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RESEARCH PAPER

T-odd observables from anomalous *tbW* couplings in single-top W associated production at the LHC

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Abstract
We investigate the possibility that an imaginary anomalous tbW coupling can be measured in the process $pp \to tW^- X$ by means of T-odd observables. One such observable is the polarization of the top quark transverse to the production plane. Another is the asymmetry between the cross sections with a positive and with a negative value of a certain T-odd correlation constructed out of observable momenta when the top quark decays leptonically. The mean value of this correlation, which is also T-odd, is the third observable. These three T-odd observables are shown to be proportional to the imaginary part of only one of the tbW anomalous couplings, the other couplings giving vanishing contributions. This imaginary part could signal either a CP-odd coupling, or an absorptive part in the effective coupling. We estimate the 1-σ limits that might be derived in the case of each of these observables for centre-of-mass energies 13, 13.6 and 14 TeV at the LHC, corresponding to Run 2, Run 3 and High-Luminosity runs, respectively.

Keywords Anomalous top couplings · tW production · T-odd observables · LHC experirments

Introduction

The Large Hadron Collider (LHC), a proton-proton collider, has seen several successes in establishing the validity of many aspects of the standard model (SM), and constraining physics beyond the standard model. With an enhanced luminosity, as in the future runs such as the High-Luminosity version (HL-LHC), results with higher precision are anticipated. Apart from investigating with great accuracy the parameters of the standard model (SM), future experiments will also strive to discover new physics—either with the discovery of new particles, or checking if the measured couplings of the SM particles violate predictions from the SM.

One of the important constituents of the SM is the top quark, the heaviest quark known. The properties of the top quark have been studied in great detail at the LHC, particularly through the most abundant process of top-pair production. However, some of the electroweak properties of the top quark can be studied through the single-top production processes For example, single-top production enables the determination of the CKM matrix element V_{tb}. Of these processes, the t-channel process has the largest cross section, followed by the tW associated production process. The s-channel single-top production process is also possible, though it has the least cross section of the three (Giammanco 2016; Andrea and Chanon 2023).

While the tW production process was measured earlier at the LHC for centre-of-mass (cm) energies of 7 and 8 TeV, the recent measurements are carried out at the cm energy of 13 TeV. The CMS collaboration at the LHC has recently measured the tW differential cross section at 13 TeV with an integrated luminosity of 138 fb^{-1}, with a central value of 79.2 for the total cross section (Tumasyan 2023). The ATLAS collaboration at the LHC has studied the tW production process recently at 13 TeV for a luminosity of 140 fb^{-1} and obtained a central value of 75 pb for the total cross section (Aad 2024).

The tW production process occurs at the lowest order through the exchange of either a t-channel virtual t quark or an s-channel b quark from an initial state of a gluon and a b quark from the protons, producing a top quark through an anomalous tbW vertex, as shown in the diagrams in Fig. 1.

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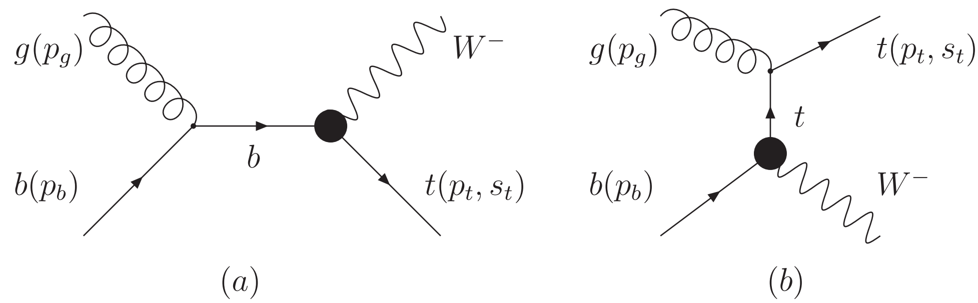

Fig. 1 Feynman diagrams contributing to $gb \to tW^-$ production. The effective *tbW* vertex is shown as a filled circle

Because of the occurrence of the weak *tbW* vertex, it is well suited for the study of a possible anomalous *tbW* coupling arising from new physics effects beyond the SM.

