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Modification of Kick Velocity of Neutron Stars Due to Non-Fermi Liquid Effects

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In this present work we have incorporated the non-Fermi liquid behavior into the expression of the neutron star (NS) kick velocity due to asymmetric neutrino emission. We have studied leading order (LO) as well as next to leading order (NLO) corrections to the velocity and compared the results with the Fermi liquid case. We have approximated our results for the case of large magnetic fields found in NS.

Key Words : Quark Matter; Neutrinos; Pulsar Kicks; Specific Heat; Magnetic Field

Introduction

Neutrinos are one of the foremost candidates responsible for the cooling of the NS composed of a normal (non-color superconducting) degenerate quark matter core. The cooling is carried out through the quark direct URCA process. For the past few years, it has been argued by several authors (Dorofeev *et al*., 1985; Sagert and Schaffner-Bielich, 2007; Sagert and Schaffner-Bielich, 2008) that asymmetric neutrino emission is responsible for the pulsar kicks during the evolution of the NS.

The underlying mechanism for the generation of such kicks is the possible polarisation of the electrons leading to neutrino emission in a preferred direction. For this to happen, the magnetic field strength has to be equal to or greater than some critical value which is given by $B_{cric} = m_i^2 c^3/(q_i \hbar)$, where m_i and q_i are the mass and charge of the corresponding particle. Such strong magnetic field forces the electrons to occupy the lowest Landau level polarising the electron spin opposite to the direction of the magnetic field. This leads to the formation of the neutrino and anti-neutrino emission cones which give rise to polarised neutrino emission along the axis opposite to the magnetic field direction. The electron polarisation for different

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conditions of magnetic field and kick velocities has been studied recently by (Sagert and Schaffner-Bielich, 2007; Sagert and Schaffner-Bielich, 2008). In this work, the dependence of the kick velocity with the quark phase temperature and radius of the star with varying quark chemical potentials has been investigated.

One of the interesting developements in recent years has been the study of the non-Fermi liquid (NFL) behavior, of quantities like neutrino emissivity (Iwamoto, 1981; Schäfer and Schwenzer, 2004; Adhya *et al*., 2012), mean free path (MFP) (Schäfer and Schwenzer, 2004; Adhya *et al*., 2013) or the specific heat (Gerhold and Rebhan, 2005) of the dense quark matter which significantly modifies the initial cooling rate of the NS via neutrino emission. The NFL effects are dominant in the relativistic regime driven by transverse or magnetic interactions which in the non-relativistic limit is $\beta = v/c$ suppressed (Holstein *et al*., 1973). In fact the quark dispersion relation in dense plasma changes significantly in presence of magnetic interactions mediated by exchange of transverse gluons (Bellac and Manuel, 1997). The quark self energy has been calculated recently both for the case of zero and low temperature by including corrections up to NLO terms which modifies the in-medium quark dispersion relation appreciably (Gerhold and Rebhan, 2005). Accordingly, some of the physical quantities, like heat capacity, pressure are also modified (Gerhold *et al*., 2004). In many calculations, such corrections appear in the modification of the phase space giving rise to appreciable contributions to the well known Fermi liquid (FL) terms. Such corrections have already been seen to be important resulting in enhanced neutrino emission from dense quark matter (Schäfer and Schwenzer, 2004; Adhya *et al*., 2012). The cooling behavior of NS with quark matter has also been studied recently by several authors (Iwamoto, 1981; Schäfer and Schwenzer, 2004; Adhya *et al*., 2012). In all these cases, the correction at the LO has been seen to involve $T\ln(1/T)$ term which has been dubbed as anomalous corrections in many of the recent literatures (Holstein *et al*., 1973; Gerhold and Rebhan, 2005). Similar behavior has also been reported while investigating quantities like drag, diffusion coefficients and thermal relaxation time of electrons for relativistic degenerate plasmas (Sarkar and Dutt-Mazumder, 2010; Sarkar and Dutt-Mazumder, 2011; Sarkar and Dutt-Mazumder, 2013). The specific heat of the QM is also modified due to the NFL corrections. In this work, we will calculate the specific heat of degenerate quark matter which receives significant correction due to the presence of the high external magnetic field while including the NFL corrections. Such high magnetic fields are supposed to be present at the core of the NS. Thus, the inclusion of the magnetic field in the specific heat in the case of interacting plasma and its effect on the kick velocity of the NS is studied.

