. We use a temperature dependent pion chemical potential and obtain the transport coefficients as a function of temperatures in the range between chemical and kinetic freeze out in heavy ion collisions.

Transport Coefficients in Chapman-Enskog Approximation

The evolution of the phase space distribution of the pions is governed by the equation

$$p^\mu \partial_\mu f(x,p) = C[f]. \tag{1}$$

For binary elastic collision $p + k \to p' + k'$, this collision integral $C[f]$is given by

$$\begin{aligned} C[f] &= \int d\Gamma_k \, d\Gamma_{p'} \, d\Gamma_{k'} [f(x,p')f(x,k')\{1+f(x,p)\}\{1+f(x,k)\} \\ &\quad - f(x,p)f(x,k)\{1+f(x,p')\}\{1+f(x,k')\}] \, W, \end{aligned} \tag{2}$$

where the interaction rate, $W = \frac{s}{2} \frac{d\sigma}{d\Omega}(2\pi)^6 \delta^4(p+k-p'-k')$ and $d\Gamma_q = \frac{d^3q}{(2\pi)^3 q_0}$. The $1/2$ factor comes from the indistinguishability of the initial state pions. For small deviation from local equilibrium we write, in the first Chapman-Enskog approximation

$$f(x,p) = f^{(0)}(x,p) + \delta f(x,p), \quad \delta f(x,p) = f^{(0)}(x,p)[1 + f^{(0)}(x,p)]\phi(x,p) \tag{3}$$

where the equilibrium distribution function is given by

$$f^{(0)}(x,p) = \left[e^{\frac{p^\mu u_\mu(x) - \mu(x)}{T(x)}} - 1 \right]^{-1}, \tag{4}$$

with $T(x)$, $u_\mu(x)$ and $\mu(x)$ representing the local temperature, flow velocity and chemical potential respectively. Putting (3) in (1) the deviation function $\phi(x,p)$ is seen to satisfy

$$p^\mu \partial_\mu f^{(0)}(x,p) = -\mathcal{L}[\phi] \tag{5}$$

where the linearized collision term

$$\begin{aligned}\mathcal{L}[\phi] = \quad & f^{(0)}(x,p)\int d\Gamma_k\ d\Gamma_{p'}\ d\Gamma_{k'} f^{(0)}(x,k)\{1+f^{(0)}(x,p')\}\{1+f^{(0)}(x,k')\} \\ & [\phi(x,p)+\phi(x,k)-\phi(x,p')-\phi(x,k')]\ W\ .\end{aligned} \tag{6}$$

Using the form of $f^{(0)}(x,p)$ as given above on the left side of (5) and eliminating time derivatives with the help of equilibrium thermodynamic laws mentioned earlier we arrive at,

$$[Q\partial_\nu u^\nu + p_\mu \nabla^{\mu\nu}(p_\sigma u^\sigma - h)(T^{-1}\partial_\nu T - Du_\nu) - \langle p_\mu p_\nu\rangle\langle\partial^\mu u^\nu\rangle] f^{(0)}(1+f^{(0)}) = -T\mathcal{L}[\phi] \tag{7}$$

In this equation $Q = -\frac{1}{3}m_\pi^2 + (p_\mu u^\mu)^2\{\frac{4}{3} - \gamma^{'}\} + \{(\gamma^{''}-1)h - \gamma^{'''}T\}(p_\mu u^\mu)$, and $\langle\partial^\mu U^\nu\rangle = [\frac{1}{2}\{\Delta^{\mu\gamma}\Delta^{\nu\delta} + \Delta^{\nu\gamma}\Delta^{\mu\delta}\} - \frac{1}{3}\Delta^{\mu\nu}\Delta^{\gamma\delta}]\partial_\gamma U_\delta$. To be a solution, ϕ must be a linear combination of the thermodynamic forces appearing on the left hand side of the transport equation.