We concentrate here on observables in the above single-top *W* associated production process which are odd under naive time reversal (T). By the term naive time reversal we mean an operation limited to reversing the signs of momenta and spins of all particles. By contrast, genuine time reversal also involves interchange of initial and final states, and is therefore a transformation difficult to achieve in actual practice. At tree level, unitarity implies that the forward and backward amplitudes have the same phase, and therefore the time reversal violation, if present, would be genuine. It would therefore occur only in the presence of CP violation, on account of the CPT theorem. On the other hand if the amplitude has an absorptive part arising from rescattering or loop diagrams, characterized by a nonzero imaginary part for an anomalous effective coupling, the forward and backward amplitudes are not the same, and the subsequent T violation would not be a genuine one. In that case the CPT theorem does not necessarily imply CP violation.

We study in the process $pp \to tW^-X$ three observables which are T odd: (1) The polarization of the *t* perpendicular to the production plane (the transverse polarization). (2) An asymmetry between the cross sections with a positive and a negative value of the combination which corresponds to the angular-variable part of the three-momentum vector triple product $\mathrm{p}_P \times \mathrm{p}_t \cdot \mathrm{p}_{\ell^-}$, where *P*, *t* and ℓ^+ correspond respectively to an initial proton, the top quark and the charged lepton produced in top decay. (3) The mean value of this combination of angular variables. These are studied in the presence of possible anomalous *tbW* couplings in an effective Lagrangian. It is customary to consider an effective Lagrangian arising in what is termed as an effective field theory, where the SM Lagrangian is supplemented by higher-dimensional terms which are suppressed by inverse powers of a cut-off scale, a scale up to which the effective field theory is valid. However, here we do not make any such specific assumption about scales, but simply write an effective Lagrangian permitted by Lorentz invariance.

The parton-level process which contributes to the process of interest is $gb \to tW^-$. While defining the T-odd variables for the parton-level process is straightforward, they average out to zero for a symmetric *pp* initial state at the LHC. We therefore make a choice for the *z* axis to be along the direction of the combined momentum of *t* and W^- in the laboratory (lab) frame. The *t* momentum is then chosen to be in the *xz* plane, with the *x* component of the *t* momentum taken as positive.

The observables we calculate arise out of interference between the SM tree-level amplitude and the amplitude which is first order in the anomalous coupling. Since we deal with spin density matrices, higher-order calculations are more difficult than those for total cross sections. Though higher-order corrections to the SM cross section are available, we have to restrict ourselves to the lowest order for consistency. It is likely that since we are dealing with ratios of quantities, the results for the observables will be reasonably accurate.

Some previous work on T-odd observables in processes involving the top quark may be found in (Tiwari and Gupta 2019, 2023, 2024; Rindani 2024). CP violation in the tW^- and $\bar{t}W^+$ production processes was studied in (Rindani and Sharma 2012).

Anomalous *tbW* couplings can also contribute to *B* physics. A recent analysis (Kozachuk and Melikhov 2020) using $B_s - \bar{B}_s$ oscillations and various *B* decay channels has obtained bounds on certain anomalous *tbW* couplings, or their combinations, assumed real. The 95% confidence level bounds are at best ± 0.05 for f_{1L} and f_{2R}.

tbW couplings: CP and T properties

The Lagrangian for effective *tbW* couplings may be written as

$$\begin{aligned}\mathcal{L}_{Wtb} \;=\; & \frac{g}{\sqrt{2}} V_{tb} \Big[W_\mu \bar{t} \gamma^\mu (f_{1L} P_L + f_{1R} P_R) b \\ & - \frac{1}{2m_W} W_{\mu\nu} \bar{t} \sigma^{\mu\nu} (f_{2L} P_L + f_{2R} P_R) b \Big] + \text{H.c.}\end{aligned} \tag{1}$$

Here, *g* is the semi-weak coupling, $P_{L,R}$ are the left (right) chirality projection matrices, and $W_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu$.

We later assume $V_{tb} = 1$ for simplicity. In the SM at tree level, $f_{1L} = 1$, and the remaining f_i are vanishing.

In general, the anomalous couplings f_{1R}, f_{2L} and f_{2R} can be complex. The presence of an imaginary part can signal either an absorptive part in the form factor or violation of CP or both. Whether the coupling is CP violating or not can be checked by a comparison of the phases in the conjugate processes involving the t and the $\bar{t}$. The CP-violating phases would be opposite in sign to each other in these conjugate processes. The absorptive phase, however, would be the same in both these processes. A discussion of these issues may be found in (Bernreuther et al. 2009; Antipin and Valencia 2009; Rindani and Sharma 2012). We recapitulate the relevant points further below.