The plan of the paper is as follows. In section II we discuss the formalism to calculate the kick velocity and calculate the velocity for different conditions of the magnetic field taking into account the NFL effect in specific heat capacity of magnetized quark matter. We present the results in section III followed by conclusion in section IV.

Formalism

The amount of pulsar acceleration depends on the polarisation of the electron spin and the momenta. The kick velocity can be written as,

$$dv = \frac{\chi}{M_{NS}}\frac{4}{3}\pi R^3 \varepsilon dt \tag{1}$$

where the polarisation fraction of the electrons has been denoted by χ and the neutrino emissivity by ε. Using the cooling equation,

$$C_v dT = -\varepsilon dt, \tag{2}$$

One can rewrite Eq. (1) in terms of the specific heat (C_v) of the quark matter core. Eq. (1) and Eq. (2) allow one to calculate the pulsar kick velocity. In the region of low temperature and high chemical potential, the Fermi liquid expression of the specific heat of quark is given as (Holstein *et al.*, 1973),

$$C_v\Big|_{FL} \;=\; \frac{N_c N_f}{3}\mu_q^2 T \tag{3}$$

where N_c and N_f are the number of color and flavor factors respectively. Thus the Fermi liquid contribution to the pulsar kick velocity as reported in can be recast into the following form,

$$v\Big|_{FL} \;\simeq\; \frac{8.3 N_C N_f}{3}\Big(\frac{\mu_q}{400MeV}\frac{T}{1MeV}\Big)^2\Big(\frac{R}{10km}\Big)^3\frac{1.4M_\odot}{M_{NS}}\chi\frac{km}{s} \tag{4}$$

The calculation of such kick velocity performed so far is restricted to the Fermi liquid results. Currently, however it is shown in a series of works that NFL behavior arising out of magnetic interactions in the relativistic regime can contribute significantly and wins over the electric or Coulombic interaction for the case of ultradegenerate matter as mentioned in the introduction. In general, in a medium, the quark dispersion relation can be obtained by solving the following equation,

$$\omega = (E_{p(\omega)} + \mathrm{Re}\Sigma(\omega, p(\omega))) \tag{5}$$

where the Σ represents the one loop quark quasiparticle self energy. Explicitly for excitation near the Fermi surface this has been derived in (Gerhold and Rebhan, 2005),

$$\begin{aligned}\Sigma(\omega) \;\simeq\; & -g^2 C_F m\Big\{\frac{\varepsilon}{12\pi^2 m}\Big[\log\Big(\frac{4\sqrt{2}m}{\pi\varepsilon}\Big)+1\Big] \\ & + \frac{i\varepsilon}{24\pi m} + \frac{2^{1/3}\sqrt{3}}{45\pi^{7/3}}\Big(\frac{\varepsilon}{m}\Big)^{5/3}(\mathrm{sgn}(\varepsilon)-\sqrt{3}\mathrm{i}) \\ & + \frac{i}{64\sqrt{2}}\Big(\frac{\varepsilon}{m}\Big)^2 - 20\frac{2^{2/3}\sqrt{3}}{189\pi^{11/3}}\Big(\frac{\varepsilon}{m}\Big)^{7/3}(\mathrm{sgn}(\varepsilon)+\sqrt{3}\mathrm{i}) \\ & - \frac{6144-256\pi^2+36\pi^4-9\pi^6}{864\pi^6}\Big(\frac{\varepsilon}{m}\Big)^3 \\ & \times \Big[\log\Big(\frac{0.928\,m}{\varepsilon}\Big) - \frac{i\pi\mathrm{sgn}(\varepsilon)}{2}\Big] + \mathcal{O}\Big(\Big(\frac{\varepsilon}{m}\Big)^{11/3}\Big)\Big\}. \end{aligned} \tag{6}$$