$$\phi = A\partial_\nu u_\nu + B_\mu \nabla^{\mu\nu}(T^{-1}\partial_\nu T - Du_\nu) - C_{\mu\nu}\langle\partial^\mu u^\nu\rangle \tag{8}$$

which on substituting on the left hand side of (7) we obtain a set of three integral equation satisfied by the coefficients, $A, B_\mu, C_{\mu\nu}$.

$$\mathcal{L}[A] = -Qf^{(0)}(p)\{1+f^{(0)}(p)\}/T \tag{9}$$

$$\mathcal{L}[B_\mu] = -\Delta_{\mu\sigma}p^\sigma(p.u-h)f^{0)}(p)\{1+f^{(0)}(p)\}/T \tag{10}$$

$$\mathcal{L}[C_{\mu\nu}] = -\langle p_\mu p_\nu\rangle f^{(0)}(p)\{1+f^{(0)}(p)\}/T \tag{11}$$

Here, $C_{\mu\nu} = C\langle p_\mu p_\nu\rangle$ and $B_\mu = B\Delta_{\mu\nu}p^\mu$. The other details are discussed in (Mitra *et al.*, 2012; Mitra and Sarkar, 2013, 2014).

In an imperfect fluid, the dissipative part of the energy momentum stress tensor is (Weinberg, 1971),

$$\Delta T^{\mu\nu} = 2\eta\langle\partial^\mu u^\nu\rangle + \zeta\Delta^{\mu\nu}\partial_\sigma u^\sigma + \lambda\{\Delta^{\mu\alpha}U^\nu + \Delta^{\nu\alpha}U^\nu\}(\partial_\alpha T - Tu.\partial u_\alpha) \tag{12}$$

The first two terms correspond to the viscous effects while the last term indicates thermal dissipation. The dissipative part of heat flow or the energy 4-flow is related to thermal conductivity by the following equation (Degroot *et al.*, 1988)

$$\Delta I^\mu = \lambda\Delta^{\mu\alpha}(\partial_\alpha T - Tu.\partial u_\alpha) \tag{13}$$

Again these quantities can be expressed in integral forms over the particle distribution function as,

$$\begin{aligned}\Delta T^{\mu\nu} &= \int d\Gamma_p f^{(0)}(1+f^{(0)})C_{\mu\nu}\langle p^\mu p^\nu\rangle\langle\partial^\mu u^\nu\rangle \\ &+ \int d\Gamma_p f^{(0)}(1+f^{(0)})QA\Delta^{\mu\nu}\partial_\sigma u^\sigma \qquad (14)\\ \Delta I^\mu &= \int \frac{d^3p}{(2\pi)^3p^0}(p.U-h)p^\sigma\Delta^\mu_\sigma f_0\{1+f_0\} \qquad (15)\end{aligned}$$

Comparing, we obtain the expressions of transport coefficients,

$$\begin{aligned}\zeta &= -\int \frac{d^3p}{(2\pi)^3p^0}QAf_0(1+f_0)\\ \lambda &= \frac{1}{3T}\int \frac{d^3p}{(2\pi)^3p^0}B_\nu p^\nu(p.u-h)f_0(1+f_0)\\ \eta &= -\frac{1}{10}\int \frac{d^3p}{(2\pi)^3p^0}f_0(1+f_0)C\langle p^\alpha p^\beta\rangle\langle p_\alpha p_\beta\rangle \qquad (16)\end{aligned}$$

Here we follow the procedure outlined in (Davesne, 1996) in which A,B_μ,$C_{\mu\nu}$ is expanded in terms of orthogonal Laguerre polynomials of half integral order. After some simplifications (discussed in detail in Refs. (Mitra and Sarkar, 2013)) the first approximation to transport coefficients comes out to be,

$$\zeta = T\frac{\alpha_2^2}{a_{22}},\ \ \lambda = -\frac{T}{3m}\frac{\beta_1^2}{b_{11}},\ \ \eta = \frac{T}{10}\ \frac{\gamma_0^2}{c_{00}}. \qquad (17)$$

