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Effect of Gluon Radiation Off Charm Quark on Transport Coefficients and the Equilibrium Distribution of Charm Quark

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The transport coefficients like drag, transverse and longitudinal diffusion have been evaluated when charm quark scatters elastically as well as suffers gluon bremsstrahlung while propagating inside Quark Gluon Plasma (QGP). The effect of the gluon radiation has been observed to be significant on the magnitude of different transport coefficients and on the equilibrium distribution function of the charm.

Key Words : QCD, QGP, Heavy Quarks, Gluon Bremsstrahlung; Drag Coefficient; Transverse and Longitudinal Diffusion Coefficients; Fokker Planck Equation; Shear Viscosity

Introduction

Heavy Quarks (HQ) originated from the early hard processes in the relativistic/ultra-relativistic heavy ion collisions can be used as a very efficient probe to extract different properties of the deconfined quark matter which is believed to be formed due to the collisions of two heavy ions. The reasons behind choosing heavy quark as the probe are:

1. Being originated from early hard collisions, HQ can experience the medium from the beginning. Therefore, their distribution function is different from that of the medium particles. Also, being heavier than the constituent particles of QGP, HQ qualifies as the Brownian particle.

2. The probability of the production of HQ (with mass M) inside the thermal medium ($T << M$) is very less. Therefore, the probability of annihilation is also very small.

3. There is less probability of their being thermalised in the medium.

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Now, we are in a position to discuss the formalism briefly. Heavy Quark propagates as a Brownian particle inside the thermalised QGP medium. The ensemble of Brownian particles immersed in identical thermal medium can be characterised by the single particle distribution function, $f(\vec{x}, \vec{p}, t)$. After integrating over the spatial volume, the time evolution of $f(\vec{p}, t)$ is governed by the Master Equation, a simplified version of which is the Fokker Planck Equation (FPE) (Svetitsky, 1988).

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial p_i}\left[A_i(\vec{p})f + \frac{\partial}{\partial p_j}[B_{ij}(\vec{p})f]\right] \ , \tag{1}$$

where the kernels are defined as

$$A_i = \int d^3\vec{k} w(\vec{p}, \vec{k}) k_i \ , \tag{2}$$

and

$$B_{ij} = \frac{1}{2}\int d^3\vec{k} w(\vec{p}, \vec{k}) k_i k_j . \tag{3}$$

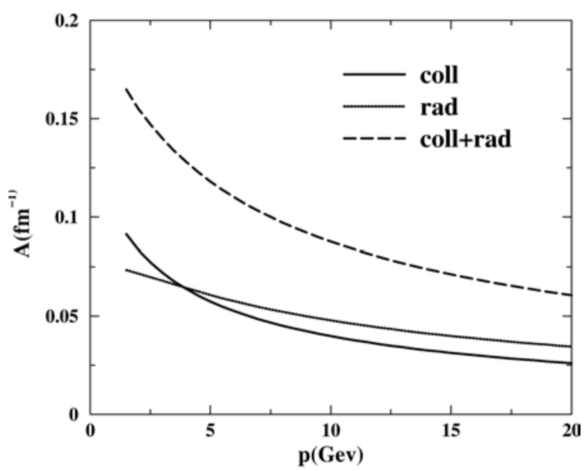

Fig. 1: Drag of charm vs momentum at bath temperature, T=525 MeV

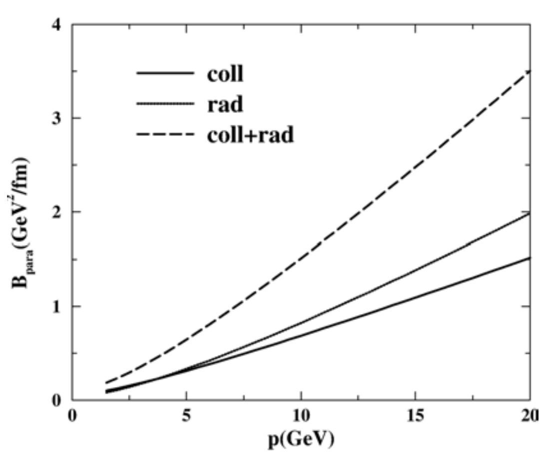

Fig. 2: Longitudinal diffusion coefficient of charm vs momentum at bath temperature, T=525 MeV

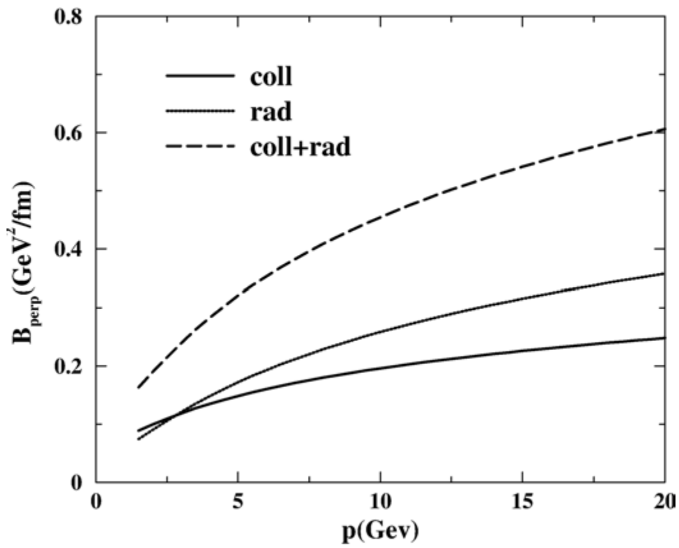

Fig. 3: Transverse diffusion coefficient of charm vs momentum at bath temperature, T=525 MeV