Here $\varepsilon = \omega - \mu_q \sim T$; ω being the quasiparticle energy; $m^2 = (N_f g^2 \mu_q^2)/(4\pi^2)$ and is related to the Debye screening mass by $m^2 = m_D^2/2$. In the above calculation of the quark self energy, the dominant contribution at one loop order arises when the gluon in the loop is soft ($\sim g\mu$). This approach requires dressing of the gluon propagator to incorporate Debye screening and Landau damping. Now, it will be interesting to investigate the effect of the exterior magnetic field on the specific heat of the quark matter. In the presence of a constant external magnetic field (B) along the z axis, the thermodynamic potential is modified as (Chakraborty, 1996; Bandopadhyay *et al.*, 1997),

$$\Omega^B = -\frac{g_d T|q|B}{2\pi^2}\sum_{\nu=0}^{\infty}\int_0^{\infty} dp_z \log(1 + e^{\beta(\mu-\epsilon)}) \tag{7}$$

where $\epsilon = \sqrt{p_z^2 + m^2 + 2\nu|q|B}$ is the single particle energy eigen value, g_d is the quark degeneracy and $\nu = 0, 1, 2, ...$ Thus, the specific heat computed from the thermodynamic potential can be written as,

$$C_v\Big|_{FL}^{B} = \frac{N_C N_f T m_q^2}{6}\Big(\frac{B}{B_{cr}^q}\Big) \tag{8}$$

Incorporating the effect of the NFL behavior in the specific heat capacity, we obtain,

$$C_v\Big|_{LO}^{B} \simeq \Big(\frac{N_C N_f C_f \alpha_s}{36\pi}\Big) m_q^2 \Big(\frac{B}{B_{cr}^q}\Big) T\Big[(-1 + 2\gamma_E) + 2log\Big(\frac{2m_B}{T}\Big)\Big] \tag{9}$$

The NLO contribution to the specific heat capacity is obtained as,

$$\begin{aligned} C_v\Big|_{NLO}^{B} \simeq & \Big(\frac{N_C N_f}{3}\Big)(C_f\alpha_s)\Big(m_q^2\frac{B}{B_{cr}^q}\Big)T\Big[c_1\Big(\frac{T}{m_B}\Big)^{2/3} \\ + & c_2\Big(\frac{T}{m_B}\Big)^{4/3} + c_3\Big(\frac{T}{m_B}\Big)^2\Big(c_4 - \log\Big(\frac{T}{m_B}\Big)\Big)\Big] \end{aligned} \tag{10}$$

where the constants are (Adhya *et al.*, 2014),

$$c_1 = -0.2752; c_2 = 0.2899; c_3 = -0.5919; c_4 = 5.007. \tag{11}$$

The Debye mass (m_B) in the QCD case in presence of magnetic field is obtained as follows,

$$m_B^2 = \frac{N_f g^2 m_q^2}{4\pi^2}\Big(\frac{B}{B_{cr}^q}\Big) \tag{12}$$

The pulsar kick velocity obtained taking into account the magnetic field effect on the specific heat capacity of the quarks reads as (Adhya *et al.*, 2014),

$$v\Big|_{FL}^{B} \simeq \frac{4.15 N_C N_f}{3}\Big(\frac{\sqrt{m_q^2(B/B_{cr}^q)}}{400MeV}\frac{T}{1MeV}\Big)^2\Big(\frac{R}{10km}\Big)^3\frac{1.4M_\odot}{M_{NS}}\chi\frac{km}{s} \tag{13}$$