The $\pi\pi$ Cross-Section with Medium Effects

The $\pi\pi$ cross-section is the key dynamical input for evaluating transport coefficients. Here the scattering is assumed to proceed via σ and ρ meson exchange in the medium. From the effective interaction (Serot and Walecka, 1986) the Lagrangian is,

$$\mathcal{L} = g_\rho\vec{\rho}^\mu\cdot\vec{\pi}\times\partial_\mu\vec{\pi} + \frac{1}{2}g_\sigma m_\sigma\vec{\pi}\cdot\vec{\pi}\sigma \qquad (18)$$

The matrix elements for $\pi\pi$ scattering are given by the following expressions where the widths of the σ and ρ mesons have been introduced in the propagators involved in the corresponding s-channel processes. We thus have

$$\begin{aligned}\mathcal{M}_{I=0} &= 2g_\rho^2\left[\frac{s-u}{t-m_\rho^2}+\frac{s-t}{u-m_\rho^2}\right]\\ &+ g_\sigma^2m_\sigma^2\left[\frac{3}{s-m_\sigma^2+im_\sigma\Gamma_\sigma}+\frac{1}{t-m_\sigma^2}+\frac{1}{u-m_\sigma^2}\right]\\ \mathcal{M}_{I=1} &= g_\rho^2\left[\frac{2(t-u)}{s-m_\rho^2+im_\rho\Gamma_\rho}+\frac{t-s}{u-m_\rho^2}-\frac{u-s}{t-m_\rho^2}\right]\\ &+ g_\sigma^2m_\sigma^2\left[\frac{1}{t-m_\sigma^2}-\frac{1}{u-m_\sigma^2}\right]. \qquad (19)\end{aligned}$$

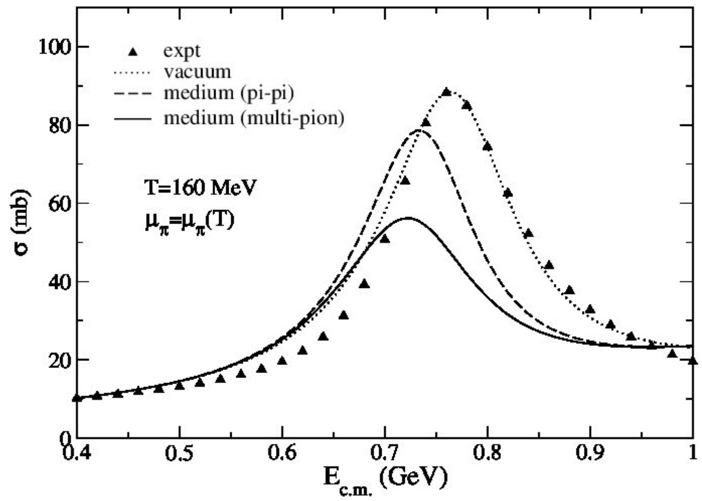

Fig. 1: The $\pi\pi$ cross-section as a function of centre of mass energy

Defining the isospin averaged amplitude as $|\mathcal{M}|^2 = \frac{1}{9}\sum_I |\mathcal{M}_I|^2$ and ignoring the non-resonant $I = 2$ contribution, the cross-section is found to agree very well (Mitra *et al.*, 2012; Mitra and Sarkar, 2013) with the estimate based on measured phase-shifts given in (Prakash *et al.*, 1993).