Numerical values of anomalous couplings, including the absorptive parts, have been evaluated in the SM by (Gonzalez-Spingberg et al. 2011), who find the values $f_{2R}((\mathrm{SM})) = -(7.17 + 1.23i) \times 10^{-3}$ and $f_{2L}((\mathrm{SM})) = -(1.21 + 0.01i) \times 10^{-3}$. They have been evaluated for certain theoretical scenarios in (Bernreuther et al., 2009), who find values for real parts of f_{2L} and f_{2R} deviating from the SM values by amounts of the order of a few times 10^{-4}, with the imaginary part corrections being somewhat smaller.

We can write the most general effective vertices up to mass dimension 5 for the four distinct processes of t decay, $\bar{t}$ decay, t production and $\bar{t}$ production as

$$V_{t\to bW^+} = -\frac{g}{\sqrt{2}} V_{tb}[\gamma^\mu (f_{1L}^* P_L + f_{1R}^* P_R) - i(\sigma^{\mu\nu} q_\nu / m_W)(f_{2L}^* P_L + f_{2R}^* P_R)], \tag{2}$$

$$V_{\bar{t}\to \bar{b}W^-} = -\frac{g}{\sqrt{2}} V_{tb}^*[\gamma^\mu (\bar{f}_{1L}^* P_L + \bar{f}_{1R}^* P_R) - i(\sigma^{\mu\nu} q_\nu / m_W)(\bar{f}_{2L}^* P_L + \bar{f}_{2R}^* P_R)], \tag{3}$$

$$V_{b^*\to tW^-} = -\frac{g}{\sqrt{2}} V_{tb}^*[\gamma^\mu (f_{1L} P_L + f_{1R} P_R) - i(\sigma^{\mu\nu} q_\nu / m_W)(f_{2L} P_R + f_{2R} P_L)], \tag{4}$$

$$V_{\bar{b}^*\to \bar{t}W^+} = -\frac{g}{\sqrt{2}} V_{tb}[\gamma^\mu (\bar{f}_{1L} P_L + \bar{f}_{1R} P_R) - i(\sigma^{\mu\nu} q_\nu / m_W)(\bar{f}_{2L} P_R + \bar{f}_{2R} P_L)], \tag{5}$$

where f_{1L}, f_{1R}, f_{2L}, f_{2R}, $\bar{f}_{1L}$, $\bar{f}_{1R}$, $\bar{f}_{2L}$, and $\bar{f}_{2R}$ are form factors, P_L, P_R are left-chiral and right-chiral projection matrices, and q represents the W^+ or W^- momentum, as applicable in each case. In the SM, at tree level, $f_{1L} = \bar{f}_{1L} = 1$, and all other form factors vanish. New physics effects would result in deviations of f_{1L} and $\bar{f}_{1L}$ from unity, and nonzero values for other form factors.

At tree level, when there are no absorptive parts in the relevant amplitudes which give rise to the form factors, the following relations are obeyed:

$$f_{1L}^* = \bar{f}_{1L}; \quad f_{1R}^* = \bar{f}_{1R}; \quad f_{2L}^* = \bar{f}_{2R}; \quad f_{2R}^* = \bar{f}_{2L}. \tag{6}$$

This can be seen to follow from the fact that in the absence of absorptive parts, the effective Lagrangian is Hermitian. Thus, the Hermiticity of the Lagrangian of Eq. (1) from which the amplitudes (2)–(5) may be derived implies the relations (6). On the other hand, if CP is conserved, the relations obeyed by the form factors are

$$f_{1L} = \bar{f}_{1L}; \quad f_{1R} = \bar{f}_{1R}; \quad f_{2L} = \bar{f}_{2R}; \quad f_{2R} = \bar{f}_{2L}. \tag{7}$$

The phases of the form factors can be expressed as sums and differences of two phases, one corresponding to a nonzero absorptive part, and another corresponding to CP nonconservation:

$$\begin{aligned} f_{1L,R} &= |f_{1L,R}| \exp(i\alpha_{1L,R} + i\delta_{1L,R}); \quad \bar{f}_{1L,R} = |f_{1L,R}| \exp(i\alpha_{1L,R} - i\delta_{1L,R}); \\ f_{2L,R} &= |f_{2L,R}| \exp(i\alpha_{2L,R} + i\delta_{2L,R}); \quad \bar{f}_{2R,L} = |f_{2L,R}| \exp(i\alpha_{2L,R} - i\delta_{2L,R}), \end{aligned} \tag{8}$$

where, for convenience, magnitudes of the couplings are assumed to be equal in appropriate pairs. In case there are no absorptive parts ($\alpha_i = 0$), we get the relations (6) as a special case. In case of CP conservation, we get the relations (7) as a special case. In what follows, we will deal with a more general case when both CP violation and absorptive parts may be present.