By including the anomalous effect, we obtain the LO contribution to the kick velocity as (Adhya *et al*., 2014),

$$v\Big|_{LO}^{B} \simeq \frac{8.8N_C N_f}{3}(C_f\alpha_s)\Big(\frac{\sqrt{m_q^2(B/B_{cr}^q)}}{400MeV}\frac{T}{1MeV}\Big)^2\Big(\frac{R}{10km}\Big)^3 \times\frac{1.4M_{\odot}}{M_{NS}}\Big[0.0635+0.05\log\Big(\frac{m_B}{T}\Big)\Big]\chi\frac{km}{s} \quad (14)$$

Now we have also extended our calculation beyond the LO in NFL correction. The NLO correction to the kick velocity can be written as (Adhya *et al*., 2014),

$$v\Big|_{NLO}^{B} \simeq \frac{8.3N_C N_f}{3}\Big(\frac{B}{B_{cr}^q}\Big)\Big(\frac{m_q}{400MeV}\frac{T}{1MeV}\Big)^2\Big(\frac{R}{10km}\Big)^3\frac{1.4M_{\odot}}{M_{NS}} \times\chi(C_F\alpha_s)\Big[a_1\Big(\frac{T}{m_B}\Big)^{2/3}+a_2\Big(\frac{T}{m_B}\Big)^{4/3} +\Big[a_3+a_4\ln\Big(\frac{m_B}{T}\Big)\Big]\Big(\frac{T}{m_B}\Big)^2\Big]\frac{km}{s} \quad (15)$$

The constants are evaluated as,

$$a_1=-\frac{12\pi\times0.04386}{8}; a_2=\frac{12\pi\times0.04613}{10}; a_3=-2.4162; a_4=-0.4595 \quad (16)$$

The long range magnetic interactions lead to an anomalous $T^2\mathrm{lnT}^{-1}$ term in the expression of the pulsar kick velocity. The kick velocity is computed with different conditions of the electron polarisation fractions.

For the case of vanishing temperature (cold neutron stars), the electron spin polarisation is given by (Sagert and Schaffner-Bielich, 2007),

$$\chi\simeq\frac{3}{2}\frac{m_e^2}{\mu_e^2-m_e^2}\Big(\frac{B}{B_{cr}^e}\Big) \quad (17)$$

where the critical value of the magnetic field is given by $B_{cr}^e\simeq4.4\times10^{13}G$.

For the case when the magnetic field strength is chosen to be much larger than the temperature, the chemical potential as well as the electron mass ($\mu_e, m_e, T\ll\sqrt{2eB}$), the electron polarisation is given as (Sagert and Schaffner-Bielich, 2007),

$$\chi \sim 1-\frac{4}{\ln(2)}\sqrt{\frac{\pi T}{2\sqrt{2eB}}}e^{-\sqrt{2eB}/T}. \quad (18)$$

For the case of large magnetic field, the kick velocity has to be solved numerically. The net contribution to the pulsar kick velocity upto NLO is obtained by the sum of the Fermi liquid result and the non-Fermi liquid correction upto NLO:

$$v\Big|_{total}^{B}=v\Big|_{FL}^{B}+v\Big|_{LO}^{B}+v\Big|_{NLO}^{B} \quad (19)$$

Results and Discussions

In our work, we have made an estimate of the quark phase radius of the NS with the temperature of the quark matter present in the core. For our purpose, we have considered kick velocity of the order of 10^2 km/s. The quark and electron chemical potentials are taken to be $400MeV$ and $10MeV$ respectively. Magnetic fields of $10^{15}G$ to $10^{19}G$ are considered relevant to the corresponding cases. In Fig. (1), for the case of highly polarised electrons, the left panel shows a comparison between FL, LO and NLO corrections to the kick velocity when the effect of external magnetic field on the specific heat is excluded for highly polarised electrons. The right panel shows a similar behavior with inclusion of the magnetic field on the specific heat capacity of quarks. In the Fig. (2), similar behavior for FL, LO and NLO are observed when the electrons are partially polarised for low external magnetic fields.

