Proceedings of the Indian National Science Academy (2023) 89:967–976
https://doi.org/10.1007/s43538-023-00211-8

RESEARCH PAPER

Finite element modeling of heat transfer in Z-shaped cavity according to constructal design

Jin Lin[1]

Received: 20 August 2023 / Accepted: 7 September 2023 / Published online: 6 October 2023

Abstract
In this study, a constructal design method is used to improve geometry of a Z-shaped cavity, which is established with a solid conducting wall with interior heat generation. This design eliminates energy from the wall by minimizing the heat resistance between the solid body and fin. The optimization is achieved by varying the size of the Z-shaped cavity as well as the ratio between its volume and the rectangular volume while maintaining the remaining geometric parameters constant. Following the validation of the finite element result, various parameters affecting the highest heat created in the system will be examined. Curves and tables will provide explanation regarding the outcomes.

Keywords Heat transfer · Fine · Z-shaped cavity · Numerical solution

Introduction

Today, due to the increasing technology in the field of electronic equipment, cooling of these devices is also of particular importance [1–3]. Extended surfaces such as fins in industry are employed to increase the rate of heat transfer in many heat exchangers and equipment [4] GPUs, CPUs, chipsets, and RAM modules are all cooled by fins in computers. When the heat dissipation of a semiconductor component, such as a power transistor, is not sufficient to maintain the temperature of a component, fins are used. With the development of advanced computers, the heat dissipation technique should be more efficient and effective than before, because there are limitations in terms of increasing the heat transfer coefficient. Therefore, the need to design fins with higher efficiency for installation on processors and other electrical equipment is fully sensed [5].

The discussion of improving heat transfer is generally reduced to increasing the thermal conductivity. Extensive studies have been conducted on the improving the heat transfer devices. In a study conducted by Sadeghian jahromi et al. [6], heat enhancement technique is classified into two general groups: active and inactive methods. Also, according to Kumar et al.[7] as well as Zhang [8], there are different approaches to intensification heat transfer, which are categorized into three types: active, passive, and combined methods. Accordingly, over the years, various issues have been studied in the field of creating various types of fins and cavities to increase the rate of heat transfer, especially in electronic components [9–11]. A study conducted by Salimpour et al.[12] investigated the application of heat transfer principles to an array of circular and non-circular channels. The channels had stable volumes and pressure drops. In a square channel with 45 inclined baffles, Promvonge et al.[13] investigated the laminar heat transfer numerically. They found that the conductive heat transfer of the fins not only increased the heat transfer with the fluid outside the tubes; it also causes thermal conduction from one pipe to the adjacent pipe. Bejan[14] first proposed the constructal design theory. He points out that the idea of constructal design theory first came to him while trying to solve the problem of minimizing the thermal resistance between a volume and heat production.Initially, this theory was applied by Bejan and Sciubba[15], [16] to optimize the arrangement of parallel plates for cooling electronic devices. By utilizing the asymptotic intersection method, they determined the optimal distances between the plates, incorporating channel length and maximum heat transfer per unit volume as a function of a dimensionless parameter.

✉ Jin Lin
linjin_wx@126.com

[1] Intelligent Manufacturing Automotive Engineering College, Wuxi Vocational Institute of Commerce, Wuxi 214153, Jiangsu, China

Shokouhmand et al. [17] explored the optimization of finned tubes using the construct design theory. The goal was to reduce general thermal resilience by designing the tube geometry. Several factors were taken into account, including the ratio of fin heights to pipe diameters, the ratio of fin conduction to air conduction, the ratio of liquid conduction inside the tube to air conduction, and the dimensionless pressure drop. They imposed two limitations on the optimization process: the constant total volume of the heat exchanger and the constant volume fraction of the fin materials. Using constructal design theory, they obtained an optimal geometry of the tube and fins, in which the number of fins was optimized. Alebrahim and Bejan [18] used constructal design theory to optimize heat transfer by considering a constant volume fraction of the fins of a heat exchanger. Kharche et al. [19] analyzed heat transfer through the fin array using natural convection and based on the constructal design theory. Biserni et al. [20] used Benjan constructal design to optimize the geometric shape of an H-shaped cavity. It should be noted that these cavities are strangely imposed on a solid wall. Studying the thermal resistance between the solid body and cavity is the purpose of this study. Heat is evenly distributed throughout the solid wall as a result of internal heat generation. On the outer surface of the solid wall, the surface of the cavity is isothermal, while the surface of the cavity is adiabatic. It is important to note that the volume of the H-shaped cavity as well as its total volume are both fixed, whereas its geometric shape may change. An optimal H-shaped body configuration outperforms an optimal T-shaped cavity based on numerical results. Rocha et al. [21] employed a T-shaped cavity placed against a solid wall and optimized its geometric configuration. The optimization process adhered to the principles outlined in the Bejan theory.

Nava-Arriaga et al. [22] in its study based on constructal design theory, using material with high conductivity coefficient, studied the constant conduction heat transfer to cool a disk, as well as stresses due to thermal expansion. The authors investigated three distinct radial arrangements: a single-stage branching tree, a tree with multiple branching stages, and a tree with dual branching within a disk. By applying numerical methods, they calculated the maximum temperature and stress generated within the disk for these scenarios, subsequently subjecting them to optimization procedures. A survey of existing research in this domain [23–26] reveals a gap in the investigation of cavity optimization within trapezoidal sections featuring Z-shaped perforations. This study aims to address this gap and serve as its primary objective.

According to heat transfer process optimization studies, no studies have been conducted on the use of cavities with a Z-shaped tree structure. In the current study, an optimal cavity structure to increase heat transfer in these devices will be presented. In order to achieve this, the subsequent structure must be such that for a given heat output power, a minimum temperature is achieved in the desired part. In addition to examining the geometry of the Z-shape cavities and their length, the dimensions of a specific channel to reduce the maximum temperature produced will also be explored. To model the temperature transfer process, the PDETools toolbox in MATLAB along with the finite element method will be employed. Following the validation of the model, an exploration of diverse parameters influencing the maximum temperature will be conducted. The outcomes will be presented and analyzed through appropriate diagrams and tables.

Finite element simulation

In this study, the cavity geometry of a two-dimensional fin under the internal heat generation with a square cross section having a Z-shaped cavity has been analyzed. Considering a square surface, the length of one side of the object is denoted as ‘d’. As depicted in Fig. 1, the interior of the object contains a cavity that takes on a Z-shaped configuration.