T-odd observables

We investigate three representative T-odd observables. The first one is the polarization of the top quark transverse to the production plane. The polarization of top quarks may be measured using the kinematic distributions of the decay particles, without contamination from the hadronization process, since the quark being heavy is expected to decay before hadronization.

The polarization can be measured experimentally from the decay distribution of one of the decay products in the rest frame of the top quark. The angular distribution of a decay product f for a top ensemble has the form

$$\frac{1}{\Gamma_f}\frac{\mathrm{d}\Gamma_f}{\mathrm{d}\cos\theta_f} = \frac{1}{2}(1 + \kappa_f P_t \cos\theta_f). \tag{9}$$

Here θ_f is the angle between the momentum of fermion f and the top spin vector in the top rest frame and P_t is the degree of polarization of the top ensemble. Γ_f is the partial decay width and κ_f is the spin power of *f*. For a charged lepton $f \equiv \ell^-$ the analyzing power is $\kappa_{\ell^-} = -1$ at tree level. We do not discuss the details of such a polarization measurement. However, it is appropriate to note that a measurement of the top polarization using the angular distribution of ℓ^- will not be affected by the contribution of an anomalous tbW coupling in the top decay process to linear order, as shown in earlier work (Godbole et al. 2019; Rindani 2000; Grzadkowski and Hioki 2000, 2000b)

The other two T-odd observables involve using the momentum of a top decay particle and constructing a triple three-vector product of its momentum with the incoming beam momentum and the *t* momentum. Since these three-vectors change sign under T, such a triple product would be odd under T. For reasons discussed later, we will use only the factors involving angular variables and not the whole triple product.

All these observables, being T odd, depend on the existence of an absorptive amplitude, as discussed earlier. In an approach with an effective Lagrangian like the one in Eq. (1), this implies that the observable will be proportional to the imaginary part of an effective coupling. In our case, it turns out that Im f_{2R} is the only one which contributes. While we could also analyze other observables which can measure the real parts of the anomalous couplings, we do not consider them here, and we set the real parts of the anomalous couplings to zero.

In order to calculate the T-odd observables, we first obtain analytic expressions for the production spin density matrix elements up to first order in the anomalous couplings.. For calculating helicity amplitudes and density matrix elements, we have made use of the algebraic manipulation software FORM (Vermaseren 2001) as well as well the procedure outlined in (Vega and Wudka 1996). We also calculated the transverse polarization of the top quark using FORM. We then evaluate numerically cross sections, expressions and asymmetries for cm energies corresponding to Run 2 and Run 3 of the LHC and to the future HL-LHC, viz., 13, 13.6 and 14 GeV. As mentioned earlier we use only expressions to leading-order in the anomalous couplings. It is expected that since the asymmetries and expectation values we calculate are ratios of cross sections, they may be somewhat insensitive to the higher-order corrections.

Though analytical expressions for all the elements of the spin density matrix ρ_{ij}, where $i, j = 1, 2$ refer respectively to the helicities $+1/2$ and $-1/2$ of the *t*, have been obtained, and used in our calculations, they are somewhat lengthy. We give below only the expression for a combination of off-diagonal elements on which the T-odd observables calculated here depend. The full expressions will be given elsewhere.

The expression for the production cross section in terms of the elements of ρ is

$$\sigma = \int dx_1\, dx_2\, g(x_1) b(x_2) \times \int d\cos\theta_t^{\mathrm{cm}}\, \frac{|\mathrm{p}_t^{cm}|}{16\pi \hat{s}^{\frac{3}{2}}} (\rho_{11} + \rho_{22}) + (x_1 \leftrightarrow x_2), \tag{10}$$

where the superscript "cm" refers to the *gb* cm frame and where we employ the five-flavour scheme. Here, $\sqrt{\hat{s}}$ is the *gb* energy in the cm frame of the pair, and θ_t^{cm} is the angle made by the *t* momentum with the *z* axis, defined by the direction of the momentum of the tW^- final-state, which is the direction of the parton (*g* or *b*) carrying the larger momentum fraction. $(x_1 \leftrightarrow x_2)$ denotes the expression obtained by an interchange of x_1 and x_2 in the previous expression, where again $\cos\theta_t^{\mathrm{cm}}$ is the polar angle of the *t*, defined with respect to the appropriate *z* axis. For a gluon carrying a momentum fraction x_1 of the proton and the *b* quark carrying a momentum fraction x_2, or vice versa, the *pp* cm energy is $\hat{s} = 4x_1x_2s$. We neglect m_b everywhere.