To obtain the in-medium cross-section we replace the vacuum width in the above expressions by the ones in the medium. The width is related to the imaginary part of the self-energy through the relation, $\Gamma(T,M) = -M\mathrm{Im}\Pi(T,M)$, where Π denotes the one-loop self energy diagrams and are evaluated using the real-time formalism of thermal field theory. The σ meson self-energy is obtained from the $\pi\pi$ loop diagram whereas in case of the ρ meson the $\pi\pi$, $\pi\omega$, πh_1, πa_1 graphs are evaluated using interactions from chiral perturbation theory. These heavy mesons, ω, h_1, a_1 having substantial 3π and $\rho\pi$ decay widths, the contributions from the loops with heavy mesons may then be considered as a multi-pion contribution to the ρ self-energy. The imaginary part of the self-energy calculated from one-loop diagrams obtained as (Mallik and Sarkar, 2009),

$$\begin{aligned}\mathrm{Im}\Pi(q_0,\vec{q}) &= -\pi\int\frac{d^3k}{(2\pi)^3 4\omega_\pi\omega_h}\times\Big[N_1\{(1-f^{(0)}(\omega_\pi)-f^{(0)}(\omega_h))\\ &\times \delta(q_0-\omega_\pi-\omega_h)+(f^{(0)}(\omega_\pi)-f^{(0)}(\omega_h))\delta(q_0-\omega_\pi+\omega_h)\}\\ &+ N_2\{(f^{(0)}(\omega_h)-f^{(0)}(\omega_\pi))\delta(q_0+\omega_\pi-\omega_h)\\ &- (1-f^{(0)}(\omega_\pi)-f^{(0)}(\omega_h))\delta(q_0+\omega_\pi+\omega_h)\}\Big] \qquad (20)\end{aligned}$$

where $f^{(0)}(\omega) = \frac{1}{e^{(\omega-\mu_\pi)/T}-1}$ is the Bose distribution function with arguments $\omega_\pi = \sqrt{\vec{k}^2+m_\pi^2}$ and $\omega_h = \sqrt{(\vec{q}-\vec{k})^2+m_h^2}$. The terms N_1 and N_2 stem from the vertex factors and the numerators of vector propagators, details of which can be found in (Mallik and Sarkar, 2009). The cross-section obtained by using the in-medium propagator in place of the vacuum propagator suffers small suppression of the peak for the $\pi\pi$ loop and a larger effect when all the loops (indicated by multi-pion) are considered. This is also accompanied by a small shift in the position of the peak as shown in In Fig. 1.

We end this section with a discussion of the pion chemical potential. It is generally accepted (Bebie *et al*., 1992) that an evolving hadronic gas gets out of chemical equilibrium early so the number-changing inelastic collisions cease at chemical freeze out and the total pion number becomes fixed, so only elastic collisions take place until the pions actually decouple later at kinetic freeze out. The pion chemical potential consequently grows from zero to a maximum at kinetic freeze out so as to keep the total number of pions fixed. Here the temperature-dependent pion chemical potential is taken from Ref. (Hirano and Tsuda, 2002) which implements the above scenario and is parametrized as $\mu_\pi(T) = a + bT + cT^2 + dT^3$, with $a = 0.824$, $b = 3.04$, $c = -0.028$, $d = 6.05 \times 10^{-5}$ and T, μ_π in MeV.

Results

In this section, let us start with the results of shear viscosity to entropy density ratio η/s. For $\mu_\pi = 0$ the upper set of curves with filled circles show the usual decreasing trend as seen, for example in (Itakura *et al*., 2008). This trend is reversed when $\mu_\pi(T)$ is used and η/s increases with T. The values in all cases remain well above $1/4\pi$.