The boundary condition involves protecting the lateral edges of the structure, which is implemented in the software by setting q = 0. It is anticipated that the combined volume of both the cavity and the body remains constant. The Z-shaped cross-sectional configuration is determined using the following equation:

$$V = L^2 W = Const., \tag{1}$$

in this context, W represents the thickness, and L signifies the length of the body. To simplify matters, it is assumed that the thickness is significantly smaller in comparison to other dimensions. This assumption allows us to neglect parameter variations in the direction of thickness. Consequently, the cross-sectional area of the square body is equal to [formula], maintaining a constant value. Across all instances, the volume of the cavity remains uniform and is calculated using the subsequent equation:

$$V_0 = (2L_0 t_0 + L_1 t_1)W. \tag{2}$$

Hence, employing the aforementioned equations, the volume restriction for the cross-sectional area of the cavities is formulated as follows

$$\phi = \frac{V_0}{V} = \frac{(2L_0 t_0 + L_1 t_1)W}{L^2 W} = Const., \tag{3}$$

here ϕ represents the volume fraction diminished by the cavities. The body is assumed to be isotropic, with a consistent and uniform heat conductivity denoted as k. Additionally, the body experienced uniform heat generation

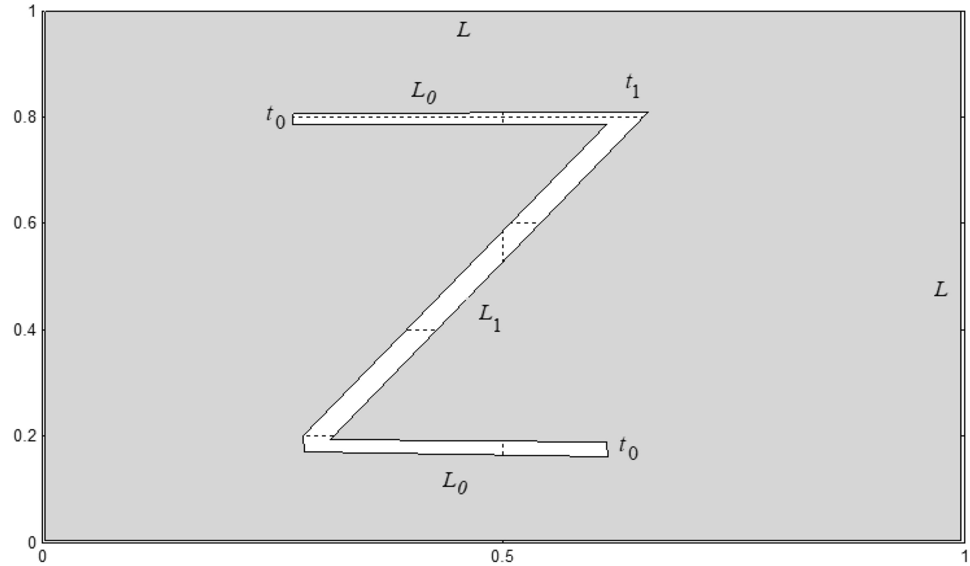

Fig. 1 Geometric characteristics of Z-shaped cavity structure in order to cool structure using heat generation

at a volume ratio i.e. $q'''(Wm^{-3})$. The outer surface of the heat generating body is completely insulated. The heat flux dissipated by the cavities is equal to $q''A(1-\phi)$ the heat transfer displaced by the cavities. Due to the thermal resistance of the body, the body temperature reaches a slightly higher temperature than the fluid inside the cavities. This temperature created in the corners of the body will have its maximum value and is denoted by T_{max}. When designing heat transfer chambers, an essential design parameter is to maintain the maximum temperature within a predefined limit, denoted as T_{max} Achieving an optimal design involves minimizing this value. Consequently, the research centers around this limiting design parameter, where the primary objective is to lower the overall system temperature through the utilization of localized heat transfer mechanisms. The numerical optimization of the geometric characteristics of both the cavity and the body is carried out by computing the maximum system error across a broad spectrum of influential system parameters.

$q'''A/(T_{\max}-T_\infty)$. Numerical optimization of the geometric parameters of the cavity and the body is done by calculating the maximum system error for a wide range of effective system parameters.

In order to optimize system parameters, it is necessary to extract and analyze the governing equations of the system. The differential equation governing two-dimensional conduction heat transfer under internal heat generation and in the steady state configuration is obtained in terms of dimensionless parameters from the following equation:

$$\frac{\partial^2 \tilde{T}}{\partial \tilde{x}^2}+\frac{\partial^2 \tilde{T}}{\partial \tilde{y}^2}+1=0, \tag{4}$$

where,

$$\tilde{T}=\frac{T-T_\infty}{q''A/k}, \tag{5}$$

and

$$(\tilde{x},\tilde{y},\tilde{n},\tilde{H},\tilde{L},\tilde{H}_e,\tilde{H}_0,\tilde{L}_0)=\frac{(x,y,n,H,L,H_e,H_0,H_0)}{A^{1/2}}, \tag{6}$$

in this context, n represents the coordinate perpendicular to the inner surface of the cavity. In accordance with the dimensionless heat transfer parameters, the heat transfer within the cavity can be expressed as follows:

$$\frac{\partial \tilde{T}}{\partial \tilde{n}}=\frac{a^2}{2}\tilde{T}, \tag{7}$$

here a, is the dimensionless parameter termed as thermal-geometric factor, and its definition is given as follows:

$$a=\left(\frac{2hA^{1/2}}{k}\right)^{1/2}. \tag{8}$$

In this study, the objective of optimization is to decrease the highest temperature generated within the system. This temperature can be expressed in terms of dimensionless parameters as follows:

$$\tilde{T}_{max} = \frac{T_{\max} - T_{\infty}}{q'''A/k}. \tag{9}$$

Solve the governing equations

In this study, the finite element method is employed to solve the differential equation that describes the temperature distribution as outlined in Eq. (4). To achieve this, the PDE Toolbox within the MATLAB software is utilized. The system's geometry is handled using lattice triangular elements, and the equation governing temperature distribution, along with suitable boundary conditions, is considered. Following an assessment of result sensitivity to mesh density, the optimal element size is determined to ensure convergence of results in accordance with the provided relationship:

$$\left|\tilde{T}^{j}_{\max} - \tilde{T}^{j+1}_{\max}\right| < 10^{-3}. \tag{10}$$

Figure 2 depicts the MATLAB software's PDE Toolbox environment, which possesses the capability to address a variety of partial differential equations. The 'Draw' function is utilized to visually represent the system's geometry, while the 'Boundary' function enables the application of diverse boundary conditions to the system. Subsequently, the 'PDE' function facilitates the assignment of specific types of differential equations, complete with numerical coefficients, to various regions. Upon selecting the 'Mesh' function, the system's geometry undergoes meshing through triangular components. Finally, the 'Solve' function is employed to solve the governing equations, taking into account the corresponding boundary conditions. The 'Plot' function allows for the visualization of results. Within this study, for extensive scenarios, this procedure is iterated, recording the highest system temperature each time. Ultimately, diverse curves are generated and analyzed.