We next proceed to evaluate the three T-odd observables.

Transverse polarization of t

The transverse top polarization, that is, the polarization transverse to the top production plane, arises for nonzero Im f_{2R}. As mentioned earlier, for a symmetric collider like the LHC, the forward and backward directions cannot be distinguished. However, it is still possible to define an asymmetry by choosing the *z* axis as the direction of the the tW^- pair in the parton cm frame. Thus, in the laboratory (lab.) frame, the *z* axis is in the direction of the parton which receives a larger Lorentz boost to take it from the parton cm frame to the lab frame, because that, by momentum conservation, is the direction of the momentum of the tW^- state. The transverse polarization, thus defined, is the asymmetry given symbolically by

$$P_t = \frac{\sigma_{x_b>x_g}(s_t^y > 0) - \sigma_{x_b>x_g}(s_t^y < 0) + \sigma_{x_g>x_b}(s_t^y > 0) - \sigma_{x_g>x_b}(s_t^y < 0)}{\sigma_{x_b>x_g}(s_t^y > 0) + \sigma_{x_b>x_g}(s_t^y < 0) + \sigma_{x_g>x_b}(s_t^y > 0) + \sigma_{x_g>x_b}(s_t^y < 0)}, \tag{11}$$

where x_b and x_g refer respectively to the momentum fractions carried by the *b* quark and the gluon. s_t^y is the *y* component of the spin of the top, the *y* axis being along $\mathrm{p}_b \times \mathrm{p}_t$ for $x_b > x_g$, the events when the *b* quark receives the larger boost. The *y* axis is along $\mathrm{p}_g \times \mathrm{p}_t$ when $x_g > x_b$, the events when the *g* parton receives the larger boost.

The numerator of this asymmetry, which may be called the polarized cross section $\sigma_{\mathrm{pol.}}$, is an integral over the phase space of the squared matrix element for the production of a top with polarization perpendicular to the production plane. It is the net polarization along the *y* direction.

The spin density matrix elements for the production of the top were evaluated using FORM (Vermaseren 2001). The result for the imaginary part of the off-diagonal density matrix element is given by the equation

$$-i(\rho_{12}-\rho_{21}) = -\mathrm{Im}\ f_{2R}\frac{g^2 g_s^2}{3m_W\hat{s}}\epsilon(p_g,p_b,p_t,s_t), \qquad (12)$$

where $\epsilon(p_g,p_b,p_t,s_t)$ is the Levi-Civita tensor ϵ contracted with three momenta and the spin four-vector of the top, chosen along the y axis as described above. The polarized top cross section is then given by

$$\sigma_{\mathrm{pol.}} = \int dx_1\, dx_2\, g(x_1)b(x_2) \times \int d\cos\theta_t^{\mathrm{cm}}\,\frac{|\mathrm{p}_t^{cm}|}{16\pi\hat{s}^{\frac{3}{2}}}[-i(\rho_{12}-\rho_{22})] + (x_1\leftrightarrow x_2), \qquad (13)$$

where, as before, $(x_1\leftrightarrow x_2)$ denotes the expression obtained by an interchange of x_1 and x_2 in the previous expression.

The denominator, which is the unpolarized cross section $\sigma_{\mathrm{unpol.}}$, is precisely the cross section given in Eq. (10). The transverse polarization, as anticipated, is proportional to Im f_{2R}. We have evaluated it to linear order in the imaginary parts of the anomalous couplings, neglecting the real parts.

We also considered the transverse polarizations arising from Im f_{1R} and Im f_{2L}. It turns out that they vanish. Thus, the measurement of a non-vanishing transverse polarization would signal specifically the presence of a nonzero coupling Im f_{2R}.

We evaluate the relevant integrals numerically. We use the values $m_t = 172.5$ GeV, $\sin^2\theta_W = 0.223$, where θ_W is the electroweak mixing angle, $m_W = 80.379$ GeV, and, to be used in the following section, $\Gamma_W = 2.085$ GeV for the W boson width. We employ CTEQL1 parton distributions with $Q = m_t$ in the five-flavour scheme.