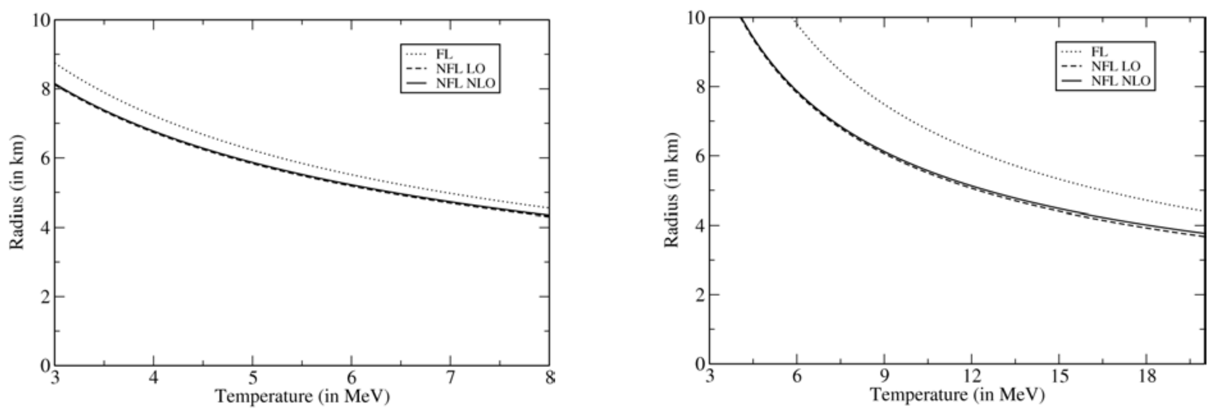

Fig. 1: The left figure shows the numerical comparison of the relationship (FL, LO and NLO respectively) where high magnetic field ($B = 10^{19}G$) has been taken into account along with vanishing temperature for kick velocity of $100\mathrm{km/s}$ for the case where external magnetic field on the specific heat is ignored. The right panel shows the corresponding case when high magnetic field effect in specific heat is included.

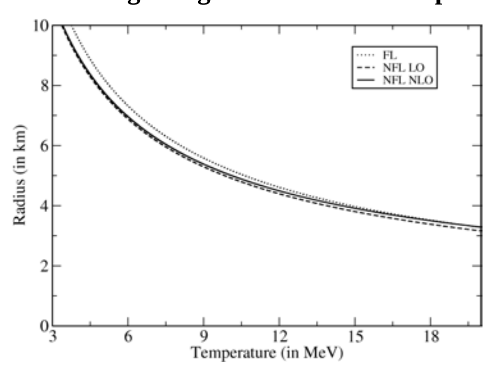

Fig. 2: This figure shows the comparison between the FL, NFL LO and NFL NLO result for the radius and temperature dependence for the case of partially polarized electrons in weak magnetic field ($B = 5 \times 10^{15}G$) for kick velocity of $100km/s$

Conclusion

In this work, we have shown that the pulsar kick velocity receives significant contribution from the logarithmic corrections at the LO in comparison to the FL result. We have computed results with LO and NLO corrections to include plasma or quasiparticle effects which are anomalous (NFL) effects. The contributions from the electron polarization (χ) for different cases have also been taken into account to calculate the velocities. In addition, comparison has been made between the NFL LO and NLO contributions to the kick velocity with the FL case. We have found that the NFL LO contributions are significant while calculating the radius-temperature relationship as seen from the graphs presented for the case of the neutron star with moderate and high magnetic field. The anomalous corrections introduced to the pulsar kick velocity due to the NFL (LO) behavior increases appreciably the kick velocity for a particular value of temperature. However, for all the cases, no appreciable change in the R-T relationship has been observed for the NLO correction with respect to the LO case.

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