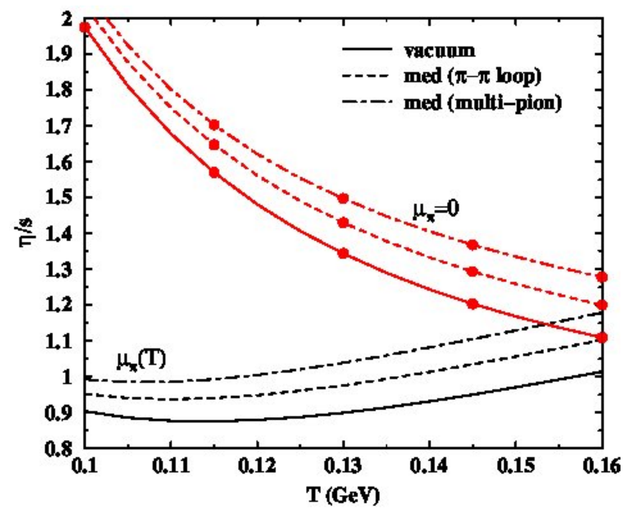

Fig. 2: **η/s as a function of T.**

Then we have the results for bulk viscosity ζ as a function of temperature T. In Fig. 3 the three sets of curves correspond to different values of the pion chemical potential. The clear separation between the curves in each set displays a significant effect brought about by the medium dependence of the cross-section. A large dependence on the pion chemical potential is also inferred since the three sets of curves appear nicely separated.

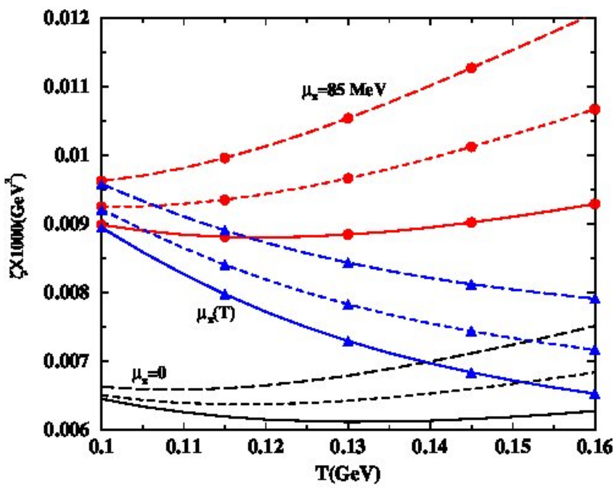

Fig. 3: ζ **as a function of** T**. In each set the solid line indicates vacuum cross-section, the dotted line for in-medium modification due to pion loop and the dashed line for loops with heavy mesons**

We next turn to the results of thermal conductivity. In Fig. 4 we plot λT as a function of T evaluated in the Chapman-Enskog approach. The effect of a hot medium (taking the contributions of heavy mesons in ρ loop) and as well as temperature dependent chemical potential is clearly visible for those plots.

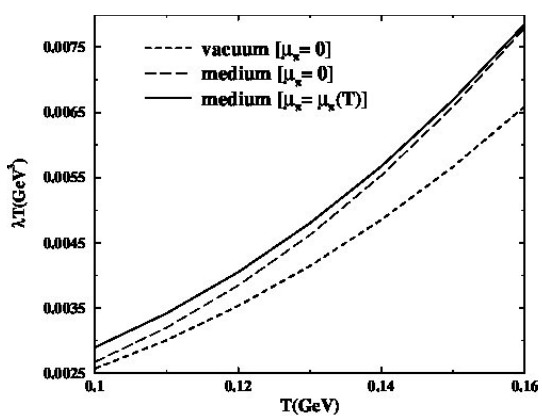

Fig. 4: λT **as a function of** T **for** $\pi\pi$ **cross-section in vacuum and in medium**

Summary and Outlook

To summarize in this work we have evaluated the transport coefficients of a pion gas by solving the Uehling-Uhlenbeck transport equation in the Chapman-Enskog approximation with an aim to study the effect of a medium dependent cross-section. In-medium effects on the $\pi\pi$ cross-section are incorporated through one-loop self-energies of the exchanged ρ and σ mesons calculated using thermal field theory. The effect of early chemical freeze out is incorporated through a temperature dependent pion chemical potential which keeps the pion number conserved. It is observed that the temperature dependence of the transport coefficients is significantly affected. It will be interesting to observe the consequences on the evolution of the late stages of heavy ion collisions by including it in the fluid-dynamical simulations.

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