To estimate how well an asymmetry A can measure Im f_{2R} for a certain integrated luminosity L, we assume that the number of asymmetric events $\sigma_{\mathrm{pol.}}L = A\sigma_{\mathrm{unpol.}}L$ is larger than the $\sqrt{N_{\mathrm{SM}}}$, the statistical fluctuation in the SM events in the absence of Im f_{2R}, thus allowing the discrimination of the polarization arising from the anomalous coupling from the statistical background at the 1-σ level. This is expressed by the inequality

Table 1 The top transverse polarization P_1 for unit Im f_{2R} and the limits Im f_{2R}^{lim} on Im f_{2R} for various cm energies and respective integrated luminosities

$\sqrt{s}$ (TeV)	L (fb^{-1})	σ_{SM} (pb)	P_1	Im f_{2R}^{lim}
13	190	24.8	4.44×10^{-2}	1.04×10^{-2}
13.6	500	27.5	4.34×10^{-2}	6.21×10^{-3}
14	3000	29.5	4.27×10^{-2}	2.49×10^{-3}

$$A\sigma_{\mathrm{SM}}L > \sqrt{\sigma_{\mathrm{SM}}L}. \qquad (14)$$

Thus, the limit on Im f_{2R} that can be placed at the 1-σ level with luminosity L is

$$\mathrm{Im}\ f_{2R}^{\mathrm{lim}} = \frac{1}{\sqrt{\sigma_{\mathrm{SM}}L}A_1}, \qquad (15)$$

where A_1 is the value of the asymmetry for Im $f_{2R}=1$.

We assume integrated luminosities of 190 fb^{-1}, 500 fb^{-1} and 3000 fb^{-1} respectively for Run 2 at 13 TeV, Run 3 at 13.6 TeV, and HL-LHC at 14 TeV. The results for the transverse polarization asymmetry for unit value of coupling Im f_{2R} and the limits on the coupling for the cm energies considered here and with respective integrated luminosities are given in Table 1, together with the cross sections in the SM. As mentioned earlier, we are only using tree-level expressions. It is likely that the asymmetries being ratios of cross sections, will be insensitive to the higher-order corrections. Since the NLO cross sections are higher, our estimate of the limits would tend to be conservative.

T-odd observables from angular variables

We now consider the two other T-odd observables arising when the t decay is taken into account. If we include the top decay $t\to b\ell^+\nu_\ell$, it is possible to construct T-odd observables without explicit recourse to top spin. For example, at the parton level, a T-odd variable which may be constructed is $\epsilon_{\mu\nu\alpha\beta}\ p_g^\mu\ p_b^\nu\ p_t^\alpha\ p_\ell^\beta$. It is a Lorentz-invariant quantity, and could be evaluated in any frame. For example, in the parton cm frame it evaluates to $\sqrt{\hat{s}}|\mathrm{p}_b||\mathrm{p}_t|E_l\sin\theta_t\sin\theta_\ell\sin\phi_\ell$, with all quantities calculated in the parton cm frame. However, as stated earlier, the charged-lepton angular distribution is independent of the anomalous couplings in the decay vertex to first order (Godbole et al. 2019; Rindani 2000; Grzadkowski and Hioki 2000, 2000b). Hence, to isolate the effect of anomalous coupling Im f_{2R} in the production process and to avoid the influence of anomalous couplings in decay, we consider only a combination of angular variables, even though it is not Lorentz invariant. We now construct two T-odd observables from

$$O = \sin\theta_t\sin\theta_\ell\sin\phi_\ell, \qquad (16)$$

all variables being measured in the lab frame. These are

(i) $A(O)$, the asymmetry in the variable O of the cross section. This is the ratio to the total number of events of the difference in the number of events for which the $A(O)$ is positive and for which it is negative:

$$A(O) = \frac{\sigma(O > 0) - \sigma(O < 0)}{\sigma(O > 0) + \sigma(O < 0)}. \tag{17}$$

The asymmetry $A(O)$ and the corresponding limits on Im f_{2R} for various cm energies and respective integrated luminosities are given in Table 2.

(ii) The expectation value $\langle O \rangle$ of the variable O.