Result discussion

Within this segment, the impact of distinct parameters on the temperature distribution in a body featuring Z-shaped cavities and subject to internal heat generation will be examined through the finite element approach. Since evaluating result consistency across varying meshes is imperative in restricted element analyses, the matter of mesh independence will be initially addressed. Subsequently, the findings will be substantiated, and ultimately, the influence of diverse parameters will be investigated.

Check mesh independency

Modeling and meshing of the structure is done in MATLAB software using PDE Toolbox. In order to study the geometry accurately, a proper grid must be made on the geometry.

The utilization of correlation techniques significantly enhances the efficacy of the solution process and the precision of the resultant outcomes. In cases where appropriate correlation is not employed for addressing the problem, not only does the solution process experience delays, but the obtained results also become riddled with

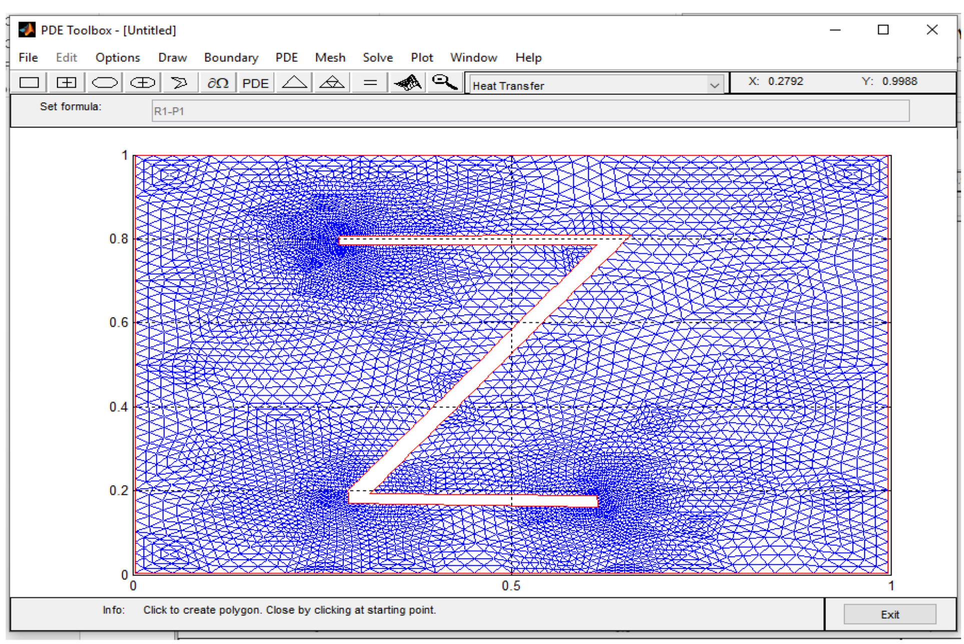

Fig. 2 MATLAB software box PDE Toolbox environment

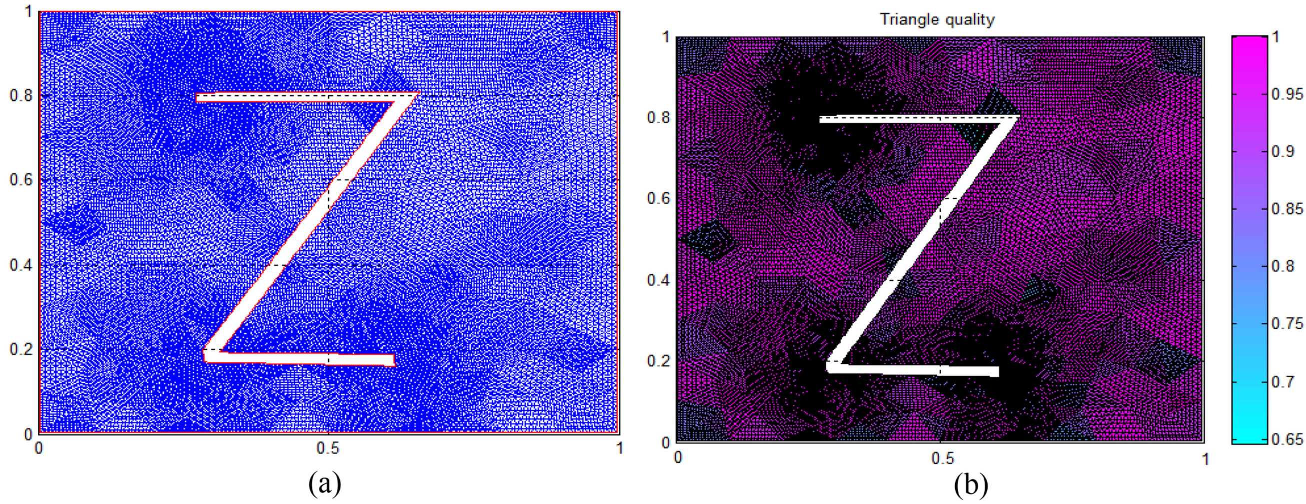

Fig. 3 **a** An example of system meshing and **b** a criterion for the worst angular deviation criterion

substantial inaccuracies. In this research, triangular elements are used to accurately meshing. Figure 3 shows an example of system meshing. This mesh was examined in terms of the worst angular deviation criterion and it was found that this value is more than 0.7 for all constraints, which is a great value. It should be noted that in the software guide it is mentioned that so as to have highly accurate outcomes, this value must be more than 0.5.