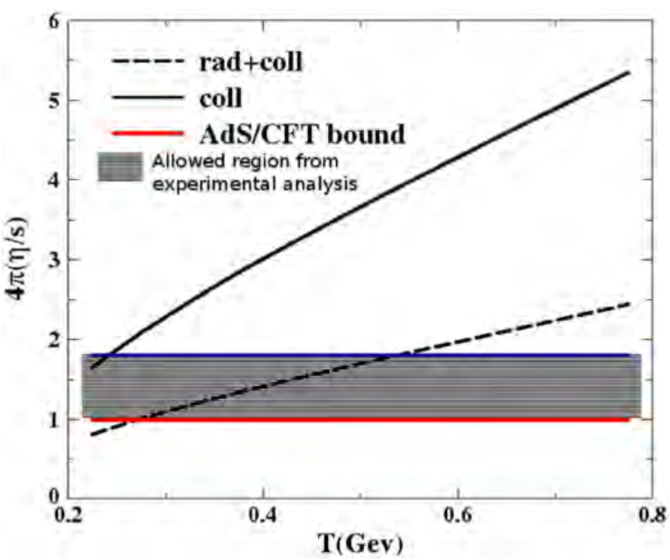

Fig. 4: For a charm quark with momentum, $< p_T >= 5$ Gev propagating in QGP of temperature, T

Physically, A_i and B_{ij} are related to the drag and the diffusion coefficients of HQ. Our motivation is to find out these coefficients from pQCD for the case when HQ undergoes only elastic collision with the bath particles as well as when it emits soft gluons in the process of suffering binary collision. In view of the equations written above, we can write down the expressions for the collisional and as well as radiative drag/diffusion coefficients. As the heavy quark is treated to be relativistic, it is important to calculate transverse and longitudinal diffusion processes along with the momentum dependence (Mazumder *et al*., 2011) of the transport coefficients. The generic transport coefficient will be written as X_{coll} and X_{rad} for elastic and radiative respectively:

$$\begin{aligned} X_{coll} &= \frac{1}{2E_p}\int\frac{d^3q}{(2\pi)^3 2E_q}\int\frac{d^3q'}{(2\pi)^3 2E_{q'}} \\ &\times \int\frac{d^3p'}{(2\pi)^3 2E_{p'}}\frac{1}{\gamma}\sum|M|^2_{2\to 2}(2\pi)^4\delta^4(p+q-p'-q') \\ &\times \hat{f}(\mathbf{q})(1\pm\hat{f}(\mathbf{q}'))X(\vec{p'}) \qquad (4)\\ &= \ll X(\vec{p'})\gg \qquad (5)\end{aligned}$$

where, $X(\vec{p'})$ is the transport part which depends on whether we are calculating drag or diffusion.

$$\begin{aligned} X_{rad} &= \frac{1}{2E_p}\int\frac{d^3q}{(2\pi)^3 2E_q}\int\frac{d^3q'}{(2\pi)^3 2E_{q'}}\int\frac{d^3p'}{(2\pi)^3 2E_{p'}} \\ &\times \int\frac{d^3k_5}{(2\pi)^3 2E_5}\frac{1}{\gamma}\sum|M|^2_{2\to 3}(2\pi)^4\delta^4(p+q-p'-q'-k_5) \\ &\times \hat{f}(E_q)(1\pm\hat{f}(E_{q'}))(1+\hat{f}(E_5)) \\ &\times \theta_1(\tau-\tau_F)\theta_2(E_p-E_5) \qquad (6)\end{aligned}$$

Therefore,

$$\begin{aligned} X_{rad} &= X_{coll}\times\int\frac{d^3k_5}{(2\pi)^3 2E_5}12g_s^2\frac{1}{k_\perp^2} \\ &\times \left(1+\frac{M^2}{s}e^{2y}\right)^{-2}[1+\hat{f}(E_5)]\theta_1\theta_2. \qquad (7)\end{aligned}$$

In the above Eq. 6, the the multiplicative factor with X_{coll} is the average of the radiated gluon multiplicity distribution (Abir *et al.*, 2012) over the phase space of the bremsstrahlung gluon. $1+\hat{f}(E_5)$ is the phase factor to take into account the absorption of the emitted gluon in the background medium. Whereas, θ_2 takes care of the fact that the energy of the radiated gluon cannot be greater than the heavy quark from which it has been emitted, θ_1 restricts our region of interest in the Bethe-Heitler limit. In this limit the gluon radiation is simply additive for multiple gluon emission as the interaction time is greater than the formation time of the radiated gluon. In the plots we can observe that the radiative transport coefficients are greater in magnitude at the higher temperatures of the bath and at larger momenta of the probe.

With the help of these radiative drag/diffusion coefficients we will now try to perform an estimation of the shear viscosity to entropy density ratio of Quark Gluon Plasma by assuming a naive relation between η/s and the transport parameter, $\hat{q}$ (Majumder *et al.*, 2007, Lacey 2009) which is the squared average transfer momentum exchange between the bath particles and the fast parton (in this case, the charm quark):

$$\frac{\eta}{s}\approx 1.25\frac{T^3}{\hat{q}} \qquad (8)$$

From a very straight forward relation $\hat{q} = 4B_{\perp}$ (Mazumder *et al.*, 2014), where $B_{\perp}$ is the transverse diffusion coefficient, we can arrive at the following plot where it has been observed that if we perform the calculation including the radiation off the heavy quark instead of taking only the elastic interactions, we can achieve a result for η/s which is closer to the band in Fig. 4 as predicted by experiments.

We have also investigated the dependence of the shape of the equilibrium distribution function of charm quark on the three transport coefficients. We find that the incorporation of radiation does not alter the picture by a substantial amount owing to the fact that the shape counts only on the ratios of the transport coefficients instead of their absolute values. But, inelastic process affects the relaxation time of charm quark substantially.

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