The contributions to these observables, as mentioned earlier, come only from the imaginary parts of the couplings. Thus, only the off-diagonal elements of ρ, already shown earlier, are relevant, the diagonal elements being real. To evaluate these observables, the density matrix for the production of the top has to be combined with the density matrix elements for the decay $t \to b\ell^+\nu_\ell$, with an integration over all variables except the charged-lepton angular variables θ_ℓ^0 and ϕ_ℓ^0. These are now written in the top rest frame. The expression for the full cross section is evaluated using the narrow-width approximation for the W. The density matrix elements Γ_{ij}, $(i, j = 1, 2)$ are (see for example (Godbole et al. 2003)).

$$\begin{array}{rcl} \Gamma_{11} & = & K(1 + \cos\theta_\ell^0) \\ \Gamma_{22} & = & K(1 - \cos\theta_\ell^0) \\ \Gamma_{12} & = & K \sin\theta_\ell^0 e^{i\phi_\ell^0} \\ \Gamma_{21} & = & K \sin\theta_\ell^0 e^{-i\phi_\ell^0} \end{array} \tag{18}$$

where

$$K = g^4(m_t^2 - 2p_t \cdot p_\ell). \tag{19}$$

In principle, the anomalous tbW couplings would contribute to the decay density matrix. However, restricting to linear order, the effect of the anomalous couplings on the normalized angular distributions has been shown to be absent (Godbole et al. 2019; Rindani 2000; Grzadkowski and Hioki 2000, 2000b). We therefore do not include anomalous couplings, assuming them to be small enough to ignore the quadratic powers. The cross section for the process in the lab frame is given by

$$\sigma = \int dx_1 g(x_1) \int dx_2 b(x_2) \int d\cos\theta_t^{\text{lab}} \int d\cos\theta_{\ell^-}^{\text{lab}} \int d\phi_{\ell^-}^{\text{lab}} \int dE_{\ell^-}^{\text{lab}} \frac{E_\ell^{\text{lab}} p_t^{\text{cm}}}{\hat{s}^{3/2}} \times \frac{1}{512(2\pi)^4 \Gamma_t \Gamma_W m_t m_W} (\rho_{11}\Gamma_{11} + \rho_{22}\Gamma_{22} + \rho_{12}\Gamma_{12} + \rho_{21}\Gamma_{21}) + (x_1 \leftrightarrow x_2). \tag{20}$$

Here, Γ_t (Γ_W) are the total widths of the t (W), $b(x)$ is the b parton distribution function in the proton, E_ℓ^{lab} is the energy of the decay lepton in the lab frame, and p_t^{cm} is the magnitude of the t three-momentum in the parton cm frame. It is understood that as before the z axis is chosen along the direction of the parton momentum which has the larger momentum fraction of the proton, and the two possibilities for choice of z axis are summed over.

Table 2 The asymmetry $A(O)_1$ in the observable O for unit Im f_{2R} and the limits Im f_{2R}^{lim} on Im f_{2R} for various cm energies and integrated luminosities

$\sqrt{s}(TeV)$	$L(\text{fb}^{-1})$	σ_{SM} (pb)	$A(O)_1$	Im f_{2R}^{lim}
13	190	2.53	0.326	4.42×10^{-3}
13.6	500	2.81	0.329	2.56×10^{-3}
14	3000	3.01	0.332	1.00×10^{-3}

Table 3 The SM cross section for t production and decay into the leptonic channel, the expectation value of O for unit coupling, the expectation value of O^2 in the SM, and the limits Im f_{2R}^{lim} on Im f_{2R} for various cm energies and respective integrated luminosities

$\sqrt{s}(TeV)$	L (fb $^{-1}$)	$\sigma_{\text{SM}}(pb)$	$\langle O \rangle_1$	$\langle O^2 \rangle_{\text{SM}}$	Im f_{2R}^{lim}
13	190	2.53	3.06×10^{-3}	8.51×10^{-2}	0.137
13.6	500	2.81	2.92×10^{-3}	8.42×10^{-3}	8.38×10^{-3}
14	3000	3.01	2.83×10^{-3}	8.37×10^{-3}	3.40×10^{-3}

The asymmetry $A(O)$ is evaluated by using the cross section of Eq. (20) in Eq. (17). As in the case of the transverse polarization asymmetry, the sensitivity of the event asymmetry of O may be measured by the 1-σ limit on the coupling given by the same Eq. (15), where now for A_1 we use $A(O)_1$, the value of $A(O)$ for Im$f_{2R} = 1$. In Table 2 we give the values of $A(O)_1$ and the limits Im f_{2R}^{lim} on Im f_{2R} for various cm energies and respective integrated luminosities.