Checking the meshing independency of the results is usually done by studying several values for different grid sizes, and the numerical solution results are checked for different mesh size values, and from a number of grids onwards, little change in the results should be observed. Given that the primary objective of the current research is to minimize the maximum temperature within the system, this parameter has been adopted as the criterion for selecting an appropriate mesh configuration. Through an exploration of the relationship between the maximum temperature and the number of meshes, it is evident that greater mesh density yields more accurate outcomes. The investigation encompasses a range of mesh quantities, spanning from 5500 to 65,000. Notably, results indicate that beyond 23,000 meshes, the maximum temperature remains relatively constant, with any further increase primarily extending computational time. Consequently, a mesh count of 23,000 was determined as optimal. Figure 4 visually portrays the results from the mesh independence assessment for maximum temperature. It is discernible that augmenting the mesh count beyond 23,000 has minimal impact on the results. Accordingly, considering that in this number of elements, the results are independent of the number of meshes, and in order to accelerate the solution time, the results are extracted and studied using 23,000 elements.

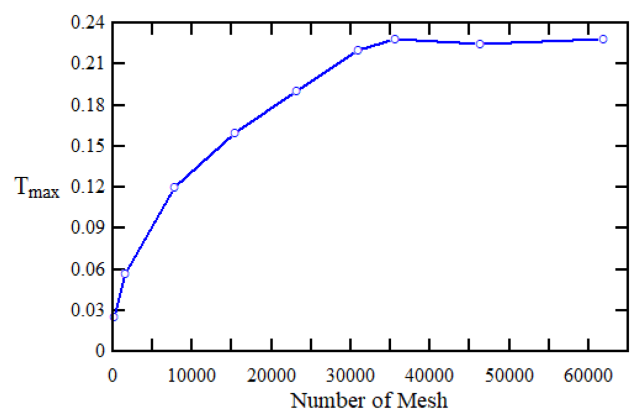

Fig. 4 The results of the mesh independency test for the maximum temperature

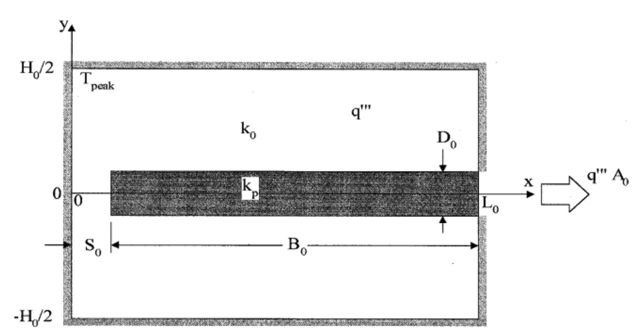

Fig. 5 Geometry under study in reference [27]

Validation of results

In the current investigation, validation of the findings has been conducted by referencing the work of Almogbel and Bejan [27]. Their study focused on the geometric enhancement of a configuration of T-shaped fins, illustrated in Fig. 5. The goal was to optimize the complex in a manner that increase the general thermal conductivity while constraining the total volume and material utilization within the fins. Their study highlighted that the optimization of the fin arrangement has limited influence on certain parameters, such as the length of the two bent ends within the fins. Notably, the study employed dimensionless parameters as outlined below:

$$\phi_0 = \frac{A_{p0}}{A_0} = \frac{D_0 B_0}{H_0 L_0}, \quad \tilde{T} = \frac{T - T(L_0, 0)}{q''' A_0 / k_0}, \quad \tilde{k} = \frac{k_0}{k_p}. \tag{11}$$

Based on the two-dimensional variables defined in Eq. (11), a comparison between the outcomes of the current study's model and those from the reference [27] is outlined in Table 1 for varying coefficients of thermal conductivity of the material. The tabulated data reveals a notably strong agreement between the outcomes presented in this study and the findings reported in reference [27]. Notably, the maximum discrepancy observed amounts to approximately 3%. Visual representation of the temperature distribution within the fin, as reported in reference [27], and as derived through the present model, is depicted in Fig. 6.

Table 1 Maximum temperature obtained from the modeling results of the present study and the results of Ref. [27] for different values of the thermal conductivity ratio

$\tilde{k}$	$\tilde{T}_{\max}$		
	Reference [24]	Present results	Error (%)
1000	0.1282	0.1244	3.05
300	0.1359	0.1331	2.10
100	0.1572	0.1563	0.58
30	0.2248	0.2287	1.70
10	0.3749	0.3827	2.04

Results for a rectangular body with a Z-shaped hole

Subsequently, an examination is undertaken to ascertain the impact of various parameters on the temperature distribution within a rectangular body housing a Z-shaped cavity. The geometric specifications of the cavity are visually detailed

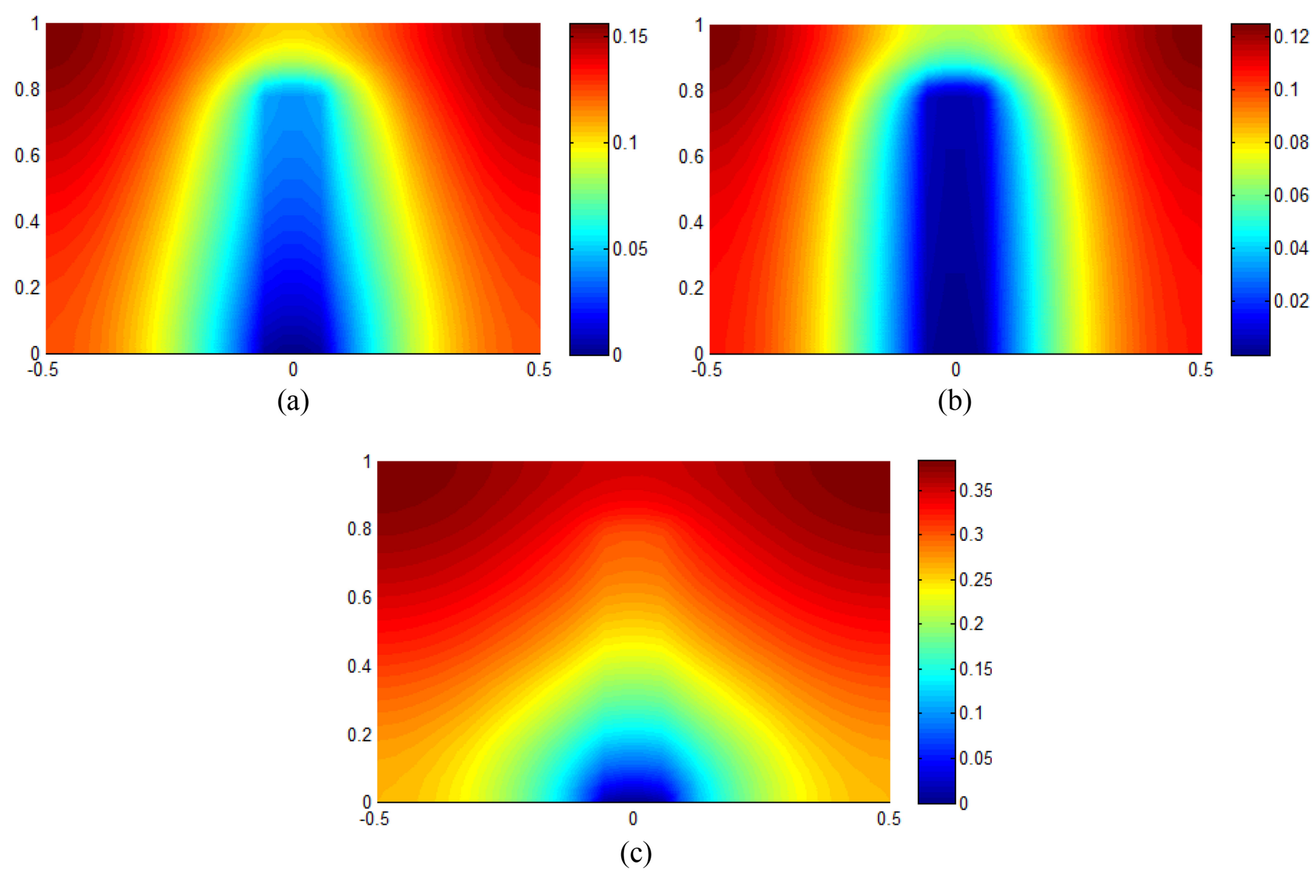

Fig. 6 Temperature distribution in the fin of the reference model [27] per $\phi_0 = 0.1$, $H_0/L_0 = 1$ and $H_0/B_0 = 0.15$ obtained using the present modeling **a** $\tilde{k} = 1000$, **b** $\tilde{k} = 100$ and **c** $\tilde{k} = 10$

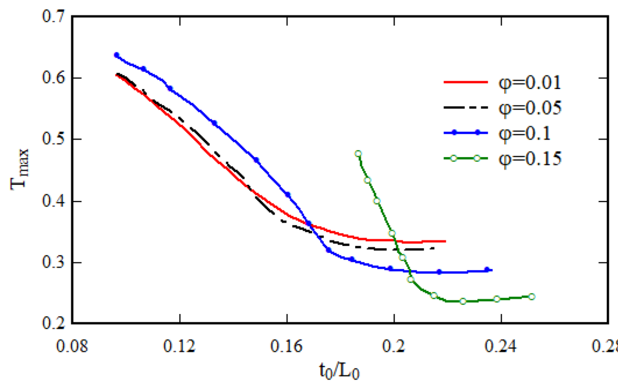

Fig. 7 The effect of the dimensionless parameter t_0/L_0 of the cavity on the maximum temperature produced in the fin with $L=1$

in Fig. 1. The objective of this endeavor is to explore how parameters such as L_0, L_1, t_0 and t_1 influence the temperature distribution across the fin structure. Adhering to structural principles, the optimization procedure must account for constraints on the cross-sectional area of the fin. For the purposes of this parametric investigation, the ratio of the cavity's area to the total area, denoted as ϕ, is employed.

The impact of the dimensional parameter of the cavity, t_0/L_0, on the maximum temperature generated within a heat-producing fin with dimensions L = 1, is illustrated in Fig. 7. This observation $t_0 = t_1$ holds true for all results. The presented figure pertains to a Z-shaped cavity and showcases varying values of the thickness-to-length ratio parameter. It is assumed that the inner surface of the cavity adheres to heat transfer boundary conditions. The findings underscore that the maximum temperature generated within the body is notably influenced by the volume fraction parameter. With a constant L1 value, an increase in the ratio t_0/L_0 corresponds to a decrease in the maximum temperature within the body. Additionally, the results indicate that when φ is less than 0.1, the impact of the ratio t_0/L_0 is exceeding 0.41 on the heat transfer rate is minor and negligible. Furthermore, when t_0/L_0 surpasses 0.4, the maximum temperature within the body diminishes with heightened volume fraction due to augmented heat transfer. Additionally, the outcomes reveal that, for a specific volume fraction magnitude, the temperature generated attains a minimum value within a particular cavity length, and elongating this value detrimentally affects the maximum temperature within the body.

Figure 8 displays the contour of temperature distribution with the production of internal heat for two states of $t_0/L_0 = 0.1, \varphi = 0.05$ and also $t_0/L_0 = 0.2, \varphi = 0.15$. As can be seen, in this case, most of the body temperature occurs in the middle areas and the temperature of the inner surface of the cavity has a minimum value of temperature and its value is almost the same in all lateral surfaces. Therefore, based on the need, an optimal design can be determined that does not change the overall performance of structure and does not affect geometric characteristics of the system much.