The expectation value of O, for the case when the gluon carries a momentum fraction x_1 of the proton and the b quark carries a momentum fraction x_2, can be calculated using the expression in Eq. (20). As in the case of the transverse polarization, it turns out the observable $\langle O \rangle$ is zero in the case of the the anomalous couplings f_{1R} and f_{2R}.

For the observable O to be measurable at the 1-σ level with an integrated luminosity L, its expectation value should satisfy

$$|\langle O \rangle - \langle O \rangle_{\text{SM}}| > \frac{\sqrt{\langle O^2 \rangle_{\text{SM}} - \langle O \rangle_{\text{SM}}^2}}{\sqrt{\sigma_{\text{SM}} L}}. \tag{21}$$

For our case, $\langle O \rangle_{\text{SM}} = 0$. The 1-$\sigma$ limit on the coupling Im f_{2R} is then given by

$$\text{Im } f_{2R}^{\text{lim}} = \frac{\sqrt{\langle O^2 \rangle_{\text{SM}}}}{(\langle O \rangle_{\text{Im} f_{2R}=1}) \sqrt{\sigma_{\text{SM}} L}}. \tag{22}$$

The expectation values and the limits on Im f_{2R} for various cm energies and integrated luminosities are given in Table 3.

The limits in Table 2 obtainable from the observable O are comparable to those in Table 1 which would result from the transverse polarization, The limits from $\langle O \rangle$ are somewhat worse. However, it would not be fair to make comparisons as the systematics are different for the different measurements and a more realistic study of the measurements of various momenta is needed. A more detailed study of the kinematics and relevant cuts for particle identification would be required to get more accurate sensitivities. However, our numbers are expected to give a reasonable indication of the true values.

Conclusions and discussion

We have considered three typical T-odd observables which might be measured in the process $pp \to tW^- X$ in an extension of the SM with anomalous tbW interactions. Use has been made of analytical expressions for the spin density matrix for the production and, when needed, the decay of the top quark to calculate the cross section and the T-odd observables. These T-odd observables are dependent on the imaginary part of the off-diagonal elements of the density matrix, and hence, the imaginary part of the anomalous couplings. Of the three anomalous couplings f_{1R}, f_{2L} and f_{2R}, only the imaginary part of f_{2R} gives rise to nonzero values for these three observables. Thus, a measurement of the T-odd observables we suggest here would serve to isolate the coupling Im f_{2R} from amongst the three anomalous couplings available.

The 1-σ limits on Im f_{2R} we derive are of the order of 10^{-2}, with for the T-odd asymmetries, and a little larger for the expectation value of O. The limit improves for higher cm energy for the same integrated luminosity. Our calculation is restricted to a single channel for the top decay. Extending it to the other two leptonic channels would improve the sensitivity.

It would be worthwhile to refine the sensitivities for the measurement of Im f_{2R} estimated here. The measurement of the detailed final state is a difficult job for the LHC detectors. A more reliable estimate would require a detailed study of the detector efficiencies and imposing of realistic kinematic cuts for identification of the particles like the t, which is not attempted here. It might be useful to resort to machine learning techniques which have been discussed for top tagging in recent times (Sahu and Ghosh 2024; Kasieczka et al. 2019). Such a study would be worthwhile to carry out.

One may construct other more complicated T-odd observables, which are products of the observable considered above with kinematic constructs which are T-even, so that the product is odd under T. It may be possible to optimize the sensitivity with the use of such a combination.

We do not consider hadronic top decays, which have a larger probability, since it would be difficult, if not impossible, to measure a triple product of momenta, which would require measurement of the charges of the quarks. However, given the advantage of an enhanced sample of events in such a case, it would be worthwhile to attempt using hadronic decays as well.

We have discussed naive T violation, not necessarily accompanied by CP violation. We could also combine conjugate production and decay channels to test and measure CP violation, if it exists. In principle, it would bring in more parameters, since the couplings $\overline{f}_{iL}$ or $\overline{f}_{iR}$ associated with the $\bar{t}W^+$ production process are independent of f_{iL} and f_{iR}. However, if some assumptions are made, as for example, those suggested in Sect. 2, the tW^- and $\bar{t}W^+$ production results could be combined to test CP. In case CP conservation is assumed, the assumptions would lead to increase in statistics for measurement of the absorptive part.

It would also be interesting to study actual realizations of the T-odd observables studied here in actual extensions of the SM, as for example two-Higgs doublet models.

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Declarations

Conflict of interest The author states that there is no conflict of interest.

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