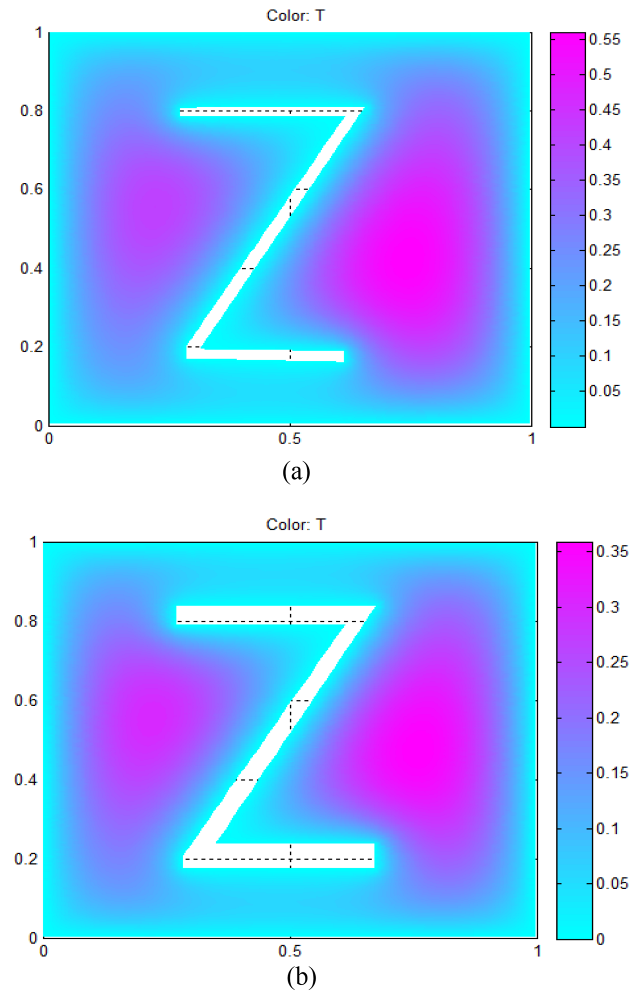

Fig. 8 Z-shaped body temperature distribution contour with internal heat generation and two states **a** $t_0/L_0 = 0.1$, $\varphi = 0.05$ and **b** $t_0/L_0 = 0.2$, $\varphi = 0.15$

The outcomes presented in the study underscore that when convection heat transfer occurs within the cavity, optimizing the geometric dimensions of the cavity becomes imperative to effectively mitigate the maximum body temperature. Determining these optimal values is essential for enhancing system performance and further reducing body temperature. Notably, substantial fluctuations in the system's maximum temperature are observed around these optimal points. Figure 9 depicts the optimal length ratio $\left(\frac{L_1}{L_0}\right)_{opt.}$, denoted as, for diverse volume fraction values. The depicted results emphasize that the influence of the ratio ϕ on reducing the maximum temperature hinges on other system parameters. For a given volume fraction value, there exists an optimal ratio t_0/L_0 for which the reduction in the

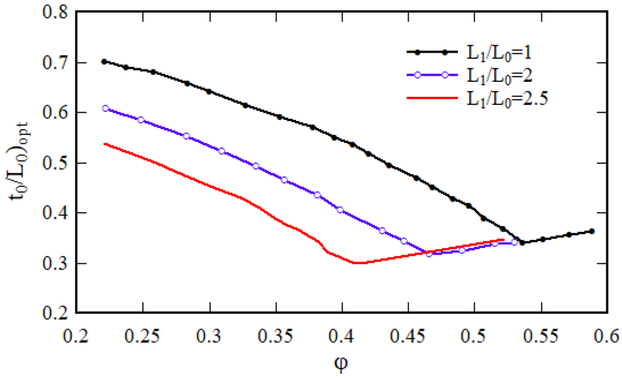

Fig. 9 Optimal length ratio $(L_1/L_0)_{opt.}$ for different volumetric fraction values

maximum system temperature is minimal. As an example, when the $L_1L_0 = 2$ volume fraction is $\phi = 0.48$, the optimal length ratio stands at 0.35. However, it's noted that the optimal length ratio can decrease with increasing volume fraction. Moreover, the findings suggest that no universal conclusion can be drawn regarding the optimal cavity length; its value is contingent upon other variables within the system. Thus, in the design of Z-shaped cavities, determining the optimal parameters should be undertaken based on the specific contextual conditions.

Figure 10 shows the temperature contour with flow and isothermal lines for the $\phi = 0.4$, $L_1/L_0 = 1$ and $L_1/L_0 = 2$. In this case, the cavity is located in the center of the body and has a convection heat transfer. As can be seen, the isothermal lines inside the chamber are symmetrical with respect to the vertical axis of the middle chamber due to the dominant effect of heat transfer and symmetry of the conductor and conduction heat transfer. A maximum dimensionless temperature of 0.09 is formed in the body, as shown in the figure.

Figure 11 provides an insight into the impact of the magnitude of interior heat creation on both the maximum temperature generated and the temperature distribution. Additionally, a comparison is drawn between scenarios with and without cavities. Evidently, the level of internal heat generation significantly influences the maximum temperature arising within the body. Importantly, the introduction of a cavity leads to a decrease in the maximum temperature. Based on the results, it is evident that for a specific dimensionless temperature value, $Q = 15$, the respective values with and without a cavity stand at 0.35 and 1.07. Consequently, the presence of the cavity results in a 40% reduction in the highest dimensionless temperature generated within the body.

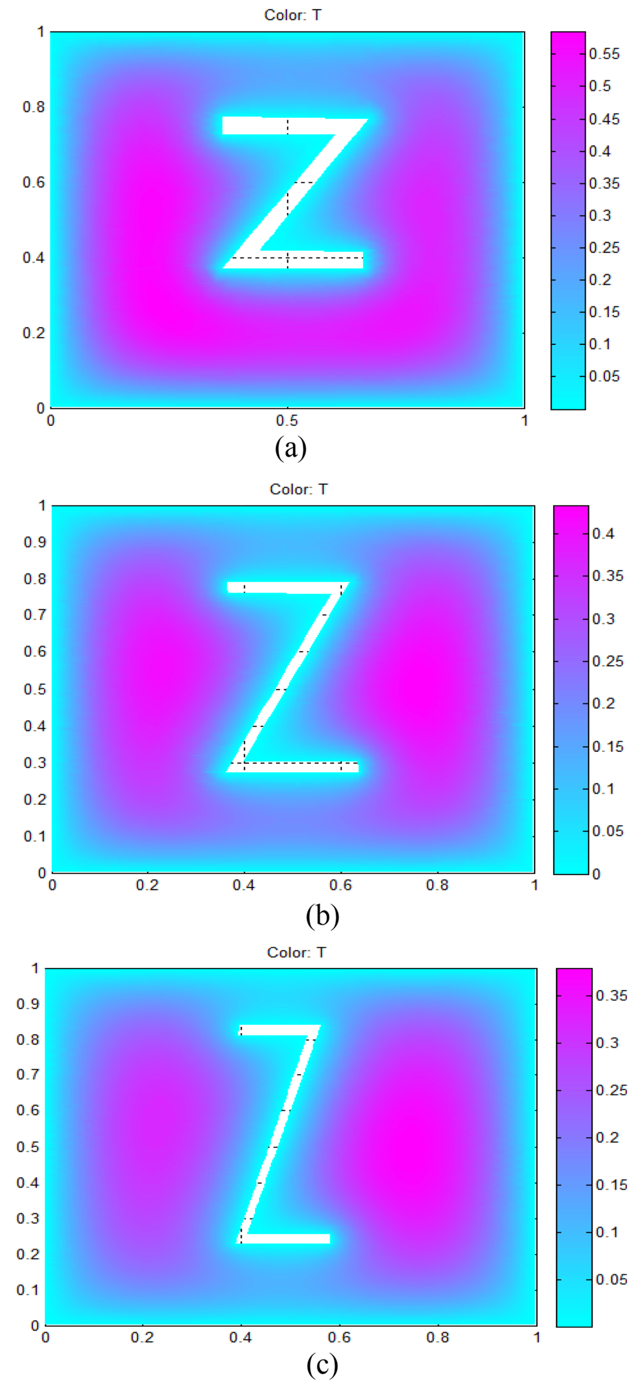

Fig. 10 Temperature contour with flow and isothermal lines for $\phi = 0.8$ and, $L_1/L_0 = 0.8$ per **a** $D_1/D_0 = 0.7$ **b** $D_1/D_0 = 0.5$ and **c** $D_1/D_0 = 1$

Conclusion

The objective of this study revolves around devising an optimal Z-shaped cavity for cooling rectangular components marked by internal heat generation. The ultimate aim is to minimize the temperature within the designated part while

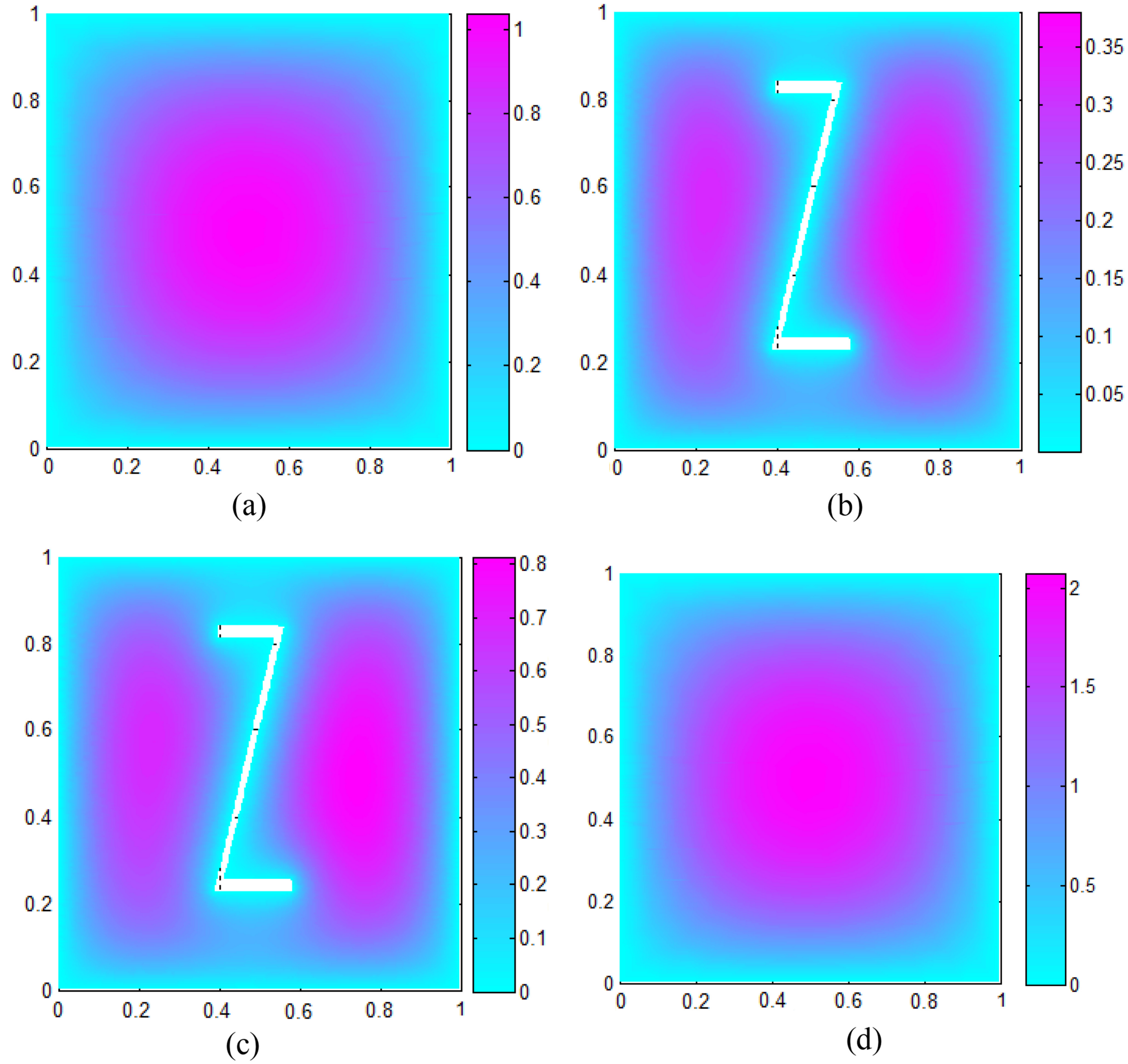

Fig. 11 Effect of internal heat generation on the maximum temperature **a** body with a cavity with the production of dimensionless heat 15, **b** body without cavity with the production of dimensionless heat 15, **c** body with a cavity with and $Q = 30$, **d** body without cavity and $Q = 30$

accommodating a specified heat output power. To achieve the goal of reducing the highest created temperature, specific cavity dimensions have been determined. Following the validation of the proposed model, which offers satisfactory approximations, the problem is tackled through numerical analysis to identify optimal conditions. The summarized outcomes of the present study are as follows:

1. Comparative analysis of outcomes from the current model and existing literature for varying thermal conductivity values exhibits a highly favorable agreement. The presented results demonstrate a maximum discrepancy of less than 3%, thus showcasing the model's accuracy.
2. Maintaining a constant value L_1, an increase in the ratio t_0/L_0 leads to a corresponding decrease in the maximum temperature within the body. For instance, for $L_1 = 0.5$, augmenting the ratio φ from 0.01 to 0.15 yields a reduction in the maximum temperature from 0.615 to 0.471.
3. The findings emphasize that the φ performance of the cavity in diminishing the maximum system temperature

is contingent upon other system parameters. Given a specific volume fraction parameter, there exists an optimal value for the ratio that results in the least reduction of the maximum system temperature.

4. The results indicate that for a heat output power of 30, the maximum dimensionless temperature of the fin decreases from 1.07 to 0.34 when transitioning from a cavity-less scenario to one with a cavity. Consequently, the presence of the cavity contributes to a 34% reduction in the maximum dimensionless temperature within the body.

Author contributions JL writing—original draft preparation, conceptualization, supervision, project administration.

Funding This work was supported by the "Blue Project" of Jiangsu Universities (RS21QL04).

Declarations

Conflicts of interest The authors declare no competing interests.

References

1. Wang, X.: Two-dimensional reconstruction of heat transfer in a flat flame furnace through computer-based tomography and tunable-diode-laser absorption spectroscopy. Fluid Dyn. Mater. Process. **16**(5), 857–869 (2020)
2. Hu, X., et al.: Assessment of the application of subcooled fluid boiling to diesel engines for heat transfer enhancement. Fluid Dyn. Mater. Process. **17**(6), 1049–1066 (2021)
3. Li, Y., Liu, Y.: A mathematical model of heat transfer in problems of pipeline plugging agent freezing induced by liquid nitrogen. Fluid Dyn. Mater. Process. **18**(3), 775–788 (2022)
4. Li, H., Zhang, S., Ji, Y., Sun, M., Li, X., Sheng, Y.: The influence of catchment scale on comprehensive heat transfer performance about tube fin heat exchanger in numerical calculation. Energy Rep. **8**, 147–155 (2022)
5. Mousavi Ajarostaghi, S.S., Zaboli, M., Javadi, H., Badenes, B., Urchueguia, J.F.: A review of recent passive heat transfer enhancement methods. Energies (Basel). **15**(3), 986 (2022)
6. Sadeghianjahromi, A., Wang, C.-C.: Heat transfer enhancement in fin-and-tube heat exchangers–a review on different mechanisms. Renew. Sustain. Energy Rev. **137**, 110470 (2021)
7. Kumar, P., Sarviya, R.M.: Recent developments in preparation of nanofluid for heat transfer enhancement in heat exchangers: a review. Mater. Today Proc. **44**, 2356–2361 (2021)
8. Zhang, J., Zhu, X., Mondejar, M.E., Haglind, F.: A review of heat transfer enhancement techniques in plate heat exchangers. Renew. Sustain. Energy Rev. **101**, 305–328 (2019)
9. Pan, M., et al.: Experimental investigation of the heat transfer performance of microchannel heat exchangers with fan-shaped cavities. Int. J. Heat. Mass. Transf. **134**, 1199–1208 (2019)
10. Al-Kouz, W.G., Kiwan, S., Alkhalidi, A., Sari, M., Alshare, A.: Numerical study of heat transfer enhancement for low-pressure flows in a square cavity with two fins attached to the hot wall using Al_2O_3-air nanofluid. Strojniški vestnik-J. Mech. Eng. **64**(1), 26–36 (2018)
11. Giwa, S.O., Sharifpur, M., Ahmadi, M.H., Meyer, J.P.: A review of magnetic field influence on natural convection heat transfer performance of nanofluids in square cavities. J. Therm. Anal. Calorim. **145**, 2581–2623 (2021)
12. Salimpour, M.R., Sharifhasan, M., Shirani, E.: Constructal optimization of the geometry of an array of micro-channels. Int. Commun. Heat Mass Transf. **38**(1), 93–99 (2011)
13. Promvonge, P., Sripattanapipat, S., Tamna, S., Kwankaomeng, S., Thianpong, C.: Numerical investigation of laminar heat transfer in a square channel with 45 inclined baffles. Int. Commun. Heat Mass Transf. **37**(2), 170–177 (2010)
14. Bejan, A., Tsatsaronis, G., Moran, M.J.: Thermal design and optimization. Wiley, Hoboken (1995)
15. Bejan, A., Sciubba, E.: The optimal spacing of parallel plates cooled by forced convection. Int. J. Heat. Mass. Transf. **35**(12), 3259–3264 (1992)
16. Bejan, A.: Constructal law, twenty years after. Proc. Rom. Acad. Ser. A Math. Phys. Tech. Sci. Inform. Sci. **18**, 309–311 (2018)
17. Shokouhmand, H., Mahjoub, S., Salimpour, M.R.: Constructal design of finned tubes used in air-cooled heat exchangers. J. Mech. Sci. Technol. **28**, 2385–2391 (2014)
18. Alebrahim, A., Bejan, A.: Constructal trees of circular fins for conductive and convective heat transfer. Int. J. Heat. Mass. Transf. **42**(19), 3585–3597 (1999)
19. Kharche, S.S., Farkade, H.S.: Heat transfer analysis through fin array by using natural convection. Int. J. Emerg. Technol. Adv. Eng. **2**(4), 595–598 (2012)
20. Biserni, C., Rocha, L.A.O., Stanescu, G., Lorenzini, E.: Constructal H-shaped cavities according to Bejan's theory. Int. J. Heat. Mass. Transf. **50**, 11–12 (2007)
21. Rocha, L.A.O., Lorente, S., Bejan, A.: Constructal law and the unifying principle of design. Springer, Cham (2012)
22. Nava-Arriaga, E.M., Hernandez-Guerrero, A., Luviano-Ortiz, J.L., Bejan, A.: Heat sinks with minichannels and flow distributors based on constructal law. Int. Commun. Heat Mass Transfer. **125**, 105122 (2021)
23. Keramat, F., Azari, A., Rahideh, H., Abbasi, M.: A CFD parametric analysis of natural convection in an H-shaped cavity with two-sided inclined porous fins. J. Taiwan. Inst. Chem. Eng. **114**, 142–152 (2020)
24. Miguel, A.F., Rocha, L.A.O.: Tree-shaped fluid flow and heat transfer. Springer, Cham (2018)
25. Estrada, E.S.D., dos Santos, E.D., Isoldi, L.A., Rocha, L.A.O.: Constructal design associated with genetic algorithm to maximize the performance of H-shaped isothermal cavities. In: Computational intelligence, optimization and inverse problems with applications in engineering, pp. 215–226. Springer, Cham (2019)
26. Avendaño, P.A., Souza, J.A., Adamatti, D.F.: Construction of conductive pathways using genetic algorithms and constructal theory. Int. J. Therm. Sci. **134**, 200–207 (2018)
27. Almogbel, M., Bejan, A.: Conduction trees with spacings at the tips. Int. J. Heat. Mass. Transf. **42**(20), 3739–3756 (1999)